# ESSENTIALS OF ARITHMETIC

## ADVANCED BOOK

WENTWORTH-SMITH

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Virgene
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WENTWORTH-SMITH MATHEMATICAL SERIES

# ESSENTIALS OF ARITHMETIC

## ADVANCED BOOK

BY
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AND
DAVID EUGENE SMITH

GINN AND COMPANY
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# PREFACE

This work furnishes the material for a thorough study of the great essentials of arithmetic in the advanced classes of the elementary school. In general, in the schools of this country the essential feature of the work of Grade VII is a thorough study of percentage and its applications, while Grade VIII considers arithmetic from the standpoint of the pupil's relation to the commercial, civic, and industrial world which he is about to enter. These two great features have been emphasized in the general plan and the contents of this book.

How to make the connection between school life and society in the later years of the elementary school is one of the most serious educational problems. Some writers in arithmetic attempt to solve this problem by substantially abandoning mathematics and devoting their attention largely to sociology — an interesting experiment but not arithmetic. Others, and the authors of this series among them, seek to preserve a worthy and perfectly graded sequence of arithmetical topics, making the connection by showing the application of each topic to some phase of the business life which the pupil is about to enter. In this series the further effort has been made to present only the great essentials of arithmetic, and to dwell constantly upon the application of these essentials to social and business life. Unreal fractions, such obsolete methods as compound proportion, and such antiquated customs as partnership involving time are all set aside; and the space thus gained is assigned to vital topics of this generation and this country. Certain other topics, such as the metric system, land measure, and foreign money, which may properly have a place in the work of the school, but which sometimes have to be omitted, are given in the supplement, to be used or not, as the teacher desires.

There can be no question that the time has passed for mere theory in elementary arithmetic. The good that is in the subject can all be secured from problems representing actual conditions of to-day, together with that drill which is essential to the fixing of habits of accurate and reasonably rapid computation. Therefore the authors have sought to establish true motives for study, to place the pupil face to face with as real issues as possible, to furnish an abundance of drill work, to suggest to the teacher the best plan of carrying out the spirit of the official courses in harmony with this idea.

Particular attention is called to two features — the Little Examinations, which furnish a review of each chapter, and the Review and Drill, which affords a cumulative review of the essentials covered in all the preceding work. In the Review and Drill will also be found numerous industrial applications and series of problems without numbers. The industrial applications relate to the great industries of the country, including prominently the greatest industry of all, the very important domain of agriculture.

The authors hope that teachers who recognize the strong features of the topical arrangement, who wish for an abundance of well-graded problems requiring thought in their solution, who are sympathetic with the movement to replace what is obsolete by the real essentials and the genuine applications of arithmetic to the American life of to-day, who are opposed to certain of the extreme attempts of the present time which are sure to result disastrously to scholarship — that such teachers will find in this work a sane, modern, and helpful treatment of the subject.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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# ESSENTIALS OF ARITHMETIC

## ADVANCED BOOK

### CHAPTER I

#### REVIEW AND PROBLEM STUDY

**1. Language of Arithmetic.** Before beginning to study a new book on arithmetic we should briefly review the important operations, and should also make certain that we know the meaning of the words which we need to use. It is not necessary that we learn the definitions of most of these terms, but we should know how to use the words properly.

First, we should review the meaning of the words used in counting, reading, and writing numbers. A *unit* is any standard by which we count or measure, like \$1, 1 ft., or the number one. A unit may be a group, like one dozen eggs or one thousand (1 M) bricks. A *number* shows how many units or what part of a unit is taken. For example, \$3 means that \$1 is taken 3 times, and $\frac{4}{5}$ means that $\frac{1}{5}$ of 1 is taken four times, so that \$3 and $\frac{4}{5}$ are both numbers. If a number bears the name of some unit of measure, it is a *concrete number*, like 5 ft.; otherwise, as in the case of 5, 10.5, and $\frac{3}{4}$, it is an *abstract number*. A number applied to a whole unit is called a *whole number* or an *integer*, but if it is applied to part of a unit, it is called a *fraction*. Thus, 5 and 7 ft. are integers, and $\frac{2}{3}$ is a fraction.

**2. Writing Numbers.** The writing of numbers is often called *notation*, and the reading of numbers is called *numeration*, although it is better to use the simpler words "writing" and "reading." There are many ways of writing numbers; but the European and American countries commonly use a method which originated in India many centuries ago, and was introduced into Europe by the Arabs about the year 1000. This system of figures is called the *Arabic* or *Hindu-Arabic notation.* We often speak of these figures as *Arabic numerals.* Many years ago the Romans had a system of *Roman numerals*, still used, as we know, on most clocks and in numbering the chapters of books.

The great advantage of our common system, the Hindu-Arabic, is that it has a *place value*; that is, in the number 52 the 2 occupies units' place and means 2 units, while the 5 occupies tens' place and means 5 tens. But in the Roman system, VII, although like 52 made up of V (five) and II (two), means only seven, because the numerals have no such place value.

In our common system the numbers have different *orders*, like the units' order, the tens' order, and so on. We also speak of units of the first order (the ones, or units), of units of the second order (the tens), and so on. The orders increase by tens (ones, tens, hundreds, and so on), and so our system is called a *decimal system*, from the Latin word *decem*, which means ten.

For convenience in reading we usually separate large numbers into groups of three figures, these groups being called *periods.* Thus, 24,376,206 has 206 in the *units'* period, 376 in the *thousands'* period, and 24 in the *millions'* period. The periods are often indicated in print by spaces, thus, 24 376 206. If a number has only four figures, the comma is usually omitted, as in the date 1921, in \$4000, and in automobile numbers. It is correct to write two thousand either 2000 or 2,000.

**3. Reading Numbers.** In reading numbers we read each period by itself and add the name of the period except the name of units' period. Thus, we read 24,376,206 "twenty-four million, three hundred seventy-six thousand, two hundred six." If there is a whole number and a fraction, we use the word "and" between them. Thus, we read 200.004 "two hundred and four thousandths," and read 0.204 "two hundred four thousandths."

We rarely use number names above millions. The names of the periods above millions are billions, trillions, quadrillions, quintillions, sextillions, septillions, octillions, and so on; but these names need not be learned.

**4. Roman Numerals.** We have already studied Roman numerals, so that only a few examples need be given, as follows:

XXXIV = 34 LXXXI = 81 CCLIX = 259
XLIX = 49 XCVIII = 98 MDCCXC = 1790
MDCCCCXVIII or MCMXVIII = 1918

**WRITTEN EXERCISE**

*Write the following in Arabic figures:*

1. XIV. 2. LIX. 3. LXXX. 4. MDCC.

*Write the following in words:*

5. 2009. 6. 40,040. 7. 125,125. 8. 7,007,007.

*Write the following in figures, arranged in periods:*

9. One hundred fifty-two thousand, eighty and one half.
10. Two hundred six thousand, four hundred and three tenths.
11. Four hundred forty thousand, one hundred six.
12. Nine hundred ninety-nine thousand, five hundred eighty.
13. One million, three hundred one thousand, one hundred one.
14. Twenty-two million, one hundred fifty-five thousand, two.

**5. Addition.** You know how to add numbers and you also know the value of addition in practical problems. We shall now briefly review the operation.

It is not necessary at this time to give concrete illustrations of the value of the operations, since the student has often seen this value demonstrated.

| | | |
|---|---|---|
| 8595 | \$74.36 | 6.87 |
| 3609 | 9.08 | .93 |
| 12204 | \$83.44 | 7.80 |

**1.** Add 8595 and 3609.

Since $9 + 5 = 14 = 1$ ten $+ 4$, we write the 4 in the units' column and add the 1 to the tens.

Since 1 ten + 9 tens = 10 tens = 1 hundred + 0 tens, we write the 0 in the tens' column and add the 1 to the hundreds.

Since 1 hundred + 6 hundreds + 5 hundreds = 12 hundreds = 1 thousand + 2 hundreds, we write the 2 in the hundreds' column and add the 1 to the thousands.

Since 1 thousand + 3 thousands + 8 thousands = 12 thousands = 1 ten-thousand + 2 thousands, we write the 2 in the thousands' column and the 1 in the ten-thousands' column.

The sum is 12,204.

The result should be checked by adding in the opposite direction.

**2.** Add \$74.36 and \$9.08.

Since we must add cents to cents, dimes to dimes, and so on, we write the numbers so that one decimal point is exactly under the other.

The addition is now performed substantially as in Ex. 1.

The sum is \$83.44.

**3.** Add 6.87 and 0.93.

Since the position of the numbers, when written in a column, indicates that we have 93 hundredths, there is no danger of error if we omit the 0 before .93. Adding in the usual way, the sum is 7.80, or 7.8.

### WRITTEN EXERCISE

*Add the following, timing yourself:*

| 1. | 2. | 3. | 4. | 5. | 6. |
|---|---|---|---|---|---|
| $4.65 | $1.35 | $3.55 | $2.95 | $8.75 | $1.25 |
| 3.25 | 2.30 | 2.90 | 4.50 | 2.90 | 6.75 |
| 4.70 | 3.60 | 4.60 | 1.20 | 3.40 | 4.20 |
| .65 | 1.25 | 5.80 | 3.75 | 2.30 | 1.10 |
| 1.85 | .75 | .75 | 1.10 | .68 | .35 |
| .75 | .30 | 3.25 | 2.00 | 9.35 | .48 |
| 3.20 | 1.90 | 4.50 | 3.65 | .75 | 6.30 |
| .48 | 1.50 | 8.75 | 5.75 | .92 | 1.30 |

The work given above is just what every one meets in adding grocery bills and in keeping accounts. Facility in this kind of addition is essential. The record of the time should be kept in all such cases, and the effort should be made to lower the record when the page is reviewed.

*Add the following, writing only the answers, and time yourself:*

| 7. | 8. | 9. | 10. | 11. |
|---|---|---|---|---|
| $7283.48 | 487.346 | 24,793 | 518,234 | 38,429.6 |
| 198.76 | 298.408 | 41,278 | 298,623 | 41,968.4 |
| 3293.49 | 377.009 | 65,092 | 436,588 | 82,396.8 |
| 4100.76 | 68.779 | 41,780 | 529,688 | 91,823.7 |
| 328.49 | 418.230 | 88,987 | 41,998 | 55,005.7 |
| 2163.41 | 42.981 | 32,600 | 55,775 | 31,028. |

*Add the following:*

12. $736 + 99$.
13. $802 + 77$.
14. $674 + 93$.
15. $808 + 65$.
16. $573 + 299$.
17. $29.7 + 3.67$.
18. $4.28 + 49.6$.
19. $7.63 + 9.98$.
20. $527 + 3.69$.
21. $749 + 56.8$.
22. $3486 + 9349$.
23. $7281 + 8799$.
24. $69.48 + 98.37$.
25. $487.6 + 6.937$.
26. $58.27 + 9.863$.

**6. Subtraction.** You know how to subtract one number from another and you also know the value of subtraction in practical problems. We shall now briefly review the operation.

| | | |
|---|---|---|
| 723 | \$9.00 | 4.70 |
| 487 | 2.73 | .68 |
| 236 | \$6.27 | 4.02 |

**1.** Subtract 487 from 723.

Since we cannot take 7 from 3, we change one of the 2 tens in 723 to units. We then have 13 units − 7 units = 6 units.

Since we cannot take 8 tens from the 1 ten remaining in 723, we change one of the hundreds in 723 to tens. We then have 11 tens − 8 tens = 3 tens.

Since we have used 1 hundred of the 7 hundreds, we have 6 hundreds − 4 hundreds = 2 hundreds.

Therefore the difference is 236.

**2.** Subtract \$2.73 from \$9.

Since we must subtract cents from cents, dimes from dimes, and so on, we write the numbers with the decimal points under each other.

We then write \$9.00 for \$9, and subtract substantially as in Ex. 1.

**3.** Subtract 0.68 from 4.7.

When the numbers are written in a column there is no chance of overlooking the decimal point if we omit the 0 before .68. Writing 4.70 for 4.7, we subtract substantially as in Ex. 1.

Teachers should recognize the fact that the method followed above is only one of several methods of subtraction in common use. That method has been selected which is most often taught in this country.

**7. Special Case.** In a case like 8000 − 2907 we may shorten the work by simply thinking of the 8000 as 7990 + 10.

| | | | |
|---|---|---|---|
| 7 | 9 | 9 | 10 |
| 2 | 9 | 0 | 7 |
| 5 | 0 | 9 | 3 |

Beginning at the left we have 7 − 2 = 5, 9 − 9 = 0, 9 − 0 = 9, 10 − 7 = 3. The remainder is 5093.

### WRITTEN EXERCISE

*Subtract, checking your work and timing yourself:*

1. 274 – 62.
2. 378 – 69.
3. 465 – 57.
4. 582 – 44.
5. 673 – 65.
6. 766 – 49.
7. 857 – 28.
8. 939 – 47.
9. 354 – 62.
10. 268 – 85.
11. 325 – 63.
12. 436 – 94.
13. 742 – 70.
14. 819 – 37.
15. 352 – 63.
16. 603 – 25.
17. 760 – 75.
18. 504 – 68.
19. 320 – 33.
20. 404 – 27.
21. 570 – 92.
22. 700 – 30.
23. 600 – 43.
24. 300 – 56.
25. 400 – 90.
26. 500 – 99.
27. 63.8 – 12.6.
28. 4.27 – 1.27.
29. 6.36 – 3.18.
30. 34.2 – 22.5.
31. 59.6 – 37.9.
32. 7.58 – 4.49.
33. 34.2 – 20.7.
34. 46.7 – 20.8.
35. 6.21 – 4.16.
36. 73.3 – 32.9.
37. 5.76 – 4.44.
38. 3.92 – 2.88.
39. 340 – 225.
40. 62.8 – 41.9.
41. 4.92 – 2.84.
42. 46.2 – 37.3.
43. 7.07 – 5.28.
44. 9.81 – 6.94.
45. 60.6 – 42.8.
46. 70.2 – 51.3.
47. 60.3 – 42.7.
48. 4.02 – 2.36.
49. 520 – 348.
50. 630 – 469.
51. 800 – 532.
52. 900 – 687.
53. 2782 – 120.
54. 3648 – 439.
55. 5276 – 168.
56. 8496 – 229.
57. 2349 – 256.
58. 4872 – 380.
59. 6463 – 291.
60. 4486 – 395.
61. 7286 – 193.
62. 2872 – 385.
63. 5342 – 264.
64. 3353 – 296.
65. 4374 – 287.
66. 6681 – 599.
67. 6720 – 535.
68. 7872 – 5785.
69. 3526 – 2239.
70. 7280 – 2385.
71. 6807 – 4938.
72. 38,265 – 23,406.
73. 34,635 – 25,296.
74. 48,406 – 26,573.
75. 62,068 – 48,679.

**8. Multiplication.** You know how to multiply one number by another and you also know the value of multiplication in practical problems. We shall now briefly review the operation.

| | | |
|---|---|---|
| 4674 | \$45.87 | 3.24 |
| 49 | 2007 | 0.21 |
| 42066 | 321 09 | 324 |
| 18696 | 9174 | 648 |
| 229026 | \$92061.09 | 0.6804 |

**1.** Multiply 4674 by 49.

Here the multiplier is 40 + 9, and so we multiply first by 9 and then by 40 and add the products.

It makes no particular difference whether we multiply first by 4 tens or by 9 units. Many good computers multiply first by the tens, but beginners usually make fewer mistakes by multiplying first by the units, and this is the common method in the schools.

**2.** Multiply \$45.87 by 2007.

The products corresponding to the zeros in the multiplier are zero, and therefore they need not be written. With this exception the multiplication is performed as in the preceding example. The decimal point is placed before the cents as in the multiplicand.

**3.** Multiply 3.24 by 0.21.

We multiply in the ordinary way, and notice that 1 (hundredth) × 4 (hundredths) = 4 (ten-thousandths), and that the 4 of the product must therefore go in the ten-thousandths' place.

From Ex. 3 we see that the following is a useful rule for multiplying decimal fractions:

*To multiply by a decimal, multiply as if by an integer and point off from the right as many decimal places in the product as there are decimal places in both factors together, prefixing zeros if necessary.*

### WRITTEN EXERCISE

*Multiply the following:*

1. $401 \times 3664$.
2. $503 \times 2763$.
3. $630 \times 4876$.
4. $580 \times 6924$.
5. $847 \times 6476$.
6. $972 \times 3502$.
7. $707 \times 8763$.
8. $5002 \times 7983$.
9. $283 \times \$48.25$.
10. $342 \times \$29.75$.
11. $276 \times \$49.42$.
12. $428 \times \$75.19$.
13. $0.5 \times 27.5$.
14. $0.8 \times 146.7$.
15. $2.3 \times 224.1$.
16. $3.2 \times 315.7$.
17. $0.35 \times 488$.
18. $3.5 \times 488$.
19. $35 \times 488$.
20. $0.35 \times 4.88$.
21. $0.85 \times 644$.
22. $8.5 \times 644$.
23. $85 \times 644$.
24. $0.85 \times 6.44$.
25. $0.85 \times 2436$.
26. $3574 \times 0.05$.
27. $0.25 \times 536$.
28. $0.25 \times 5.36$.
29. $0.25 \times 0.536$.
30. $704 \times \$0.25$.
31. $172 \times \$0.25$.
32. $776 \times \$0.12\frac{1}{2}$.
33. $0.175 \times 424$.
34. $1.75 \times 424$.
35. $175 \times 424$.
36. $175 \times 4.24$.
37. $0.425 \times 584$.
38. $4.25 \times 584$.
39. $4.25 \times 5.84$.
40. $0.425 \times 0.584$.
41. $592 \times \$0.275$.
42. $368 \times \$0.225$.
43. $336 \times \$4.33$.
44. $297 \times \$4.33$.
45. $372 \times \$2.65$.
46. $492 \times \$2.67$.
47. $558 \times \$5.16$.
48. $174 \times \$5.16$.
49. $225 \times \$3.12$.
50. $640 \times \$1.25$.
51. $0.4 \times 3.21$.
52. $3.17 \times 4.22$.
53. $7.39 \times 5.81$.
54. $9.75 \times 6.92$.
55. $0.475 \times 728$.
56. $0.475 \times 7.28$.
57. $4.75 \times 72.8$.
58. $4.78 \times 72.8$.
59. $475 \times 728$.
60. $2.66 \times 546$.
61. $26.6 \times 546$.
62. $5.66 \times 546$.
63. $2952 \times 0.07$.
64. $248 \times 0.07$.
65. $24.8 \times 0.7$.
66. $2.48 \times 0.7$.
67. $24 \times 6.5$.
68. $2.4 \times 6.5$.
69. $2.4 \times 0.65$.
70. $0.225 \times 0.16$.
71. $0.9 \times 2.735$.
72. $4.35 \times 5.006$.
73. $8.25 \times 9.901$.
74. $8.24 \times 0.0009$.
75. $5.75 \times 0.0707$.

**9. Aliquot Part.** A number that will exactly divide another is called an *aliquot part* of that number.

Thus 5, 10, 25, $12\frac{1}{2}$, $16\frac{2}{3}$, and $33\frac{1}{3}$ are aliquot parts of 100.

We may also have aliquot parts of numbers besides 100. For example, $2\frac{1}{2}$, $3\frac{1}{3}$, and 5 are important aliquot parts of 10; 125, $333\frac{1}{3}$, and 250 are important aliquot parts of 1000; and so on.

**10. Important Aliquot Parts.** The following fractional parts of \$1 should be memorized:

| | |
|---|---|
| $5¢ = \$\frac{1}{20} = \$0.05$ | $40¢ = \$\frac{2}{5} = \$0.40$ |
| $6\frac{1}{4}¢ = \$\frac{1}{16} = \$0.06\frac{1}{4}$ | $50¢ = \$\frac{1}{2} = \$0.50$ |
| $8\frac{1}{3}¢ = \$\frac{1}{12} = \$0.08\frac{1}{3}$ | $60¢ = \$\frac{3}{5} = \$0.60$ |
| $10¢ = \$\frac{1}{10} = \$0.10$ | $62\frac{1}{2}¢ = \$\frac{5}{8} = \$0.625$ |
| $12\frac{1}{2}¢ = \$\frac{1}{8} = \$0.125$ | $66\frac{2}{3}¢ = \$\frac{2}{3} = \$0.66\frac{2}{3}$ |
| $16\frac{2}{3}¢ = \$\frac{1}{6} = \$0.16\frac{2}{3}$ | $70¢ = \$\frac{7}{10} = \$0.70$ |
| $20¢ = \$\frac{1}{5} = \$0.20$ | $75¢ = \$\frac{3}{4} = \$0.75$ |
| $25¢ = \$\frac{1}{4} = \$0.25$ | $80¢ = \$\frac{4}{5} = \$0.80$ |
| $30¢ = \$\frac{3}{10} = \$0.30$ | $83\frac{1}{3}¢ = \$\frac{5}{6} = \$0.83\frac{1}{3}$ |
| $33\frac{1}{3}¢ = \$\frac{1}{3} = \$0.33\frac{1}{3}$ | $87\frac{1}{2}¢ = \$\frac{7}{8} = \$0.875$ |
| $37\frac{1}{2}¢ = \$\frac{3}{8} = \$0.375$ | $90¢ = \$\frac{9}{10} = \$0.90$ |

The most important aliquot parts of 100, including those that should be memorized, are as follows:

| | | |
|---|---|---|
| $50 = \frac{1}{2}$ of 100 | $33\frac{1}{3} = \frac{1}{3}$ of 100 | $20 = \frac{1}{5}$ of 100 |
| $25 = \frac{1}{4}$ of 100 | $16\frac{2}{3} = \frac{1}{6}$ of 100 | $10 = \frac{1}{10}$ of 100 |
| $12\frac{1}{2} = \frac{1}{8}$ of 100 | $8\frac{1}{3} = \frac{1}{12}$ of 100 | $5 = \frac{1}{20}$ of 100 |
| $6\frac{1}{4} = \frac{1}{16}$ of 100 | $4\frac{1}{6} = \frac{1}{24}$ of 100 | $2 = \frac{1}{50}$ of 100 |

Other important fractional parts of 100, not aliquot parts, are as follows:

| | | |
|---|---|---|
| $75 = \frac{3}{4}$ of 100 | $62\frac{1}{2} = \frac{5}{8}$ of 100 | $66\frac{2}{3} = \frac{2}{3}$ of 100 |
| $37\frac{1}{2} = \frac{3}{8}$ of 100 | $87\frac{1}{2} = \frac{7}{8}$ of 100 | $83\frac{1}{3} = \frac{5}{6}$ of 100 |

As in the case of aliquot parts, these should be memorized.

**11. Short Methods in Multiplication.** We shall now review those short methods of multiplication which are particularly valuable because of certain multipliers which are so often used.

*To multiply by 10, move the decimal point one place to the right, annexing a zero if necessary.*

*To multiply by 100 or 1000, move the decimal point to the right two or three places respectively, annexing zeros if necessary.*

*To multiply by 5, multiply by 10 and divide by 2.*

*To multiply by 25, multiply by 100 and divide by 4.*

*To multiply by 125, multiply by 1000 and divide by 8.*

*To multiply by $33\frac{1}{3}$, multiply by 100 and divide by 3.*

*To multiply by 9, multiply by 10 and subtract the multiplicand.*

*To multiply by 11, multiply by 10 and add the multiplicand.*

*To find 50% of a number, divide the number by 2.*

*To find 25% of a number, divide the number by 4.*

**WRITTEN EXERCISE**

*Multiply, in turn, by 10, by 100, by 5, by 25, and by 125:*

| | | | | | |
|---|---|---|---|---|---|
| 1. | 5656 | 8448 | 17,184 | 15,048 | 14,120 |
| 2. | 7368 | 8.776 | 11,840.8 | 17.008 | 50.744 |
| 3. | 4976 | 16.424 | 48,245.6 | 15.032 | 30.832 |

*Multiply, in turn, by $33\frac{1}{3}$, by 9, and by 11:*

| | | | | | |
|---|---|---|---|---|---|
| 4. | 16,974 | 64,050 | 12,210 | 14,601 | 10,803 |
| 5. | 141.51 | 136.71 | 1600.2 | 154.05 | 99.819 |
| 6. | 180.36 | 160.26 | 1594.2 | 17.163 | 99.603 |

*Find, in turn, 50% and 25% of the following:*

| | | | | | |
|---|---|---|---|---|---|
| 7. | 11,380 | 16,816 | 83,864 | 396.56 | 77.768 |
| 8. | 46,588 | 43,748 | 97,144 | 567.68 | 47.376 |

### WRITTEN EXERCISE

*Multiply the following:*

1. $10 \times \$0.25$.
2. $10 \times \$8251$.
3. $10 \times \$6.25$.
4. $10 \times \$5.50$.
5. $100 \times \$0.37$.
6. $100 \times \$2.87\frac{1}{2}$.
7. $1000 \times \$0.25$.
8. $1000 \times \$6.37\frac{1}{2}$.
9. $50 \times \$64{,}224$.
10. $50 \times \$752.20$.
11. $50 \times \$861.55$.
12. $25 \times \$144$.
13. $25 \times \$640$.
14. $25 \times \$325$.
15. $675 \times \$25$.
16. $25 \times \$25.50$.
17. $25 \times \$75.25$.
18. $12\frac{1}{2} \times \$6400$.
19. $12\frac{1}{2} \times \$4880$.
20. $125 \times \$640$.
21. $125 \times \$6400$.
22. $125 \times \$2800$.
23. $33\frac{1}{3} \times 36$.
24. $33\frac{1}{3} \times 48$.
25. $33\frac{1}{3} \times 144$.
26. $33\frac{1}{3} \times \$312.30$.
27. $16\frac{2}{3} \times 12{,}630$.
28. $16\frac{2}{3} \times \$636.30$.
29. $5 \times 47{,}664.80$.
30. $5 \times \$210{,}650$.
31. $125 \times \$84{,}480$.
32. $125 \times \$844.80$.
33. $16\frac{2}{3} \times 4{,}800{,}000$.
34. How much will 25 books cost at \$1.20 each?
35. How much will 25 wagons cost at \$116 each?
36. How much will 50 cows cost at \$54 a head?
37. How much will 100 books cost at $62\frac{1}{2}¢$ each?
38. How much will 25 tons of hay cost at \$12.40 a ton?
39. How much will $12\frac{1}{2}$ yards of cloth cost at $36¢$ a yard?
40. How much will 25 doz. pencils cost at \$0.48 a dozen?
41. How much will 48 office desks cost at \$25 each?
42. At \$5 an acre, how much is a tract of 14,264 acres of mountain land worth?
43. How much will a man earn in 36 weeks at \$25 a week? at $\$33\frac{1}{3}$ a week?
44. How much will a dozen tables weigh at 25 lb. each? at 50 lb. each?

**12. Division.** You know how to divide one number by another and you also know the value of division in practical problems. We shall now briefly review the operation.

**1.** Divide \$21.15 by 9.

We see that $21 \div 9 = 2$, with remainder 3, and now have 3.15 to be divided. We see that $3.1 \div 9 = 0.3$, with remainder 0.4, and now have 0.45 to be divided. We see that $0.45 \div 9 = 0.05$. Therefore the quotient is \$2.35.

$$9\overline{)\$21.15}\quad \$2.35$$

*Check.* $9 \times \$2.35 = \$21.15$.

The above method, known as *short division*, should be used for all divisors less than 12, and may easily be used for many divisors greater than 12.

| | | |
|---|---|---|
| 172 | $17\frac{19}{37}$ | \$3.12 |
| $42\overline{)7224}$ | $37\overline{)648}$ | $18\overline{)\$56.16}$ |
| 42 | 37 | 54 |
| 302 | 278 | 21 |
| 294 | 259 | 18 |
| 84 | 19 | 36 |
| 84 | | 36 |

**2.** Divide 7224 by 42.

Since 72 hundreds $\div$ 42 = 1 hundred, with remainder 30 hundreds, we write the 1 in the hundreds' place in the quotient. We now have 3024 to be divided. Since 302 tens $\div$ 42 = 7 tens, with remainder 8 tens, we write the 7 in the tens' place in the quotient. We now have 84 to be divided, and we see that $84 \div 42 = 2$. Therefore the quotient is 172.

**3.** Divide 648 by 37.

Proceeding as before, we have 17 for the quotient, with remainder 19. Since the 19 is still to be divided by 37, we write the full quotient $17\frac{19}{37}$.

**4.** Divide \$56.16 by 18.

The process is sufficiently clear from the above work.

### WRITTEN EXERCISE

*Find the quotient, and the remainder if any:*

1. $50{,}568 \div 602$.
2. $13{,}020 \div 620$.
3. $19{,}251 \div 621$.
4. $76{,}109 \div 629$.
5. $33{,}750 \div 125$.
6. $51{,}736 \div 116$.
7. $26{,}288 \div 106$.
8. $56{,}826 \div 231$.
9. $61{,}202 \div 202$.
10. $38{,}668 \div 151$.
11. $153{,}186 \div 422$.
12. $237{,}975 \div 501$.
13. $972{,}360 \div 222$.
14. $322{,}926 \div 321$.
15. $324{,}360 \div 153$.
16. $140{,}850 \div 450$.
17. $183{,}750 \div 350$.
18. $297{,}110 \div 407$.
19. $469{,}802 \div 503$.
20. $208{,}575 \div 103$.
21. $435{,}336 \div 102$.
22. $\$34.51 \div 17$.
23. $\$47.25 \div 35$.
24. $\$208.65 \div 65$.
25. $82{,}127 \div 75$.
26. $59{,}333 \div 78$.
27. $63{,}481 \div 55$.
28. $82{,}603 \div 82$.
29. $86{,}509 \div 98$.
30. $43{,}782 \div 263$.
31. $48{,}343 \div 821$.
32. $62{,}196 \div 333$.
33. $48{,}277 \div 608$.
34. $128{,}321 \div 129$.
35. $236{,}482 \div 421$.
36. $527{,}309 \div 562$.
37. $603{,}409 \div 488$.
38. $500{,}682 \div 328$.
39. $400{,}028 \div 475$.
40. $600{,}000 \div 325$.
41. $500{,}000 \div 324$.
42. $700{,}000 \div 987$.

Teachers should use only so much of a page of this kind as is necessary to accomplish the purpose. If the pupils can divide accurately and with fair rapidity they should pass on to the next topic. The large number of examples allows the teacher to select different ones from time to time. The textbook should be a servant, not a master.

**13. Division by a Decimal.** Since we may multiply both divisor and dividend by the same number without affecting the quotient, we may, if the divisor contains decimal places, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer. That is,

*In the division of decimals, multiply both divisor and dividend by such a power of 10 as shall make the divisor an integer, and divide as by an integer, placing the decimal point in the quotient over the decimal point in the dividend.*

$$\begin{array}{r} 14.4 \\ 23\overline{)331.2} \\ 23 \\ \hline 101 \\ 92 \\ \hline 92 \\ 92 \\ \hline \end{array}$$

Thus, in dividing 33.12 by 2.3, we multiply both divisor and dividend by 10, and then have $331.2 \div 23$. The divisor is now an integer, and we proceed as shown, finding the quotient to be 14.4.

*Check.* $2.3 \times 14.4 = 33.12$.

**14. Second Method.** If we prefer, we may leave the decimal points as they are, and determine the position of the decimal point in the quotient as follows:

*In the division of decimals, draw a vertical line as many places to the right of the decimal point in the dividend as there are decimal places in the divisor. Then divide as by an integer, placing the decimal point in the quotient above this vertical line.*

$$\begin{array}{r} 14.4 \\ 2.3\overline{)33.1|2} \\ 23 \\ \hline 101 \\ 92 \\ \hline 92 \\ 92 \\ \hline \end{array}$$

Thus, in dividing 33.12 by 2.3, we place a vertical line one place to the right of the decimal point in the dividend. Proceeding as in ordinary division, we find that the quotient is 14.4.

Both of these plans (§§ 13, 14) are in common use. The school should adopt one of them and adhere to it in teaching, but should not require a pupil to change from the plan that he already knows.

### WRITTEN EXERCISE

*Divide the following:*

1. $74.4 \div 24$.
2. $74.4 \div 2.4$.
3. $7.44 \div 2.4$.
4. $62.9 \div 37$.
5. $6.29 \div 37$.
6. $62.9 \div 3.7$.
7. $6.29 \div 3.7$.
8. $80.5 \div 35$.
9. $80.5 \div 3.5$.
10. $8.05 \div 3.5$.
11. $46.2 \div 22$.
12. $46.2 \div 2.2$.
13. $4.62 \div 2.2$.
14. $600 \div 0.6$.
15. $600 \div 0.06$.
16. $6 \div 0.0006$.
17. $627 \div 0.3$.
18. $6.27 \div 0.3$.
19. $62.7 \div 0.03$.
20. $305 \div 6.1$.
21. $3.05 \div 6.1$.
22. $62.32 \div 8.2$.
23. $623.2 \div 8.2$.
24. $6.232 \div 8.2$.
25. $1156 \div 3.4$.
26. $1.156 \div 3.4$.
27. $7.14 \div 4.2$.
28. $71.4 \div 4.2$.
29. $7.14 \div 0.42$.
30. $62.32 \div 7.6$.
31. $6.232 \div 0.76$.
32. $8190 \div 0.9$.
33. $61.20 \div 1.2$.
34. $7.200 \div 1.2$.
35. $78.4 \div 0.08$.
36. $3.12 \div 0.04$.
37. $7.81 \div 1.1$.
38. $8.91 \div 0.11$.
39. $4.55 \div 0.07$.
40. $2.73 \div 0.03$.
41. $8.96 \div 12.8$.
42. $4.41 \div 0.21$.
43. $17.28 \div 14.4$.
44. $7.44 \div 0.24$.
45. $7.44 \div 24$.
46. $2.002 \div 0.22$.
47. $3.003 \div 7.7$.
48. $2.65 \div 0.53$.
49. $60.06 \div 0.33$.
50. $72.5 \div 0.22$.
51. $7.368 \div 2.4$.
52. $0.279 \div 0.09$.
53. $12.312 \div 24$.
54. $1231.2 \div 2.4$.
55. $1.2312 \div 2.4$.
56. $62.75 \div 123$.
57. $6.275 \div 1.23$.
58. $36.08 \div 32.8$.
59. $3.608 \div 0.328$.
60. $1.036 \div 0.560$.
61. $882.32 \div 0.82$.
62. $4.4116 \div 0.41$.
63. $11.300 \div 5.65$.
64. $6.51021 \div 3.207$.
65. $651.021 \div 320.7$.
66. $176.4 \div 0.0072$.
67. $5.44 \div 0.0068$.
68. $717.75 \div 4.125$.
69. $4.41 \div 0.002$.
70. $4.752 \div 2.2$.
71. $4.752 \div 0.022$.
72. $32.4714 \div 4.6$.
73. $883.88 \div 0.19$.
74. $32.4714 \div 2.3$.
75. $324.714 \div 9.2$.

**15. Short Methods in Division.** *To divide by 10, move the decimal point one place to the left, prefixing a zero if necessary.*

*To divide by 100 or 1000, move the decimal point to the left, two or three places respectively, prefixing zeros if necessary.*

*To divide by 5, multiply by 2 and divide by 10.*

*To divide by 25, multiply by 4 and divide by 100.*

*To divide by 125, multiply by 8 and divide by 1000.*

*To divide by $12\frac{1}{2}$, multiply by 8 and divide by 100.*

*To divide by $33\frac{1}{3}$, multiply by 3 and divide by 100.*

*To divide by $66\frac{2}{3}$, multiply by 3 and divide by 200.*

*To divide by $16\frac{2}{3}$, multiply by 6 and divide by 100.*

### WRITTEN EXERCISE

*Divide by short methods, writing only the answers:*

| | | |
|---|---|---|
| **1.** $7945 \div 5$. | **6.** $9486 \div 66\frac{2}{3}$. | **11.** $3978 \div 16\frac{2}{3}$. |
| **2.** $3875 \div 25$. | **7.** $3739 \div 16\frac{2}{3}$. | **12.** $7168 \div 66\frac{2}{3}$. |
| **3.** $7925 \div 12\frac{1}{2}$. | **8.** $8347 \div 33\frac{1}{3}$. | **13.** $9375 \div 12\frac{1}{2}$. |
| **4.** $9676 \div 33\frac{1}{3}$. | **9.** $7969 \div 12\frac{1}{2}$. | **14.** $4873 \div 33\frac{1}{3}$. |
| **5.** $3375 \div 125$. | **10.** $59{,}625 \div 125$. | **15.** $67{,}500 \div 125$. |

**16.** Divide by 5: 125, \$375, \$67,295, \$575,265.

**17.** Divide by 25: 650, \$175, 98,750, \$482,000.

**18.** Divide by 125: \$250,125, \$375,375, \$276,525, \$452,625.

**19.** At 25¢ a yard, how many yards of cloth can be bought for \$16.25? for \$42.25? for \$63.75? for \$17.50?

**20.** At $33\frac{1}{3}$¢ a dozen, how many dozen eggs can be bought for \$17? for \$56? for \$92? for \$125?

**21.** Divide 452,346,125 by 125 and also by 25, using long division, and see how many seconds it takes. Do the same by the short method.

### WRITTEN EXERCISE

*Add the following, timing yourself:*

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $2.48 | $1.25 | $4.45 | $5.66 | $24.68 |
| 3.42 | .46 | 6.50 | 7.44 | 32.49 |
| 6.81 | 3.84 | 7.28 | 8.21 | 38.80 |
| 4.88 | 2.95 | .66 | 9.64 | 56.42 |
| 5.75 | 6.72 | 8.34 | 4.34 | 94.75 |
| 6.42 | 3.88 | 5.07 | 2.56 | 80.80 |
| 3.96 | 4.06 | 6. | 1.79 | 52.68 |
| 8.64 | 9.87 | 8.96 | .36 | 3.92 |
| 7.63 | 8.49 | 1.42 | 5.26 | 54.22 |

*Subtract the following, timing yourself:*

| 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|
| $9.20 | $7.60 | $6.12 | $5.23 | $62.81 |
| 4.36 | 5.85 | 4.63 | 4.70 | 29.93 |

*Multiply the following, timing yourself:*

**11.** $2.8 \times 5.9$.
**12.** $7.6 \times 9.7$.
**13.** $3.2 \times 0.9$.
**14.** $6.4 \times 12.9$.
**15.** $4.2 \times 3.47$
**16.** $68 \times 0.426$.
**17.** $0.48 \times 0.926$.
**18.** $2.37 \times 6.532$.
**19.** $8.07 \times 93.64$.

*Divide, carrying the results to three decimal places:*

**20.** $526.8 \div 31.4$.
**21.** $73.42 \div 62.8$.
**22.** $5.821 \div 43.9$.
**23.** $12.964 \div 3.41$.
**24.** $648.29 \div 52.8$.
**25.** $7234.6 \div 2.99$.

It is a good plan to review a page like this frequently, having the pupils keep a record of the time required to do all the work each time. The records should be compared as the pupils progress, so that they may see that the time is reduced as a result of such drills.

### ORAL EXERCISE

1. If you have 5 pt. of cream, how many quarts have you?

2. If you buy 3 qt. of milk, how many pints do you buy?

3. A man buys 1500 lb. of coal. What part of a ton is this?

4. If a horse eats 8 qt. of oats a day, how long will a bushel of oats last? How many bushels of oats will he eat in 4 wk.?

*Fill in the parentheses in the following:*

5. 2 lb. = ( ) oz.
6. 4 T. = ( ) lb.
7. 7 ft. = ( ) in.
8. 36 ft. = ( ) yd.
9. 3 gal. = ( ) qt. = ( ) pt.
10. 5 yd. = ( ) ft. = ( ) in.
11. 8 pt. = ( ) gal. = ( ) qt.
12. 2 yd. = ( ) in. = ( ) ft.

### WRITTEN EXERCISE

1. The distance between two places is 5808 ft. Express this in miles and the decimal of a mile.

2. A man buys 3200 lb. of coal. Express this in tons.

3. A man buys $\frac{3}{4}$ T. of coal. Express this in pounds.

4. How many feet are there in $\frac{5}{8}$ mi.? how many yards?

*Fill in the parentheses in the following:*

5. 11,088 ft. = ( ) mi.
6. 2.7 mi. = ( ) ft.
7. 1936 yd. = ( ) mi.
8. 1.3 mi. = ( ) yd.
9. 17 rd. = ( ) ft. = ( ) yd.
10. 17 yd. = ( ) ft. = ( ) rd.
11. $2\frac{1}{2}$ bu. = ( ) qt. = ( ) pk.
12. $17\frac{3}{4}$ bu. = ( ) pk. = ( ) qt.

*Add the following:*

13. 2 mi. 348 ft. + 5 mi. 4875 ft. + 1 mi. 250 ft. + 75 ft.
14. 17 ft. 9 in. + 16 ft. 7 in. + 37 ft. 6 in. + 48 ft. 4 in. + 8 ft.

### ORAL EXERCISE

*Recite the following tables:*

1. Length.
2. Square measure.
3. Cubic measure.
4. Weight.
5. Liquid measure.
6. Dry measure.

7. How many cubic feet are there in a cord?
8. A pint is what part of a quart? of a gallon?
9. A quart is what part of a gallon?
10. A quart is what part of a peck? of a bushel?
11. How many quarts in 3 pt.? in $2\frac{1}{2}$ pk.? in 15 pk.?
12. How much is 50% of \$24? of \$36? of \$64? of \$400?

### WRITTEN EXERCISE

1. Some boys are laying out a quarter-mile track. How many feet long is it?

2. Suppose the boys mentioned in Ex. 1 have only a yard-stick with which to measure, how many yards must they allow?

3. Land surveyors often use a chain which is 66 ft. long. How many chains must the boys mentioned in Ex. 1 allow?

4. There are 10 square chains in an acre, the chain being the length mentioned in Ex. 3. If a man has a rectangular garden 90 ft. by 121 ft., what part of an acre does it contain?

5. Find what part of an acre is contained in a rectangular piece of land 270 ft. long and 121 ft. wide.

6. If a 10-acre field is 400 ft. wide, how long is it?

The pupils should use the information given in Ex. 4.

7. A pile of 4-foot wood is 6 ft. high and 80 ft. long. How many cords does it contain?

### WRITTEN EXERCISE

*Find the sums of the following:*

1. $\frac{1}{2}, \frac{3}{8}$.
2. $\frac{1}{4}, \frac{3}{16}$.
3. $\frac{1}{4}, \frac{5}{16}$.
4. $\frac{1}{4}, \frac{9}{16}$.
5. $\frac{3}{4}, \frac{13}{16}$.
6. $\frac{1}{8}, \frac{3}{16}$.
7. $\frac{3}{8}, \frac{7}{16}$.
8. $\frac{3}{8}, \frac{15}{16}$.
9. $\frac{5}{8}, \frac{15}{16}$.
10. $\frac{7}{8}, \frac{13}{16}$.
11. $\frac{1}{2}, \frac{7}{32}$.
12. $\frac{1}{2}, \frac{15}{32}$.
13. $\frac{1}{4}, \frac{25}{32}$.
14. $\frac{3}{4}, \frac{31}{32}$.
15. $\frac{3}{8}, \frac{5}{32}$.
16. $\frac{3}{4}, \frac{7}{24}$.
17. $\frac{3}{4}, \frac{17}{24}$.
18. $\frac{5}{8}, \frac{5}{24}$.
19. $\frac{7}{8}, \frac{23}{24}$.
20. $\frac{5}{16}, \frac{1}{64}$.
21. To the sum of $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{8}$, add the sum of $\frac{3}{5}$, $\frac{7}{15}$, and $\frac{2}{3}$.
22. To the sum of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{3}{16}$, add the sum of $\frac{1}{2}$, $\frac{7}{8}$, and $\frac{9}{16}$.

*Find the sums of the following:*

23. $8\frac{3}{4}, 5\frac{1}{3}$.
24. $6\frac{3}{4}, 8\frac{2}{3}$.
25. $2\frac{1}{3}, 7\frac{1}{8}$.
26. $6\frac{1}{3}, 5\frac{3}{8}$.
27. $8\frac{2}{3}, 9\frac{5}{8}$.
28. $7\frac{2}{3}, 8\frac{7}{8}$.
29. $3\frac{3}{4}, 5\frac{5}{12}$.
30. $4\frac{3}{8}, 6\frac{5}{16}$.
31. $62\frac{5}{6}, 29\frac{3}{5}$.
32. $75\frac{7}{8}, 62\frac{15}{16}$.
33. $92\frac{5}{6}, 75\frac{4}{15}$.
34. $73\frac{2}{3}, 68\frac{7}{15}$.
35. $52\frac{4}{5}, 75\frac{13}{15}$.
36. $48\frac{9}{10}, 73\frac{11}{12}$.
37. $52\frac{1}{5}, 61\frac{7}{20}$.
38. $37\frac{4}{15}, 32\frac{11}{20}$.
39. $5\frac{4}{5}, 6\frac{2}{3}, 4\frac{2}{15}$.
40. $6\frac{7}{8}, 3\frac{1}{2}, 5\frac{15}{16}$.
41. $4\frac{1}{3}, 3\frac{3}{5}, 5\frac{1}{30}$.
42. $2\frac{7}{8}, 5\frac{13}{16}, 6\frac{5}{32}$.
43. $3\frac{5}{6}, 2\frac{5}{12}, 6\frac{7}{24}$.
44. $4\frac{7}{8}, 3\frac{2}{3}, 5\frac{11}{24}$.
45. $3\frac{1}{16}, 5\frac{3}{32}, 7\frac{11}{64}$.
46. $9\frac{2}{15}, 13\frac{13}{30}, 15\frac{17}{20}$.

*Subtract the following as indicated:*

47. $\frac{3}{4} - \frac{1}{8}$.
48. $\frac{5}{8} - \frac{1}{2}$.
49. $\frac{5}{8} - \frac{3}{16}$.
50. $\frac{7}{8} - \frac{5}{16}$.
51. $\frac{1}{2} - \frac{5}{32}$.
52. $\frac{7}{8} - \frac{5}{32}$.
53. $\frac{15}{32} - \frac{3}{64}$.
54. $32 - 4\frac{7}{8}$.
55. $63 - 5\frac{3}{4}$.
56. $72 - 8\frac{9}{10}$.
57. $48 - 3\frac{7}{32}$.
58. $67 - 5\frac{3}{16}$.
59. $71 - 6\frac{5}{24}$.
60. $80 - 9\frac{5}{48}$.
61. $9\frac{7}{16} - 7\frac{3}{8}$.
62. $8\frac{9}{16} - 3\frac{3}{8}$.
63. $6\frac{11}{16} - 4\frac{5}{8}$.
64. $7\frac{15}{16} - 3\frac{7}{8}$.
65. $9\frac{31}{32} - 6\frac{3}{4}$.
66. $8\frac{29}{32} - 5\frac{3}{8}$.
67. $23\frac{1}{8} - 8\frac{3}{16}$.

### ORAL EXERCISE

1. To multiply by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$ is the same as to divide by what numbers? Illustrate in each case.

2. To divide by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{8}$ is the same as to multiply by what numbers? Illustrate in each case.

3. To multiply by $\frac{2}{3}$ is the same as to multiply by what number and divide by what number? Illustrate by a problem.

4. To divide by $\frac{2}{3}$ is the same as to divide by what number and multiply by what number? Illustrate by a problem.

5. How do you find $\frac{7}{8}$ of a number? Illustrate.

6. How do you divide a number by $\frac{7}{8}$? Illustrate.

7. Since $\frac{1}{5} = \frac{2}{10}$, $\frac{2}{5} = \frac{4}{10}$, $\frac{3}{5} = \frac{6}{10}$, and $\frac{4}{5} = \frac{8}{10}$, state short methods of multiplying by $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

8. State short methods of dividing by $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$.

9. How do you multiply by a mixed number?

### WRITTEN EXERCISE

1. A piece of paper $7\frac{3}{8}$ in. wide is folded lengthwise into two equal parts. How wide is each part?

2. In a certain game a baseball team was entitled to one third of the gate receipts. If the gate receipts were \$285.75, to how much was the team entitled?

3. How many sheets of cardboard $\frac{3}{64}$ in. thick will it take to make a pile $3\frac{3}{16}$ in. thick?

*Solve as indicated, writing only the answers:*

4. $\frac{2}{3} \times \frac{7}{8}$.
5. $\frac{3}{4} \times \frac{1}{8}$.
6. $\frac{3}{4} \times \frac{2}{3}$.
7. $\frac{2}{3} \times \frac{1}{5}$.
8. $\frac{2}{3} \times \frac{4}{5}$.
9. $\frac{2}{3} \div \frac{3}{8}$.
10. $\frac{3}{8} \div \frac{2}{3}$.
11. $\frac{1}{5} \div \frac{7}{8}$.
12. $\frac{7}{8} \div \frac{1}{5}$.
13. $\frac{7}{8} \div \frac{3}{8}$.
14. $3\frac{3}{4} \times 46\frac{1}{8}$.
15. $17\frac{3}{4} \div 2\frac{1}{2}$.

### WRITTEN EXERCISE

*Add the following:*

| | | | | |
|---|---|---|---|---|
| 1. $\frac{1}{2}+\frac{1}{4}$. | 3. $\frac{1}{2}+\frac{1}{8}$. | 5. $\frac{1}{2}+\frac{5}{8}$. | 7. $\frac{1}{4}+\frac{1}{8}$. | 9. $\frac{3}{4}+\frac{3}{8}$. |
| 2. $\frac{1}{2}+\frac{3}{4}$. | 4. $\frac{1}{2}+\frac{3}{8}$. | 6. $\frac{1}{2}+\frac{7}{8}$. | 8. $\frac{1}{4}+\frac{7}{8}$. | 10. $\frac{3}{4}+\frac{5}{8}$. |

*Subtract the following:*

| | | | | |
|---|---|---|---|---|
| 11. $\frac{1}{2}-\frac{1}{4}$. | 13. $\frac{1}{2}-\frac{1}{8}$. | 15. $\frac{7}{8}-\frac{1}{2}$. | 17. $\frac{1}{4}-\frac{1}{8}$. | 19. $\frac{3}{4}-\frac{3}{8}$. |
| 12. $\frac{3}{4}-\frac{1}{2}$. | 14. $\frac{1}{2}-\frac{3}{8}$. | 16. $\frac{5}{8}-\frac{1}{2}$. | 18. $\frac{7}{8}-\frac{1}{4}$. | 20. $\frac{3}{4}-\frac{5}{8}$. |

*Multiply the following:*

| | | | | |
|---|---|---|---|---|
| 21. $\frac{1}{2}\times\frac{1}{4}$. | 23. $\frac{1}{2}\times\frac{1}{8}$. | 25. $\frac{5}{8}\times\frac{1}{2}$. | 27. $\frac{1}{4}\times\frac{1}{8}$. | 29. $\frac{3}{4}\times\frac{3}{8}$. |
| 22. $\frac{1}{2}\times\frac{3}{4}$. | 24. $\frac{1}{2}\times\frac{3}{8}$. | 26. $\frac{7}{8}\times\frac{1}{2}$. | 28. $\frac{7}{8}\times\frac{3}{4}$. | 30. $\frac{3}{4}\times\frac{5}{8}$. |

*Divide the following:*

| | | | | |
|---|---|---|---|---|
| 31. $\frac{3}{4}\div\frac{3}{8}$. | 33. $\frac{3}{4}\div\frac{5}{8}$. | 35. $\frac{3}{4}\div\frac{7}{8}$. | 37. $\frac{2}{3}\div\frac{3}{4}$. | 39. $\frac{2}{3}\div\frac{7}{8}$. |
| 32. $\frac{3}{8}\div\frac{3}{4}$. | 34. $\frac{5}{8}\div\frac{3}{4}$. | 36. $\frac{7}{8}\div\frac{3}{4}$. | 38. $\frac{3}{4}\div\frac{2}{3}$. | 40. $\frac{3}{5}\div\frac{2}{3}$. |

*Copy in horizontal rows, and add without writing in columns:*

41. $24+36+48+29$.
42. $72+30+92+68$.
43. $60+93+49+77$.
44. $83+29+76+55$.
45. $88+34+56+74$.
46. $23+31+99+87$.

*Write the following in common numerals:*

| | | | |
|---|---|---|---|
| 47. XIV. | 49. XXIX. | 51. LX. | 53. CLIV. |
| 48. XVI. | 50. XXXI. | 52. XL. | 54. CCCIV. |

*Multiply the following:*

| | | | |
|---|---|---|---|
| 55. $2\times41\frac{2}{3}$. | 56. $32\times68\frac{5}{8}$. | 57. $2\frac{1}{2}\times31\frac{1}{3}$. | 58. $3\frac{3}{4}\times5\frac{2}{5}$. |

*Divide the following:*

| | | | |
|---|---|---|---|
| 59. $18\frac{2}{3}\div2\frac{2}{3}$. | 60. $27\frac{1}{2}\div5\frac{1}{2}$. | 61. $61\div7\frac{5}{8}$. | 62. $3\frac{5}{8}\div4\frac{3}{4}$. |

### WRITTEN REVIEW

1. A merchant bought 38 pieces of cloth of $36\frac{1}{2}$ yd. each and paid \$1.48 a yard for it. What was the total cost?

2. Which will cost the more, 56 cows at \$42.50 each or 25 horses at \$95 each? how much more?

3. If a cubic foot of ice weighs 58 pounds, what will be the weight of 26 cakes, each containing $3\frac{1}{2}$ cu. ft.?

4. If a man earns \$3.75 a day and works 6 days in the week, how much will he earn in 26 weeks?

5. The earth moves about the sun 101,090 ft. in a second. Find to the nearest mile the distance it moves in 15 min.

6. If a steam engine uses $3\frac{1}{2}$ T. of coal a day, how many tons will it consume in February, 1928, if it runs every day?

7. A grocer sold 16 boxes of oranges, each box containing 196 oranges, at an average price of $4\frac{1}{2}$¢ an orange. How much did he receive for all?

8. If electricity travels on a wire at the rate of 288,000 mi. per second, how long would a wire have to be for an electric current to travel from one end to the other in $2\frac{1}{2}$ min.?

9. A man pays \$202.95 for coal at \$6.15 a ton. How many tons does he buy?

10. A shipper paid \$92.16 freight on some goods at \$0.64 per thousand pounds. How many tons were there?

11. The product of two numbers is 13.02042, and one of the numbers is 6.414. What is the other number?

12. Wheat that cost 96¢ a bushel is sold for \$1.02 a bushel. How much must be sold to realize a profit of \$99?

13. A contractor bought a shipment of bricks for \$377.40, paying \$7.40 a thousand. How many bricks did he buy?

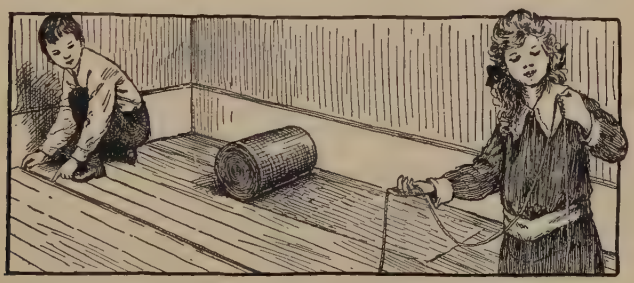
**14.** The product of two numbers is 468.75 and one of the numbers is 500. What is the other number?

**15.** How many times will a roller 6.8 ft. in circumference turn in rolling across a street that is 74.8 ft. wide?

**16.** If the wind in a hurricane blows at the rate of 8796.6 ft. per minute, what is the rate per hour? per second?

**17.** If a sewing machine makes 360 stitches of $\frac{1}{8}$ in. each in a minute, how many yards of goods can be pulled through in 4 min.?

**18.** If a sewing machine makes 424 stitches of $\frac{1}{32}$ in. each in a minute, how many yards of goods can be pulled through in 12 min.?



**19.** This boy and girl have measured the room. They find it to be 24 ft. long and 18 ft. wide. They know that the carpet which their mother bought has a pattern 1 yd. long, that the carpet is 27 in. wide, and that it costs 70¢ a yard, but can be bought at 10% discount. Their mother has $125 in the bank. Now make up a good practical problem and solve it.

**20.** Farmers have a simple rule for finding the number of bushels in a bin. The rule is to take $\frac{4}{5}$ of the product of the number of feet in the length, breadth, and depth of the bin. How many bushels of corn can be put in a bin $8\frac{1}{2}$ ft. long, $6\frac{1}{4}$ ft. wide, and 5 ft. deep?

**21.** An Ohio farmer found that he was raising on the average 65 bu. of corn per acre. He determined to see if he could not by proper selection of seed increase the yield. The result was an increase of 15%. The corn being worth 50¢ a bushel, what was the increase in the value of the product on 40 acres?

**22.** The farmer mentioned in Ex. 21 decided to try still more careful selection. In the first seven years he averaged 77 bu. per acre, and in the last seven he averaged 10% more. What was the latter yield per acre? What was it for the 40 acres? What was the gain in value at 50¢ a bushel?

**23.** Flies often lay their eggs in the filth of stables. It has been found that 1200 flies will hatch in a pound of this filth. If even $\frac{1}{10}$ of this rate were kept up, how many flies would hatch in a summer from 6 tons of this filth?

**24.** If a man earns \$3.50 a day when he is well, but loses his wages and has to pay \$1.30 a day for care and medicines when he is ill, how much more is his net weekly income when he is well and is working 6 da. in the week than when he is ill?

**25.** If the list prices of some goods are \$100, \$200, \$350, and \$50, what is the discount on the whole bill at 10%? at 5%?

**26.** A bicycle was marked \$45, but was sold at 12% discount. What was the net price?

**27.** A piano was listed at \$550, but was sold at 30% discount. What was the selling price?

**28.** If the drivewheel of a sewing machine has a diameter of 1 ft., and the circumference is $3\frac{1}{7}$ times the diameter, what is the circumference in inches?

**29.** Water in freezing expands 10% in volume. How many cubic feet of ice would 825 cu. ft. of water make?

**16. Interpreting Problems.** We have done much work in the interpretation of problems, that is, in finding how to solve them, in estimating the results in advance, in saving labor by determining exactly the thing that should be done, and in using short methods of operation. In the last two years of arithmetic you should give even more attention to this subject. A few examples will now be given to illustrate the study and interpretation of ordinary business problems.

**1.** A dealer bought some goods for \$2500 and sold them for \$3000. What is his gain per cent?

Exactly what must I find? The gain per cent on \$2500.

How shall I find the sum gained? By taking \$2500 from \$3000.

Do I need pencil and paper for this work? No, for the gain is \$500, and \$500 is $\frac{1}{5}$ of \$2500, and $\frac{1}{5}$ is 20%.

How shall I check my result? See if $\frac{1}{5}$ of \$2500 is \$500.

**2.** A dealer in horses bought a shipment of ponies for \$2500. He sold them at a loss of 20%, and with the proceeds he bought some horses which he sold at a profit of 20%. Did he gain or lose by the whole transaction, and how much?

What must I find? First whether he gained or lost, and then how much.

Can I estimate the result in advance? Yes, for he lost 20% on \$2500 and gained 20% on a smaller sum, so there was a loss. Furthermore, not only can I estimate the result, but I can easily work it out mentally.

The dealer lost $\frac{1}{5}$ of \$2500, or \$500; therefore he received \$2000 for the ponies. He gained $\frac{1}{5}$ of \$2000, or \$400, on the horses. He lost \$500 and gained \$400, so he is \$100 out of pocket on the whole deal.

**3.** A boy is offered a salary of \$6.50 a week for the first year in a store, 20% more the second year, and an increase of 50% of this amount the third year. If he works 50 weeks in a year, what is his total salary for the first three years?

How shall I find the boy's income the first year? How much more was it the second year? How much was his salary for the third year?

**4.** A dealer sold two cars for \$960 each. On the first he lost $\frac{1}{5}$ of what he paid, and on the second he gained $\frac{1}{5}$ of what he paid. Did he gain or lose on the whole transaction, and how much?

Problems of this nature are not likely to be met in actual business, but the solution of one such example is interesting. At first reading it looks as if the dealer came out even, but this cannot be true because he did not gain and lose $\frac{1}{5}$ of the same amount.

We are now ready for the work and for the full analysis of the problem.

*Analysis*

Since he lost $\frac{1}{5}$ of the cost of the first car, and $1 - \frac{1}{5} = \frac{4}{5}$, he sold this car for $\frac{4}{5}$ of the cost.

Therefore $\frac{4}{5}$ of the cost = \$960, and $\frac{1}{5}$ of the cost = \$960 ÷ 4 = \$240.

Therefore the loss on the first car was \$240.

*Work*

$4)\overline{\$960}$ \$240 loss

$6)\overline{\$960}$ \$160 gain

\$240
160
\$80 net loss

Since he gained $\frac{1}{5}$ of the cost of the second car, and $1 + \frac{1}{5} = \frac{6}{5}$, he sold this car for $\frac{6}{5}$ of the cost.

Therefore $\frac{6}{5}$ of the cost = \$960,

and $\frac{1}{5}$ of the cost = \$960 ÷ 6 = \$160.

Therefore the gain on the second car was \$160.

Therefore the net loss was \$240 − \$160 = \$80.

**5.** A dealer bought 32 horses for \$4320. How much would 12 horses cost at the same rate?

*Analysis*

Since 32 horses cost \$4320, 1 horse costs \$4320 ÷ 32, and 12 horses cost 12 times this amount.

Indicating these operations and canceling as shown, we find that the cost of the 12 horses would be \$1620.

*Work*

$$\frac{3 \quad 540}{\cancel{12} \times \cancel{\$4320}} = \$1620$$
$$\cancel{32}$$
$$8$$

### WRITTEN EXERCISE

1. If a man works 312 da. a year at \$3.75 a day, and saves \$117, what per cent of his income does he spend?

2. A man receiving a salary of \$24 a week worked 50 wk. last year. He paid \$24 a month rent. How much money had he left for other expenses?

3. A laborer earns \$2.25 a day for 6 da. in the week. If he works for 48 wk. and spends 22% of his wages for rent, how much money is left for other expenses?

4. A man works 6 da. in the week for 43 wk., and 5 da. in the week for 5 wk. He takes two rides on the trolley car each day that he works, at 5¢ a ride. How much does he pay in a year for trolley rides?

5. In casting brass hinges the brass shrinks, in cooling, $\frac{1}{64}$ of its length when melted. How long must be the mold for a hinge that is to be $3\frac{5}{16}$ in. long?

6. The wooden pattern from which an iron casting is made weighs $\frac{1}{16}$ as much as the iron. If the pattern weighs 7 lb. 3 oz., how much does the casting weigh?

7. An iron founder knows that an iron casting will weigh 16 times as much as the wooden pattern. If he has four patterns for the parts of a given piece that he wishes to cast, and these weigh 7 lb. 9 oz., 9 lb. 3 oz., 26 lb. 14 oz., and 13 lb. 8 oz., what will the casting weigh?

8. A manufacturer is building a chimney with a square flue for his boiler room. He has two horizontal boilers, each containing 96 tubes with a cross-section area of 3 sq. in. each. What must be the cross-section area of the chimney in order to equal the sum of all the cross-section areas of the tubes of the boilers?

**9.** Allowing 1000 shingles to 120 sq. ft., find how many thousand shingles will cover a pitched roof each side of which is 58 ft. long and 24 ft. wide.

**10.** A corner lot has 96 ft. front and is 180 ft. deep. Find the cost of laying a 4-foot stone walk on the front and side, at 18¢ per square foot.

**11.** Find in inches the depth of a rectangular tank, 9 ft. square at the base, that has the same capacity as a rectangular cistern 8 ft. square at the base and 6 ft. deep.

**12.** A man receives \$3.60 for every 8 hr. of work, and works during 6 da. of a week the following number of hours: $7\frac{1}{2}$, 8, $7\frac{3}{4}$, 8, 8, $7\frac{1}{4}$. How much does he receive in all?

**13.** A certain kind of wheat produces 85% of its weight in flour. How many pounds of flour will be produced from 75 bu. of this wheat, allowing 60 lb. to the bushel?

**14.** If 15 burners, each consuming 6 cu. ft. of gas per hour, are used, on an average, 4 hr. per day for 365 days, what will be the gas bill at 80¢ per thousand?

**15.** Find to the nearest pound the weight of the air in a schoolroom $30' \times 16' \times 12'$, if water is 770 times as heavy as air and weighs 1000 oz. per cubic foot.

**16.** Bronze contains 9 parts of copper by weight to 2 parts of tin. How many pounds of tin in a bronze statue that weighs 2662 lb.?

**17.** A bell weighing 1250 lb. is composed of 77% copper, 21% tin, and the rest zinc. How many pounds of each metal does the bell contain?

**18.** Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 150 lb. of flint.

**19.** If a certain gun metal is composed of 84% copper and the rest tin, how many pounds of tin will be needed in making 750 lb. of the gun metal?

**20.** A man, computing the cost of 54 articles, multiplied by mistake the cost of each by 45 instead of by 54, and obtained \$121.50. What was the cost of the 54 articles?

**21.** Find the number of dry quarts (67.2 cu. in.) in a measure containing 1 cu. ft.

**22.** At what rate will \$1000 yield \$60 interest in 1 yr.? At what rate will \$1000 yield \$100 interest in 2 yr.? At what rate will \$1000 yield \$210 interest in 3 yr.?

**23.** What fraction of the year 1920 is the time from January 20 to March 15, including both of these days?

**24.** On seven successive days at noon the thermometer indicated respectively 66°, 72°, 61°, 68°, 73°, 62°, and 65°. What was the average for the week?

**25.** On seven successive days the barometer indicated respectively 29.22 in., 29.18 in., 29.65 in., 30.02 in., 30.12 in., 30.18 in., and 30.21 in. What was the average for the week?

**26.** A cubic inch of iron weighs $4\frac{1}{9}$ oz. Find the weight in pounds of an iron bar 1 in. square at the end and 1 yd. long; 1 in. square at the end and 15 ft. long.

**27.** Water is flowing at the rate of 2 cu. ft. in 3 min. into a rectangular tank, which is 6 ft. 8 in. long and 4 ft. 6 in. wide. How long will it take the water to fill the tank to a depth of 6 ft.?

Teachers will find it an excellent exercise to have a pupil read the problem silently, state what it means, state what is given, and then state what is required. When the problem allows, the pupil should be encouraged to tell in what situation such a problem might arise. Another pupil may then be called upon to plan the solution, and a third may be asked to solve it upon the blackboard. Short methods should always be commended.

**17. Unitary Analysis.** Examples like the following are easily solved by a particular kind of analysis called *unitary analysis.*

If 5 T. of coal cost $27.50, what will 7 T. cost?

Since 5 T. cost $27.50, 1 T. costs $\frac{1}{5}$ of $27.50, or $5.50.

Therefore 7 T. cost 7 times $5.50, or $38.50.

Notice that the problem was solved by finding the *unit,* the cost of *1 ton.* Hence the name "unitary analysis."

### WRITTEN EXERCISE

**1.** If 3000 ft. of lumber cost $96.60, what will 16,100 ft. cost at the same rate?

**2.** At $460 per 100 bbl., how many barrels of flour can be bought for $73.60? for $782? for $8556?

**3.** At $5.60 a ton of 2000 lb., what will 2775 lb. of coal cost? 1575 lb.? 1850 lb.? 3225 lb.? 6450 lb.?

**4.** At $12.50 per hundred pounds, what will $4\frac{1}{2}$ lb. of cheese cost? $7\frac{1}{4}$ lb.? 9 lb. 4 oz.? 12 lb. 12 oz.?

**5.** At $5.20 a hundred pounds, how many pounds of meat can be bought for $639.60? for $587.60? for $4004?

**6.** When the conditions of the air are such that sound travels 3696 ft. in 3.3 sec., how far does it travel in 7 sec.?

**7.** If 15 bales of cotton weigh 7305 lb., what will 4 such bales weigh? 7 bales? 16 bales? 25 bales?

**8.** If a tree 25 ft. high casts a shadow 20 ft. long, how long a shadow will a tree 30 ft. high cast at the same time?

**9.** If your schoolhouse casts a shadow 40 ft. long when a post 6 ft. high casts a shadow 8 ft. long, how high is the schoolhouse?

**10.** If a pole 6 ft. high casts a shadow 8.3 ft. long, how high is an adjacent tree that casts a shadow 74.7 ft. long at the same time?

**18. Comparison of Numbers.** A comparison of the numbers often leads to a short solution, as in the following examples:

1. If 3 tons of hay cost \$36, how much will 6 tons cost?

Since $6 = 2 \times 3$, 6 tons will cost $2 \times \$36$, or \$72.

2. If 20 A. of land cost \$1600, how much will 80 A. cost?

Since $80 = 4 \times 20$, 80 A. will cost $4 \times \$1600$, or \$6400.

### ORAL EXERCISE

1. If 3 readers cost \$1.50, how much will 9 readers cost?
2. If 4 horses cost \$500, how much will 16 horses cost?
3. If 7 doz. eggs cost \$2.10, how much will 21 doz. cost?
4. If 9 yd. of cloth cost \$4.50, how much will 3 yd. cost?
5. If 15 bbl. of sugar cost \$150, how much will 5 bbl. cost?
6. If you walk 2 mi. in 40 min., how long will it take you to walk 4 mi. at the same rate?

### WRITTEN EXERCISE

1. If 4 M ft. (4000 board feet) of lumber cost \$128, how much will 20 M ft. cost? How much will 60 M ft. cost?
2. If 7 bu. of apples cost \$5.25, how much will 35 bu. cost?
3. If 5 sets of furniture cost \$325, how much will 75 sets cost?
4. If a half-pound stick of solder is used to seal 30 cans of corn, how many pounds of solder are needed for 21,000 cans?
5. If $\frac{3}{8}$ of a farm is worth \$2100, how much is $\frac{3}{4}$ of the farm worth at the same rate?
6. If $\frac{3}{10}$ of the value of a mill is \$21,000, how much is $\frac{3}{5}$ of the mill worth? How much is $\frac{3}{20}$ of it worth?
7. If $2\frac{1}{2}$ bu. of apples cost \$1.75, how much will 10 bu. cost?

**19. Stating a Problem in One Step.** It is often a good plan to indicate the solution of a problem in a single step, so as to shorten the work by canceling, as in the following example:

If 1 cu. ft. of water weighs $62\frac{1}{2}$ lb., and gold is 19.5 times as heavy as water, how much is 1 cu. ft. of gold worth at \$18.60 an ounce avoirdupois?

1 cu. ft. of gold weighs $19.5 \times 62\frac{1}{2}$ lb., or $19.5 \times 62\frac{1}{2} \times 16$ oz.

Therefore the value of 1 cu. ft. of gold is

$$19\tfrac{1}{2} \times 62\tfrac{1}{2} \times 16 \times \$18.60.$$

But $19\frac{1}{2} = \frac{39}{2}$, and $62\frac{1}{2} = \frac{125}{2}$.

We therefore have $\dfrac{39 \times 125 \times 16 \times \$18.60}{2 \times 2}$.

We may now cancel and thus complete the solution more easily than by multiplying step by step.

**WRITTEN EXERCISE**

1. Finish the solution of the above problem.

2. When sound travels 1125 ft. per second, how many miles away is a lightning flash which is seen 13.2 sec. before the thunder is heard?

3. Soldiers marching quickstep take 120 paces a minute, averaging 28 in. each. At this rate, how many miles will a company of soldiers march in an hour?

4. If the circumference of a wheel is $3\frac{1}{7}$ times the diameter, and the drive wheel of a locomotive is 6 ft. in diameter, how many revolutions will it make in going from Albany to New York, 143 mi.?

5. What is the expense of covering a floor with plain matting 1 yd. wide, the room being 18 ft. by $21\frac{1}{2}$ ft., and the matting being worth 30¢ a yard, allowing 6 in. extra on each strip for turning under? Draw a plan of the floor.

**6.** If a piece of cast iron $10'' \times 8'' \times 2''$ weighs 41.6 lb., what is the weight of a piece $16'' \times 10'' \times 3''$?

**7.** If 475 horse power from several engines costs \$13,490 a year, what will 60 horse power cost at the same rate?

**8.** A dealer bought 25 bu. of grass seed at \$2.90 a bushel. He paid \$1.20 for paper bags, and sold the seed in bags at 10¢ a quart. How much did he gain?

**9.** Two steamers started for Europe on the same route at the same time. After 4 days one had averaged 448.7 mi. a day, and the other 502.1 mi. a day. How far apart were they?

**10.** A box of arithmetics cost a dealer \$53.94, including \$1.48 freight. He sold the arithmetics for 50¢ each, thus gaining 7¢ on the list price of each book. How many did he buy?

**11.** A man bought two adjacent farms containing the same number of acres. For one he paid \$95 per acre, and for the other \$130 per acre. The two farms cost him \$31,950. How many acres did he buy?

**12.** If it costs \$702 to excavate a cellar $27' \times 39' \times 9'$, how much will it cost to excavate another cellar $30' \times 36' \times 12'$ at the same rate?

**13.** If 100 lb. of bituminous (soft) coal contain 76.05 lb. of carbon and 2.32 lb. of ash, how many pounds of each are there in 15 T.?

**14.** If 2000 lb. of anthracite (hard coal) contain 1828 lb. of carbon, how much carbon will 100 lb. contain? What is the per cent of carbon?

**15.** A gallon contains 231 cubic inches. Alcohol is 0.84 as heavy as water. Find the number of cubic inches of alcohol that will weigh as much as a gallon of water.

## LITTLE EXAMINATIONS

**I.** 1. \$275 is what per cent more than \$200?
2. \$200 is what per cent less than \$275?
3. If 17 cars cost \$13,600, how much will 9 cars cost?
4. How much is 72% of 350 lb.? of \$3500? of 35,000 lb.?
5. How much is 175% of \$2500? $\frac{1}{2}$% of \$2000?

**II.** 1. 39,987 + 46,296.
2. 73,203 − 59,827.
3. $34\frac{1}{2} \times$ \$42,346.
4. \$29.75 ÷ 25.
5. $\frac{7}{8}$ of 25 ft. 4 in.
6. CDXL = (?).
7. $32\frac{1}{2}$ ft. = (?) yd.
8. $18\frac{1}{2}$ gal. = (?) qt.
9. $4\frac{3}{4}$ bu. = (?) pk.
10. 66 ft. = (?) rd.

**III.** 1. \$500 is what per cent of \$400?
2. \$500 is what per cent more than \$400? than \$250?
3. \$400 is what per cent less than \$500? than \$800?
4. If 40 mules cost \$5000, how much will 15 mules cost?
5. At \$120 for $\frac{3}{4}$ A., how much will 1 A. of land cost?

**IV.** 1. $41\frac{2}{3}$ in. + $1\frac{1}{2}$ in.
2. $5\frac{3}{8}$ in. − $2\frac{3}{4}$ in.
3. $2\frac{3}{4} \times 7\frac{1}{2}$ in.
4. $6\frac{7}{8}$ ft. ÷ $3\frac{7}{16}$ ft.
5. 3.75 ÷ $2\frac{1}{2}$.
6. 25% of \$1.60.
7. \$2.75 ÷ $1\frac{1}{4}$.
8. $\frac{2}{3}$ of $4\frac{1}{2}$ ft.
9. $\frac{7}{8}$ of 9 ft. 4 in.
10. 75% of 6 lb. 4 oz.

These Little Examinations occur at the end of each chapter. They furnish brief written tests relating to the work of the chapter.

## REVIEW AND DRILL

### I. DRILL SECTION

*Add the following:*

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $27.56 | $18.96 | $41.63 | $15.92 | $17.26 |
| 32.75 | 26.45 | 18.99 | 17.86 | 18.32 |
| 41.07 | 33.87 | 17.46 | 33.40 | 15.88 |
| 9.89 | 91.82 | 33.81 | 29.69 | 27.40 |
| 10.07 | 5.50 | 40.00 | 8.72 | 82.60 |
| 12.50 | 17. | 7.80 | .50 | 82.74 |
| 16.75 | 8.96 | 16.15 | 3.00 | 81.68 |
| 8.92 | 14.73 | 27.75 | 27.60 | 84.12 |

**6.** In Ex. 1, add the first four numbers, then the second four, and then these two sums, thus checking the result of Ex. 1.

**7.** In Ex. 2, proceed as in Ex. 6, thus checking the result.

**8.** In Ex. 3, proceed as in Ex. 6, thus checking the result.

**9.** In Ex. 4, proceed as in Ex. 6, thus checking the result.

**10.** In Ex. 5, proceed as in Ex. 6, thus checking the result.

**11.** From the sum of $175.50 and $87.75 take the sum of $27.85 and $52.25.

**12.** Multiply $175.50 by 146, and divide the result by 73. What is the shortest way of obtaining the result?

**13.** Find the number of cubic inches in a box that is $8\frac{1}{4}$ in. long, $4\frac{3}{8}$ in. wide, and $3\frac{1}{2}$ in. deep. Use the shortest method.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

**1.** What is the best way of multiplying one mixed number by another?

**2.** If one writing pad costs a certain aliquot part of a dollar, what is the best way of finding the cost of a given number of these pads?

**3.** At a given price per yard, how do you find the number of yards of cloth, carpet, or linoleum that can be bought for a given amount of money?

**4.** If you know the price of pens per gross, how do you find the price per pen?

**5.** When a train is traveling a certain number of miles in a certain number of minutes, how do you find its rate per hour?

**6.** If you know what an automobile dealer pays for a car and the per cent above cost at which he sells it, how do you find the selling price?

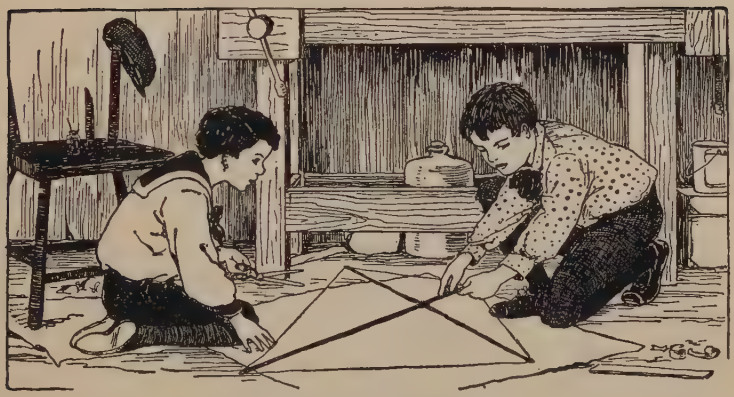
**7.** If you know the length and width of this room in feet and inches, how do you find the area of the floor?

**8.** If you know the three dimensions of this room in feet and inches, how do you find the number of cubic feet in the room? Why should any one wish to find this result?

**9.** If you were asked by your mother to find how many yards of carpet were needed for your sitting room or stairs, how would you find it?

**10.** If you knew or were told the number of cubic inches in a bushel, how would you find, by first measuring the dimensions, the number of bushels of wheat that could be put into a bin?

## III. INDUSTRIAL PROBLEMS



THE BOYS' WORKSHOP

1. The two cross sticks of this kite are 19 in. and 18 in. in length. They are joined at right angles in the middle of the shorter piece and 7 in. from the end of the longer one. Draw a plan of the kite to the scale $\frac{1}{4}$.

2. In your drawing in Ex. 1 measure the four sides of the kite and then compute their lengths in the real kite.

3. By dividing the kite mentioned in Ex. 1 into two triangles, compute its area.

4. Find the area of a rectangle if the dimensions are the lengths of the two cross sticks of the kite mentioned in Ex. 1.

5. The frame of another kite is made of five pieces of wood, the top one being 3 in. long, each of the other two short sides $8\frac{1}{4}$ in., and each of the two long sides $13\frac{1}{2}$ in. Find the perimeter.

The pupils should be encouraged to bring into class any problems relating to handwork of this kind in which they may be interested.

### THE RAILWAY INDUSTRY

1. If railway ties are 8″ wide and are placed 18″ apart, how many ties are there to a mile?

2. Our railways use 90,000,000 new ties a year, averaging 8″ × 8′ 6″ × 6″. How many cubic feet of timber do they use?

3. The modern coal car usually has a capacity of 100,000 lb. At 35 cu. ft. to the ton of 2000 lb., what is the volume?

4. If a rail is 30′ long and weighs 120 lb. to the yard, and if a man can carry 160 lb., how many men will it take to carry a rail?

5. The standard gauge of railways in America and England is 4′ $8\frac{1}{2}$″. Express this in meters, as used in most European countries, the meter being 39.37″.

6. The standard American and English gauge (see Ex. 5) is $\frac{1}{5}$ more than the gauge of a certain mountain road. What is the gauge of the latter?

7. If the cost of maintaining the single track of a certain railway averages $846 a mile, what is the cost of maintaining its 450 mi. of double track?

## CHAPTER II

### PRACTICAL MEASUREMENTS

**20. Tables.** Before beginning the study of practical measurements, it is necessary to recall the principal tables which we shall use. They are as follows:

LENGTH

**12 inches (in. or ″) = 1 foot (ft. or ′)**
**3 feet = 1 yard (yd.)**
**$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)**
**320 rods, or 5280 feet = 1 mile (mi.)**

Surveyors use a *chain* 66 ft. long, composed of 100 *links* each 7.92 in. long.

AREA

**144 square inches (sq. in.) = 1 square foot (sq. ft.)**
**9 square feet = 1 square yard (sq. yd.)**
**$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)**
**160 square rods = 1 acre (A.)**
**640 acres = 1 square mile (sq. mi.)**

We occasionally write sq.′ for square feet and sq.″ for square inches.

VOLUME

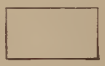
**1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)**
**27 cubic feet = 1 cubic yard (cu. yd.)**
**128 cubic feet = 1 cord (cd.)**

A cubic yard of earth is considered a load. A perch of stone or masonry is usually considered as $24\frac{3}{4}$ cu. ft., but it varies in different parts of the country.

**21. Areas.** You have already learned how to find the areas of several kinds of figures, so it is necessary at this time only to repeat the rules which we studied in the Intermediate Book.

*The area of a rectangle equals the product of the base and height.*

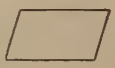
In all such cases abstract numbers are understood.



For example, if the base is 3 ft. and the height 2 ft., the area is found thus:

$$2 \times 3 \text{ sq. ft.} = 6 \text{ sq. ft.}$$

*The area of a parallelogram equals the product of the base and the height.*



For example, if the base is 2.4 in. and the height 2 in., the area is 4.8 sq. in. That is,

$$2 \times 2.4 \text{ sq. in.} = 4.8 \text{ sq. in.}$$

*The area of a triangle equals half the product of the base and the height.*

For example, if the base is $3\frac{1}{2}$ in. and the height $2\frac{1}{2}$ in., the area is found thus:

$$\tfrac{1}{2} \text{ of } 2\tfrac{1}{2} \times 3\tfrac{1}{2} \text{ sq. in.} = 4\tfrac{3}{8} \text{ sq. in.}$$

The actual work is as follows:

$$\frac{5 \times 7}{2 \times 2 \times 2} = \frac{35}{8} = 4\tfrac{3}{8}.$$

*The area of a trapezoid equals half the product of the height and the sum of the parallel sides.*

This brief form of stating the rule will now be understood. The longer form, used by beginners, says that *the area of a trapezoid equals half that of a rectangle with the same altitude and with a base equal to the sum of the two parallel sides.*

**22. Volumes.** We have learned that *the volume of a rectangular solid equals the product of its three dimensions.*

For example, if a box is 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. deep, the volume is found as follows:

$$3 \times 2 \times 1\tfrac{1}{2} \text{ cu. ft.} = 9 \text{ cu. ft.}$$

### WRITTEN EXERCISE

**1.** How much will it cost, at \$1.40 a yard, to cover the floor of a room 20 ft. 3 in. by 28 ft. 6 in. with 27-inch carpet, allowing 2 yd. for matching the pattern?

Similar practical problems should be secured by the pupils at home.

**2.** How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 72 yd. long, each sheet being $2\frac{1}{2}$ yd. long with a top hem $3\frac{1}{4}$ in. and a bottom hem $2\frac{1}{4}$ in., an allowance of $\frac{1}{4}$ in. for turning under being made for each hem?

In buying sheeting the width of the material is given in quarters of a yard. Sheeting $\frac{6}{4}$ wide means 6 quarters of a yard wide. Sheeting $\frac{6}{4}$ wide is used on single beds, $\frac{8}{4}$ wide on three-quarter beds, and $\frac{10}{4}$ wide on double beds.

**3.** A piece of sheeting of $\frac{8}{4}$ width contains 64 yd. How many sheets for three-quarter beds can be made from it, allowing the same measurements as in Ex. 2?

**4.** A piece of sheeting of $\frac{10}{4}$ width contains 72 yd. How many sheets for double beds can be made from it, allowing the same measurements as in Ex. 2?

The class should study the simple measurements needed in the household, including the measurement of materials used in food and clothing. The cost of furnishing a kitchen, laundry, dining room, living room, and bedroom, with the expense of renovating them, should also be considered.

Where electricity or gas is used, the meter should be read by the class, and the cost of lighting computed. The relative cost of different systems of heating and lighting in the school and home should be considered.

**5.** The ice chamber in an ice box is 16 in. by 24 in. by 18 in. Ice weighs 8% less than water, and 1 cu. ft. of water weighs $62\frac{1}{2}$ lb. If the ice comes in rectangular pieces weighing 10, 25, 50, 100, 150, and 200 lb., which one or which two of these pieces would come nearest to filling the chamber?

6. The bottom of an ordinary farm-wagon box is 3 ft. by 10 ft., and the box is from 24 in. to 26 in. deep. Find the contents in cubic feet and in cubic inches for each of these depths.

7. Such a wagon box 24 in. deep is estimated to contain 50 bu. of shelled corn. How many bushels of shelled corn will such a box 26 in. deep contain?

8. Certain farmers estimate that $2\frac{2}{5}$ cu. ft. of corn in the ear will produce 1 bu. of shelled corn. How many bushels of shelled corn in a crib 12 ft. by 20 ft. by 8 ft., filled with corn in the ear?

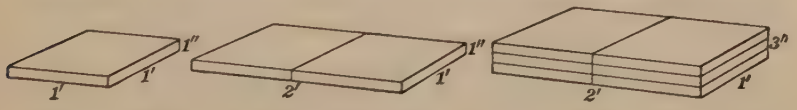
9. If a dirt car is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep, how many cubic yards will it carry?

10. How many cubic yards of earth must be removed in digging a tunnel 492 ft. long, 39 ft. wide, and 19 ft. 6 in. high?

*An irregular city block is divided into lots with measurements correct to one decimal place, as shown above; find the areas in square feet of the lots numbered as follows:*

| | | | | |
|---|---|---|---|---|
| **11.** I. | **13.** II. | **15.** III. | **17.** IV. | **19.** V. |
| **12.** VI. | **14.** VII. | **16.** VIII. | **18.** IX. | **20.** X. |

**23. Board Measure.** One *board foot* of lumber means a piece that is 1 sq. ft. on one surface and is 1 in. or less in thickness. In speaking of lumber, a board foot is often called simply one foot.




1 BOARD FOOT 2 BOARD FEET 6 BOARD FEET

For example, a board 2′ long, 1′ wide, and 1″ (or less) thick contains 2 board feet (usually written as 2 bd. ft.).

A plank 12′ long, 1′ wide, and 3″ thick contains 36 bd. ft.

A board 18′ long, 8″ wide, and 1″ (or less) thick contains $18 \times \frac{8}{12}$ bd. ft., or 12 bd. ft. A joist $12' \times 6'' \times 2\frac{1}{2}''$ contains $2\frac{1}{2} \times 12 \times \frac{1}{2}$ bd. ft., or 15 bd. ft.

A joist $12' \times 6'' \times 2\frac{1}{2}''$ means and is read "a joist 12 ft. by 6 in. by $2\frac{1}{2}$ in." In such cases always use cancellation; thus, since $2\frac{1}{2} = \frac{5}{2}$, we have

$$2\frac{1}{2} \times 12 \times \frac{1}{2} = \frac{5 \times \overset{3}{\cancel{12}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}}} = 15.$$

### WRITTEN EXERCISE

*Find the number of feet, board measure, in the following:*

1. 32 planks 3 in. thick, 16 ft. long, and 8 in. wide.
2. 4 sticks of timber 16 ft. long, 1 ft. wide, and 8 in. thick.
3. 36 joists 3 in. by 4 in. and 16 ft. long.
4. 12 beams 10 in. by 12 in. and 14 ft. long.
5. 60 boards $\frac{7}{8}$ in. thick, 8 in. wide, and 16 ft. long.

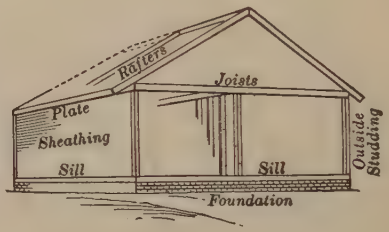
In such measurements, a board $\frac{7}{8}$ in. thick is considered to be 1 in. thick.

6. What is the cost of 30 spruce joists 2 in. by 6 in. and 14 ft. long, at $34 per M (that is, per 1000 bd. ft.)?

**24. House Building.** The house shown in the picture below is to be $27\frac{1}{2}' \times 34'$. A builder estimates the cost of materials and labor. Part of the computation of cost and amount of materials is given in the following examples; the rest is left for the class. The specifications are taken from a recent government report.

Call a fraction of a foot a whole foot in any result.

The pupils should know, from the figure at the right, the meaning of the terms used.




**WRITTEN EXERCISE**

1. For sills the builder needs the following:

3 pieces, $6'' \times 8''$, $16'$ long, 192 bd. ft.
4 pieces, $6'' \times 8''$, $14'$ long, (?) bd. ft.
6 pieces, $6'' \times 8''$, $12'$ long, (?) bd. ft.
1 piece, $2'' \times 8''$, $14'$ long, 19 bd. ft.
1 piece, $2'' \times 10''$, $8'$ long, (?) bd. ft.

2. The builder needs the following joists:

30 pieces, $2'' \times 10''$, $16'$ long, (?) bd. ft.
18 pieces, $2'' \times 8''$, $12'$ long, (?) bd. ft.
13 pieces, $2'' \times 8''$, $6'$ long, (?) bd. ft.
27 pieces, $2'' \times 8''$, $16'$ long, (?) bd. ft.
27 pieces, $2'' \times 8''$, $12'$ long, (?) bd. ft.
27 pieces, $2'' \times 6''$, $16'$ long, (?) bd. ft.

3. The builder also needs the following studding:

5 pieces, 4″ × 6″, 14′ long, (?) bd. ft.
100 pieces, 2″ × 4″, 14′ long, 934 bd. ft.
35 pieces, 2″ × 4″, 10′ long, 234 bd. ft.
25 pieces, 2″ × 4″, 18′ long, (?) bd. ft.
45 pieces, 2″ × 6″, 12′ long, (?) bd. ft.
50 pieces, 2″ × 4″, 8′ long, 267 bd. ft.
50 pieces, 2″ × 4″, 14′ long, 467 bd. ft.
40 pieces, 2″ × 6″, 18′ long, (?) bd. ft.

4. For the roof the builder needs 1050 bd. ft., and this, with the amounts in Exs. 1–3, should make a total of 8021 bd. ft.

5. How much will the lumber mentioned in Exs. 1–4 cost at $60 per M?

6. The builder also needs 2000 bd. ft. of sheathing at $65 per M; 1950 bd. ft. of flooring at $45; 125 bd. ft. of flooring for the porch at $65; 1500 shingles for the gables at $5.25 per 1000; 6500 shingles for the roof at $7. What will this material cost?

7. He also needs 28 foundation posts at 40¢ each; water tables, cornice, corner boards, etc., $35; porch columns, $7; 16 doors at an average cost of $5.75 each; 16 windows at an average cost of $5.50 each; hardware, $50; chimney and plastering, $200; painting, $148; tin work, $25.50; the work of 4 carpenters for 30 days at $4 a day each; excavating, $130; grading, $25. What is the total cost of these items?

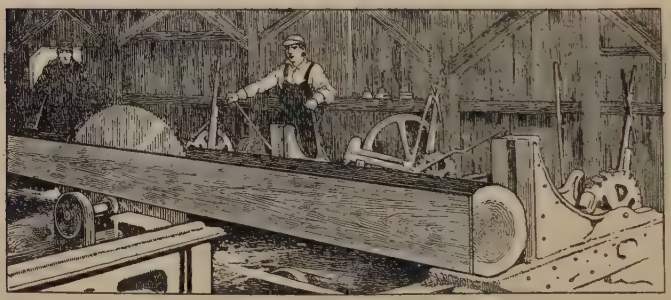
8. How much will the flooring for a house cost, taking the measurement of each floor as 28′ × 32′, and allowing for two floors, the lumber costing $40 per M?

One good way of treating the general topic of practical measurements is to work out problems in connection with building and equipping a house or the school building. If a building is being erected in the neighborhood it should be used for this work in order that the problems may be real.

**25. Measuring Lumber in the Log.** We could find by careful measurement almost the exact number of cubic inches in a log, but for practical purposes a rule known as Doyle's Rule gives the number of board feet with sufficient accuracy.

*From the number of inches in the smallest diameter of the log take 4, multiply the remainder by half itself, multiply this result by the number of feet in the length of the log, and divide by 8.*

The rule need not be learned, but it may be used in solving the problems.



For example, find the number of feet of lumber in a log 16 ft. long, with its smallest diameter 18 in. Since $18 - 4 = 14$, and $\frac{1}{2}$ of 14 is 7, the work is as here shown. The result is 196 ft. Here, as in all similar cases, cancellation should be employed.

$$\frac{\overset{2}{\cancel{14} \times 7 \times \cancel{16}}}{\underset{1}{\cancel{8}}} = 196$$

### WRITTEN EXERCISE

*Determine the number of feet, board measure, of lumber in logs of the following lengths and smallest diameters:*

| | | |
|---|---|---|
| **1.** 24′, 12″. | **4.** 18′, 18″. | **7.** 28′, 30″. |
| **2.** 20′, 14″. | **5.** 16′, 24″. | **8.** 26′, 18″. |
| **3.** 26′, 16″. | **6.** 20′, 22″. | **9.** 22′, 20″. |

**26. Plastering.** The unit for estimating the cost of plastering is usually the square yard. The rule varies greatly in different parts of the country. Teachers should explain to the pupils the method in common use in their vicinity.

### ORAL EXERCISE

1. If a room is 9 ft. high and 20 ft. square, what is the area of the four walls, not allowing for openings?

2. What is the area of the ceiling of a room 18 ft. by 20 ft.? How many square yards in the ceiling?

3. What will it cost to plaster 60 sq. yd. at 30¢ a square yard? at 33¢ a square yard? at 35¢ a square yard?

4. How many square yards of plastering are needed for the two sides and ceiling of a corridor 30 ft. long, 4 ft. wide, and 9 ft. high, not allowing for any openings?

### WRITTEN EXERCISE

1. How many square yards of plastering are there in the walls and ceiling of a room 16 ft. by 20 ft., and 9 ft. high, allowing 14 sq. yd. for openings and baseboard?

2. How many square yards of plastering are there in the walls and ceiling of a hall 60 ft. long, 40 ft. wide, and 28 ft. high, allowing 36 sq. yd. for openings and baseboard?

3. How much will it cost to plaster a schoolroom 32 ft. long, 18 ft. wide, and 12 ft. high, at 32¢ a square yard, making an allowance of 30% for blackboards, baseboard, and openings?

4. On account of danger from falling plaster, it is desired to use pressed-steel ceilings in a schoolhouse. There are 742 sq. yd. of ceiling, and a contract is made for putting up steel ceilings at 10¢ a square foot. How much will the ceilings cost?

**27. Carpeting.** Carpet is usually either 27 in. or 1 yd. wide, and in estimating the amount needed for a room, allowance should in general be made for matching the pattern.

In allowing for waste in matching the pattern, no allowance need be made for loss on the first strip, since the other strips are cut to match that one. In these examples the strips are to run lengthwise of the room.

### ORAL EXERCISE

1. How many strips of $\frac{3}{4}$-yard carpet are needed for a room 18 ft. wide, not allowing for matching the pattern?

2. A carpet has an 18-inch pattern. Will there be any loss in matching if the strips are 15 ft. long? if they are 14 ft. long?

### WRITTEN EXERCISE

1. How many yards of carpet, $\frac{1}{2}$-yard pattern, are needed for a room 13 ft. 6 in. wide and 16 ft. long, the carpet being 27 in. wide? How much is wasted on each strip? Draw a plan.

*Find how many yards of carpet are needed for the following, allowing for matching each strip, except the first, as indicated:*

2. 18 ft. by 20 ft., 27-inch carpet, allowing 9 in.
3. 16 ft. by 18 ft., 27-inch carpet, allowing $\frac{1}{2}$ yd.
4. 14 ft. by 15 ft., 36-inch carpet, allowing $\frac{1}{4}$ yd.

*Find the cost, at $1.20 a yard, of carpeting these rooms with 27-inch carpet, allowing 4 inches on each strip, except the first:*

5. 15 ft. by 18 ft.
6. 14 ft. 3 in. by 18 ft.
7. 12 ft. by 13 ft. 6 in.
8. 17 ft. 6 in. by 18 ft.
9. $13\frac{1}{2}$ ft. by 16 ft. 3 in.
10. 4 ft. 6 in. by 23 ft. 8 in.
11. 15 ft. 9 in. by 25 ft. 6 in.
12. 22 ft. 3 in. by 28 ft. 6 in.

**28. Papering.** The standard width for estimating wall paper is 18 in., although it varies greatly. A single roll is considered to contain 8 yd. of paper. In estimating, paper hangers usually count 3 rolls to 100 sq. ft. Parts of rolls cannot be bought.

### ORAL EXERCISE

1. What will the border for a room 16 ft. by 20 ft. cost, at 25¢ a yard?

2. Estimate the cost of the paper for a ceiling 15 ft. by 20 ft., at 40¢ a roll.

3. A room has nearly 500 sq. ft. of wall to be papered. Estimate the cost of the paper at 50¢ a roll.

4. A plain wall 14 ft. long and 9 ft. high above the baseboard is to be papered. How many rolls should you allow?

### WRITTEN EXERCISE

1. The walls of a room are 8 ft. 6 in. above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40¢ a roll?

2. How many rolls of plain paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, the paper being 24 in. wide, allowing $1\frac{1}{2}$ rolls for windows and doors? At 35¢ a roll, what will it cost?

3. The walls of a room are to be papered with plain paper worth 30¢ a roll. They are 9 ft. high above the baseboard, and the total length of the four walls, allowing for windows and doors, is 60 ft. At the usual estimate, what will the paper cost?

The cost of papering rooms at home should be considered.

### WRITTEN EXERCISE

**1.** The house described on page 46 was built on a lot 100 ft. wide and 200 ft. deep. The owner took a piece 40 ft. by 80 ft. for a vegetable garden, and one 60 ft. by 80 ft. for a fruit orchard. How many square feet were there in each of these two pieces?

**2.** Express the area of the lot and of all three parts as fractions of an acre.

**3.** What was the area occupied by the house?

**4.** The excavation for the cellar was 5 ft. deep. How much earth was removed?

**5.** If the excavation for the cellar had been 6 ft. 4 in. deep, how much earth would have been removed?

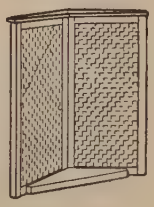
**6.** If the owner fences the garden, the orchard, and the part on which his residence stands, how many feet of fencing will he need, not allowing for gates? how many rods of fencing? how many yards of fencing?

**7.** The owner planted a bed of asparagus 4 ft. wide and 36 ft. long; two beds of parsnips and two of beets, each $1\frac{1}{2}$ ft. wide and 36 ft. long; two beds of tomatoes and two of squash, each 3 ft. wide and 36 ft. long. How many square feet of area did he give to each of the beds?

**8.** He planted shrubs about the lawn. These, with the walks, occupied 2200 sq. ft. How many square feet were left for the lawn? See if you can improve this design.

### DRAWING PLANS

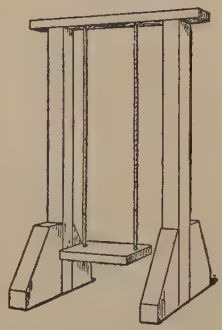
**1.** A class in school made this lamp screen. It is $18\frac{3}{4}''$ high, and each wing is $9\frac{3}{4}''$ wide. Draw a plan of either wing of the screen to the scale $\frac{1}{2}$.



**2.** What is the total area of the side of the screen shown in the figure?

**3.** The woodwork is filled in with plaited ribbon, the spaces being $16\frac{1}{4}''$ by $7\frac{3}{4}''$. Draw a plan to the scale $\frac{1}{2}$, and find the sum of the two areas.

**4.** Jack made a swing for his sister's doll house. The crosspiece at the top is $7''$ long, $1''$ wide, and $\frac{1}{2}''$ thick. Make a drawing of the top of this crosspiece, first full size, and then to the scale $\frac{1}{2}$.



It may be necessary to remind the class again that drawing to the scale $\frac{1}{2}$ means that every inch is to be represented in the drawing by $\frac{1}{2}''$. All drawings should be made with a ruler and a sharp pencil and should look neat and clean.

**5.** The seat in the swing is $3''$ long, $2''$ wide, and $\frac{1}{2}''$ thick. Make three full-size drawings, one showing the seat as seen from the top, one showing it as seen from the front, and one showing it as seen from the end.

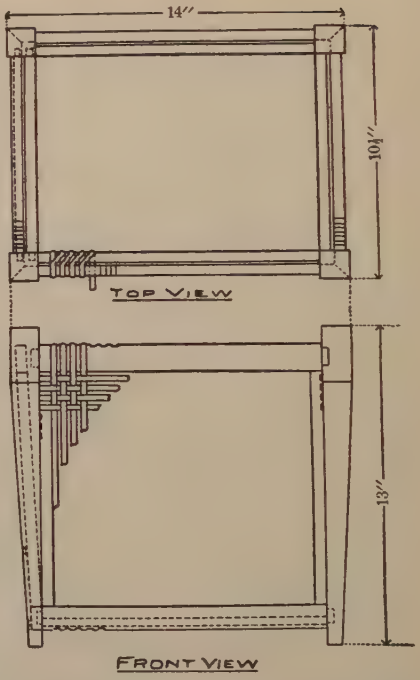
**6.** Measure the top of your desk to the nearest inch. Make a drawing of the top, as seen from above, using the scale $\frac{1}{8}$.

**7.** Measure this page of the book. Make a drawing of the page, disregarding the print, using the scale $\frac{1}{4}$.

**8.** A room is $24'$ long and $16'$ wide. Draw a plan of the floor to the scale $1''$ to $8'$.

Pupils may be required to measure their living rooms at home and to draw a plan of the floor to some convenient scale.

**9.** The boys in the class made a wastebasket, these being the working drawings for the top and the front. Measure the drawings and determine the scale that was used.

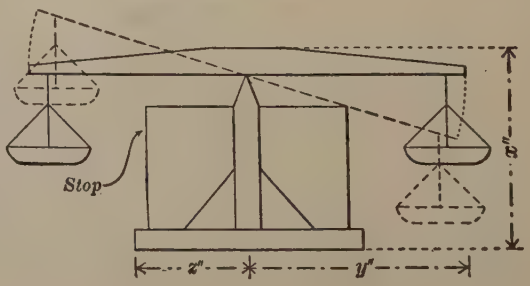



**10.** Taking the length and width as given, find the perimeter at the top, expressing it in feet and inches.

**11.** The height of the basket is how many inches more than the width? By what fraction of the width is the height greater than the width?

**12.** Measure the area to be filled in by wickerwork in the front of the basket. This is easily done because the scale was determined in Ex. 9, so you have only to measure the figure carefully, find the width and height, and then find the area.

**13.** The boys also made some wooden balances. These are the drawings, and they are made to the scale $\frac{1}{8}$. Determine the lengths marked $x$, $y$, and $z$.



*Using a ruler and a sharp pencil, make drawings of the following to any convenient scale:*

**14.** The front of your schoolroom, showing the blackboard and any doors or windows.

This will require measuring the schoolroom. In general, it is better to have the pupils take their own measurements, but if necessary the teacher may give the measurements to the class.

**15.** The floor plan of your schoolroom, showing the position and size of the platform, if there is one.

**16.** The school grounds, showing the location and size of the schoolhouse.

The measurements necessary for this drawing may take a whole class period or longer, but the time is profitably spent in this way. The pupils should first make a rough drawing of the plan, and should write upon it the necessary measures as they are taken, these measurements having been decided upon before the class goes outdoors.

**17.** Make a drawing, not to scale, of the front elevation of your schoolhouse, showing the position of the doors and windows.

The teacher should explain that the front elevation is the picture of the front of the building, not in perspective and without shading; that the side elevation is a similar picture of the side of the building; and that the plan means the ground plan. Such terms need not be strictly defined, but they should be used intelligently.

**18.** Draw a plan of the house in which you live and bring it to school to-morrow. Show both the first, or ground, floor, and the floor above it (the second floor).

**19.** Draw the side elevation of the schoolhouse.

**20.** Draw a plan of the first floor of the schoolhouse.

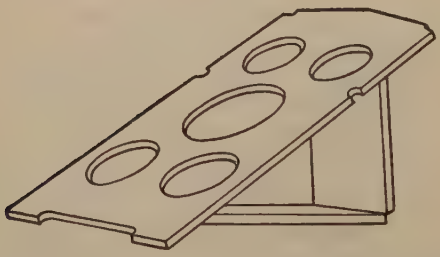
**21.** Let the boys draw a plan of a baseball field, showing the diamond, pitcher's box, and the position of the players.

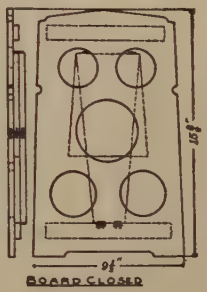
**22.** Let the boys draw a plan of a football field, showing the location of the goal posts, the two 25-yard lines, and the places for the kick-offs.

### HANDWORK

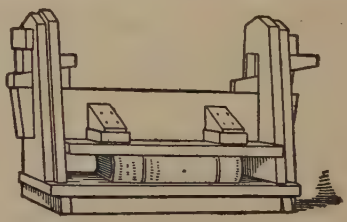
1. The boys made some bookracks. For the shelf they used a piece of wood 1′ 5″ long. How long a piece would they need for 7 shelves?

2. The boys of the class made this bean-bag board for the girls to use at recess. It was 1′ $3\frac{3}{4}$″ long. How long a piece of lumber would be needed to make the fronts of 2 such boards?






3. In making this bookpress the class found that 5 pieces of board 14″ long were needed. How many feet long was the board needed to make these pieces?



4. One piece $18\frac{3}{8}$″ long and one $20\frac{3}{4}$″ long were needed for the bookpress. Express the sum of these lengths in feet and inches.

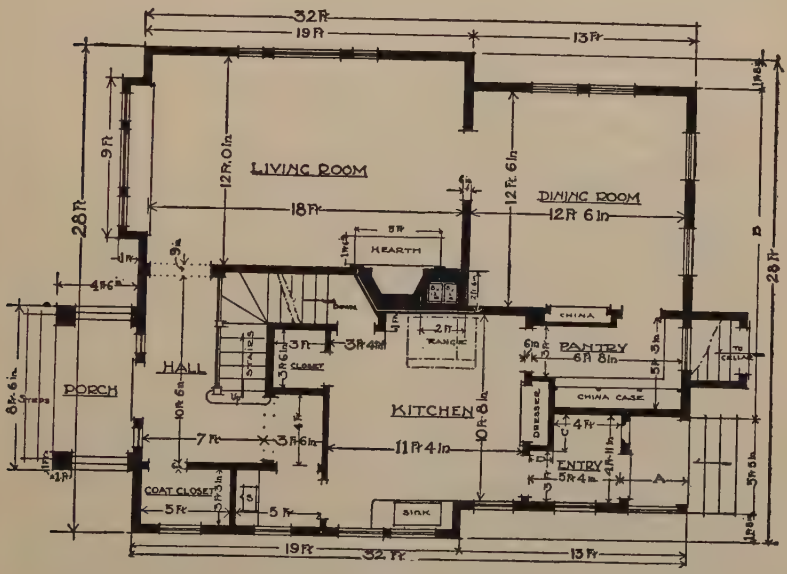
5. Each piece was $10\frac{3}{4}$″ wide. This is what part of a foot? Express the result both as a common fraction and as a decimal.

6. Using 10-inch board, $\frac{7}{8}$″ thick, for the bookpress, what is the cost of 5 pieces 14″ long, 1 piece 18″ long, and one piece 20″ long, at $85 per M for best oak?

## HOUSE PLANS

**1.** In this floor plan of a house, how many board feet of lumber, 1″ thick, are needed for the dining-room floor, allowing $\frac{1}{5}$ extra for waste and joining?

**2.** In the same plan, how many square feet of floor space are there in the living room, not allowing for fireplace and hearth?




**3.** In the same plan, how many square feet of plaster must be allowed for the ceilings of the pantry and of the dining room?

**4** In the same plan, how many cubic feet are there in the coat closet, the ceiling being 10 ft. above the floor?

**5.** In the same plan, how many square inches of tiling are needed for the hearth in the living room?

### BUILDING A BARN

**1.** A farmer employs a carpenter to build a barn. The carpenter figures on the following as part of the materials: 4 pieces $2'' \times 10'' \times 18'$; 8 pieces $2'' \times 10'' \times 10'$; 8 pieces $2'' \times 8'' \times 24'$; 36 pieces $2'' \times 10'' \times 14'$; 60 pieces $2'' \times 8'' \times 22'$; 12 pieces $2'' \times 8'' \times 20'$; 10 pieces $2'' \times 8'' \times 18'$; 30 pieces $2'' \times 8'' \times 16'$; 220 pieces $2'' \times 8'' \times 12'$; 20 pieces $2'' \times 8'' \times 10'$; 40 pieces $2'' \times 8'' \times 8'$; 22 pieces $2'' \times 6'' \times 24'$; 8 pieces $2'' \times 6'' \times 12'$; 5000 sq. ft. of siding $1''$ thick; 22,000 shingles. Find the number of feet of lumber (board measure), its cost at \$42 per M, and the cost of the shingles at \$5.10 per M.

**2.** The carpenter also figured on needing 100 lb. of 4d nails, 140 lb. of 8d nails, and 650 lb. of spikes of various sizes. At 6¢ per pound, what was the cost of the nails and spikes?

A 4d nail is called a fourpenny nail, an old name which has nothing to do with the cost. Specimens may profitably be brought to class.

**3.** The carpenter bought his windows ready made, needing ten at \$2.60 each, and allowed \$30 for foundation work. He also bought the necessary hardware, such as door hinges, latches, hangers, tracks for the large doors, and window fasteners, the bill amounting to \$12.75. What was the cost of this material?

**4.** The carpenter figured his work at \$3.75 a day, and the work of another man at \$2.50 a day, both for a period of 37 days. How much was allowed for this labor?

**5.** The farmer decided to put in a concrete floor. The floor area was 1860 sq. ft. and the floor was 4 in. thick. How many cubic yards of concrete did he require? If 1 cu. yd. of the concrete required 1.3 bbl. of cement at \$2.25 per barrel, how much would the cement for the floor cost?

In such cases figure as a whole barrel any fraction of a barrel in the total.

**29. Ratio.** The relation of one number to another number of the same kind, as expressed by division, is called the *ratio* of the first to the second. Without using the name, we studied about ratio in studying fractions.

The ratio of \$2 to \$4 is $\frac{2}{4}$, or, in its simplest form, $\frac{1}{2}$; the ratio of 6 ft. to 5 ft. is $\frac{6}{5}$; the ratio of 10 mi. to 2 mi. is $\frac{10}{2}$, or 5.

In the exercise below, when we speak of the ratio of $E$ to $D$ we mean the ratio of their number values; that is, of 1 to 2, the ratio being $\frac{1}{2}$. When we speak of the ratio of $E$ to $2B$ we mean the ratio of 1 to $2 \times 4$, which is $\frac{1}{8}$.

### ORAL EXERCISE

**1.** In this picture, what is the ratio of $E$ to $D$? of $E$ to $C$?

**2.** In the same picture, what is the ratio of $E$ to $B$? of $E$ to $A$?

*Referring to the picture, state the following ratios:*

**3.** $E$ to $2B$ $A$ to $E$ $D$ to $C$

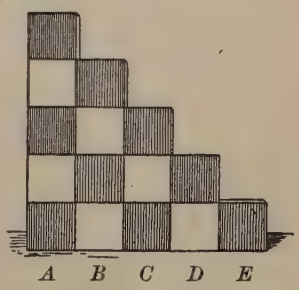
**4.** $D$ to $2A$ $C$ to $B$ $C$ to $A$

*State also the following ratios:*

**5.** $2E$ to $D$ $C$ to $3E$ $4E$ to $A$

**6.** $2D$ to $B$ $A$ to $2D$ $5E$ to $A$

**7.** What is the ratio of any two equal numbers?




**8.** In the picture, what is the ratio of $A$ to $B+E$? of $A$ to $B+C$? of $B$ to $C+E$? of $B$ to $C+D+E$?

The old method of solving business problems by ratio and proportion is no longer used to any extent. The subject of ratio has, however, a value of its own, and there is a special field in which proportion is peculiarly useful.

In this book we use the letter $x$ to represent the number to be found, but teachers may prefer to use an interrogation point (?), vacant parentheses ( ), or an asterisk (*). The use of the equation is considered in the Supplement.

**30. Writing a Ratio.** A ratio may be written like a fraction, $\frac{2 \text{ ft.}}{3 \text{ ft.}}$, or with a sign of ratio (:), as 2 ft. : 3 ft.

We think of the ratio of 20 ft. to 4 ft. as $\frac{20 \text{ ft.}}{4 \text{ ft.}}$, or 20 ft. : 4 ft., or $\frac{20}{4}$, or 20 : 4, or simply 5. The word *ratio* is used for all.

**31. Ratios always Abstract.** Any number divided by a like number, as feet by feet, has an abstract quotient. Therefore,

*A ratio is always abstract, and its terms may be written as abstract numbers.*

Teachers should use the fractional form at first, because the pupils are already familiar with common fractions. Teachers should also recall the fact that all number is essentially abstract. In work in ratios it is better not to label with any denomination the numbers involved.

### ORAL EXERCISE

*Find the value of the missing number, designated by $x$, in each of the following ratios:*

1. $\frac{x}{2} = 10.$
2. $\frac{x}{3} = 3.$
3. $\frac{x}{4} = 2.$
4. $\frac{x}{5} = 3.$
5. $\frac{x}{10} = 1.$
6. $\frac{x}{10} = \frac{1}{2}.$
7. $\frac{x}{7} = \frac{3}{7}.$
8. $\frac{x}{6} = \frac{2}{3}.$
9. $\frac{x}{5} = \frac{4}{5}.$
10. $\frac{2}{x} = \frac{1}{3}.$
11. $\frac{20}{x} = 0.4.$
12. $\frac{10}{x} = 1.$

13. What is the number whose ratio to 5 is 6?

14. What is the number to which 75 has the ratio 3?

15. What is the number whose ratio to $72\frac{7}{8}$ is 1?

16. The ratio of the height of a certain kitchen door to the width is $3\frac{1}{2}$. The door is 7 ft. high. How wide is it?

17. The ratio of the height of Fred's father to Fred's height is $\frac{3}{2}$. Fred's father is 6 ft. tall. How tall is Fred?

**32. Separating a Number in a Given Ratio.** Required to separate 25¢ in the ratio of 2 to 3.

With 2 parts in one group and 3 in the other, there are 5 parts in all.
Since one group has $\frac{2}{5}$, it has $\frac{2}{5}$ of 25¢, or 10¢.
The other group has $\frac{3}{5}$ of 25¢, or 15¢.
*Check.* $10¢ + 15¢ = 25¢$; $10 : 15 = 2 : 3$.

### ORAL EXERCISE

1. Separate 9 in the ratio of 2 to 1.

2. What is the ratio of the number of boys in your class to the number of girls?

3. What is the ratio of the time you are in school to the whole 24 hours of the day?

4. What is the ratio of the length of summer, 3 months, to the whole year? to the remainder of the year?

5. There were 6 cloudy days in September, the rest being clear. What was the ratio of cloudy days to clear days?

6. In playing ringtoss, Will has a score of 15 and Mollie has one of 18. What is the ratio of Will's score to Mollie's?

### WRITTEN EXERCISE

1. Divide \$100 in the ratio of 4 : 1; of 7 : 3; of 1 : 9.

2. In a year having 165 cloudy days and 200 clear days, what is the ratio of cloudy days to clear days?

3. In November the ratio of clear days to cloudy days was 3 : 2. How many were there of each?

4. Four boys arrange a three weeks' camping trip. The first stays 18 da.; the second comes late and stays 12 da.; the third is in camp the last two weeks; the fourth stays the three weeks. The total expense is \$32.50. What is the share of each?

**33. Proportion.** An expression of equality between two ratios is called a *proportion.*

For example, the ratio \$2 : \$3 equals the ratio 10 ft. : 15 ft. Therefore

$$\$2 : \$3 = 10 \text{ ft.} : 15 \text{ ft.}$$

is a proportion. It is read, "\$2 is to \$3 as 10 ft. is to 15 ft." It may, of course, be written simply $2 : 3 = 10 : 15$, or $\frac{2}{3} = \frac{10}{15}$.

**34. Extremes and Means.** The first and last terms of a proportion are called the *extremes;* the second and third terms are called the *means.*

In the proportion $\frac{3}{7} = \frac{15}{35}$,

we see that $7 \times 15 = 3 \times 35$.

That is, in any proportion,

*The product of the means equals the product of the extremes.*

We also see that if $x : 21 = 35 : 105$,

or $$\frac{x}{21} = \frac{35}{105},$$

then $$x = \frac{21 \times 35}{105}.$$

Therefore, in any proportion,

*The product of the means divided by one extreme equals the other extreme.*

1. If $x : 7 = 13 : 21$, what is the value of $x$?

We have $$x = \frac{7 \times 13}{21} = 4\tfrac{1}{3}.$$

2. If $21.3 : 17.1 = 3 : x$, find the value of $x$.

We have $$x = \frac{3 \times 17.1}{21.3} = 2\tfrac{34}{71}.$$
$$7.1$$

### WRITTEN EXERCISE

*Find the value of $x$ in each of the following proportions:*

1. $x : 51 = 12 : 17$.
2. $x : 17 = 54 : 51$.
3. $x : 22 = 41 : 11$.
4. $x : 250 = 35 : 50$.
5. $x : 27 = 36 : 243$.
6. $x : 1001 = 8 : 143$.
7. $24 : x = 3 : 20$.
8. $35 : x = 7 : 91$.
9. $111 : x = 37 : 38$.
10. $19 : x = 209 : 143$.
11. $x : 18.2 = 9.1 : 7.3$.
12. $x : 4.1 = 82.7 : 82$.
13. $x : 21.7 = 2.4 : 3.3$.
14. $x : 1.37 = 2.4 : 13.7$.
15. $x : 3.21 = 4.12 : 6.6$.
16. $x : 0.32 = 7.21 : 1.8$.
17. $3.17 : x = 4.28 : 3.2$.
18. $2.4 : x = 3.8 : 123$.
19. $8.21 : 3 = 4.105 : x$.
20. $3.9 : 4 = x : 3200$.

**21.** I am thinking of a number which has to 17 the ratio 35 : 119. What is the number?

**22.** A miller uses 18 bu. of wheat for making 4 bbl. of flour. How many barrels of flour can he make from 207 bu. of wheat?

$$x : 4 = 207 : 18.$$

**23.** A certain circle has a circumference of 183.33 in. What is the circumference of a circle whose radius is to that of the first as 1.7 : 2.91?

The ratio of two circumferences equals the ratio of their radii. That is, in this example, $x : 183.33 = 1.7 : 2.91$.

**24.** If 4 men can do a piece of work in 10 da., how many men would it take to do the work in 8 da.?

In all such problems the words "at the same rate" are evidently to be understood.

It will take more men to do the work in fewer days. Therefore $x$ must be greater than 4, and the proportion is $x : 4 = 10 : 8$. Solving, $x = 5$. Therefore 5 men can do the work in 8 da.

**25.** How long will it take a train to go 148 mi., at the rate of $55\frac{1}{2}$ mi. in 1 hr. 30 min.?

**26.** If 9 men can complete some work in 20 hr., how many less men can complete it in 16 hr. more time?

**27.** If a man can skate 75 yd. in $8\frac{1}{5}$ sec., how long would it take him to skate 200 yd. if he could maintain this rate?

**28.** If a man can skate 220 yd. in 19 sec., how long would it take him to skate 5 mi. if he could maintain this rate?

**29.** If 750 lb. can be transported 100 mi. for a certain sum, how far should $\frac{1}{2}$ T. be transported for the same sum?

**30.** If 6 men can complete some work in 15 hr., how many men must be added to complete it in 5 hr. less time?

**31.** If 10 men can complete some work in 30 hr., how many men must be added to complete it in 10 hr. less time?

**32.** If a crew is rowing at the average rate of 1 mi. in 5 min. 30 sec., how long will it take to row 1 mi. 1540 yd.?

**33.** If a man can swim 45 yd. in 23 sec., how long would it take him to swim 2 mi. if he could maintain this rate?

**34.** A prize of \$60 was divided between two persons in the ratio of $\frac{2}{3}$ to $\frac{5}{6}$. How much was the share of each?

**35.** A map is drawn to the scale of 1 in. to 0.8 mi. How many acres of land are represented by a portion of the map 1 in. square? by a portion 3 in. square?

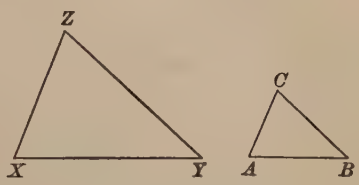
**36.** When a sum of money is divided equally among 7 persons, each receives \$16.80. How much would each receive if the same sum were divided equally among 8 persons?

**37.** One man can do a piece of work in 10 da., and another can do the same work in 9 da. If the wages of the first are \$3.60 a day, what should be the wages of the second?

**35. Proportional Numbers.** Numbers which form a proportion are called *proportional numbers* and are said to be *proportional.*

**36. Similar Figures.** Figures which are of exactly the same shape are called *similar figures* and are said to be *similar.*

For example, these two triangles are similar.




**37. Proportional Lines.** The lengths of corresponding lines in similar figures are proportional.

We express this more briefly by saying that the lines are proportional.

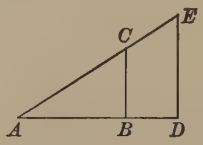
For example, in the above triangles $XY : YZ = AB : BC$, where $XY$ represents the numerical length of the line $XY$; in two circles the circumferences and radii are proportional, the circumference of the first being to that of the second as the radius of the first is to that of the second.

### ORAL EXERCISE

1. In the above triangles, if $XY$ is twice as long as $AB$, how does $ZX$ compare in length with $CA$?

2. In the same triangles, state four proportions that exist among the sides $AB$, $BC$, $XY$, and $YZ$.

3. In this figure, state two proportions that exist among $AB$, $AD$, $AC$, and $AE$?




4. In this figure, if $AB$ is $\frac{2}{3}$ of $AD$, what is the ratio of $AC$ to $AE$?

5. In the same figure, if $DE$ represents the height of a man 6 ft. tall, $BC$ the height of a boy, $DA$ the length of the shadow cast by the man, and $BA$ the length of the shadow cast by the boy, show how to find the height of the boy by measuring the lengths of the shadows.

**38. Measuring Heights and Distances.** The height of a tree is often measured by proportion, as shown below.

The man holds a right triangle in which $AB = BC$. He holds $AB$ level and walks toward the tree until he just sees the top along $AC$. Then because

$$AB = BC,$$

and

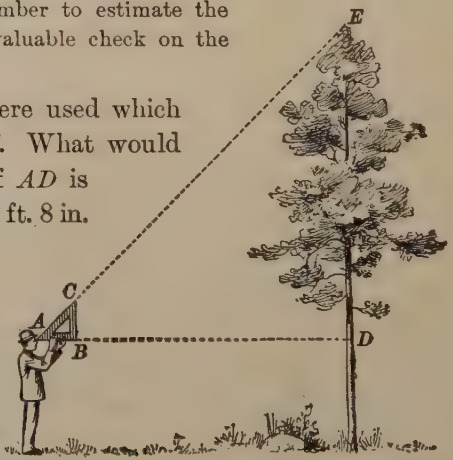
$$AB : BC = AD : DE,$$

the height *above* $D$ will equal the distance $AD$.

### WRITTEN EXERCISE

**1.** What is the height of the tree in the picture if the triangle is 5 ft. 4 in. from the ground, and $AD$ is 23 ft. 8 in.?

In all such problems remember to estimate the result in advance. This is a valuable check on the accuracy of the work.




**2.** Suppose a triangle were used which had $AB$ equal to twice $BC$. What would be the height of the tree if $AD$ is 75 ft., and if the point $D$ is 5 ft. 8 in. above the ground?

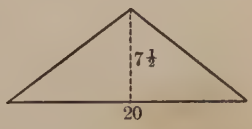
**3.** Suppose the triangle used has $AB = 2$ ft., $BC = 1$ ft. 6 in. What is the height of the tree if $AD$ is equal to 4 rd., and if $D$ is 5 ft. 6 in. above the ground? if $AD$ is equal to 61 ft., and if $D$ is 5 ft. 3 in. above the ground?

**4.** What is the height of a tree that casts a shadow 88 ft. long at the same time that a man 5 ft. 9 in. high casts a shadow 7 ft. 8 in. long?

**5.** A boy whose eye is 5 ft. above the ground sights over the top of a 10-foot pole and just sees the top of a tree. The boy is 6 ft. from the pole and 42 ft. from the tree. How high is the tree? Draw the picture.

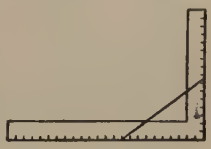
**6.** To find the height of a church spire a boy measured its shadow, and found it to be 152 ft. 3 in. At the same time he found that a post 3 ft. 6 in. high was casting a shadow 5 ft. 3 in. long. How high was the spire?

**7.** A carpenter wishes to construct a roof as here shown, the house being 20 ft. wide and the gable $7\frac{1}{2}$ ft. high. He wishes to know the length of slope. He draws a plan to the scale $\frac{1}{30}$. How many inches on the plan will represent the $7\frac{1}{2}$ ft., or "rise," as the carpenter calls it?




**8.** The carpenter found that the plan of Ex. 7 was not convenient, so he drew a right triangle representing half the plan, and represented by a line 10 in. long the 10 ft., or "run," as he calls it. Draw such a plan, representing the $7\frac{1}{2}$ ft. correctly, and measure the slope. What did the carpenter find to be the length of the slope of the roof?

**9.** But practically the carpenter would proceed in another way. He would take his square, and on the tongue, or short arm, he would take a point $7\frac{1}{2}$ in. from the corner; then on the blade, or long arm, he would take a point 10 in. from the corner. He would then measure the distance between these points, and the number of inches would be the same as the number of feet in the slope of the roof. Draw the figure to scale and write out the reason why this plan gives the correct result.



**10.** Two stockmen, A and B, maintain a common drinking trough and share the annual cost of repairs in proportion to the number of cattle owned by each. A has 1253 head of cattle and B has 2747. If the annual cost of repairs is \$80, how much should each contribute?

**11.** If $\frac{3}{4}$ in. on a map represents a distance of 750 mi., how many miles will $2\frac{1}{2}$ in. represent?

**12.** If a distance of 650 mi. is represented on a map by $1\frac{5}{8}$ in., how many inches will represent a distance of 680 mi.?

**13.** If a tree casts a shadow 60 ft. long when a post 4 ft. high casts a shadow 4.8 ft. long, how high is the tree?

**14.** If a spire casts a shadow 200 ft. long when a post $4\frac{1}{2}$ ft. high casts a shadow $5\frac{5}{8}$ ft. long, how high is the spire?

**15.** In finding the height of the school building, one boy stood a yard measure upright and another boy lay down with his eye 4 in. from the ground where he could just see the top of the building in line with the top of the stick. The distance from his eye to the stick was 4 ft. 6 in., and the distance to the building was 45 ft. How high was the building?

**16.** If a building casts a shadow 75 ft. long when a post 6 ft. 2 in. high casts a shadow 12 ft. 6 in. long, how high is the building?

**17.** Edward and Henry wished to find the height of the school building. Edward suggested standing a yardstick upright, measuring its shadow, and at the same time measuring the shadow of the school. The yardstick shadow was 4 ft., and the shadow of the building was 36 ft. How high was the school building? Draw a picture.

It is very easy to measure the height of trees and buildings in this way, or by sighting from the ground across the top of a pole to the top of a building, and pupils will find such work interesting and profitable.

**18.** These boys find that this post is 3 ft. 6 in. high and casts a shadow 3 ft. 2 in. long. The shadow of the tree is 34 ft. 10 in. long at the same time. Required the height of the tree.

**19.** If a building casts a shadow 68 ft. long at the same time that a post 4 ft. 2 in. high casts a shadow 8 ft. 6 in. long, how high is the building?

**20.** If the railway fare for a journey of 75 mi. is \$1.50, what will be the fare for 275 mi.?

Are you making use of short methods in your work whenever possible?

**21.** Two boys weighing respectively 100 lb. and 80 lb. sit 9 ft. apart on the ends of the plank of a see-saw. Not counting the weight of the plank, how far from the heavier boy must the stick on which the see-saw rests be placed so that they just balance?

**22.** How much pressure must you exert on the handles of a pair of shears just 3 in. from the *fulcrum*, or screw, to exert a pressure of 5 lb. at a point 5 in. from the fulcrum?

**23.** Three men pay \$848 for some water power. The first uses 28 horse power, the second 36 horse power, and the third 42 horse power. How much should each pay? How much should each pay if the total amount were \$1007?

## LITTLE EXAMINATIONS

I. 1. 17 rd. = (?) ft.
2. $\frac{3}{4}$ mi. = (?) rd.
3. 227 yd. = (?) ft.
4. $\frac{1}{4}$ sq. mi. = (?) A.
5. 320 A. = (?) sq. mi.
6. 216 sq. in. = (?) sq. ft.
7. 27 sq. yd. = (?) sq. ft.
8. 7 cd. = (?) cu. ft.
9. 864 cu. in. = (?) cu. ft.
10. 121 sq. yd. = (?) sq. rd.

II. *Express each of the following ratios in simplest form:*

1. $\frac{34}{51}$. 2. $\frac{46}{115}$. 3. $\frac{\frac{4}{5}}{6}$. 4. $\frac{13}{3\frac{1}{4}}$. 5. 58:145.

*Find the missing term x in each of the following proportions:*

6. $\frac{x}{4} = 2$. 7. $\frac{x}{9} = \frac{1}{3}$. 8. $\frac{10}{x} = \frac{5}{3}$. 9. $\frac{x}{9} = 2$.

10. Two men rent a pasture for \$132. One puts in 7 cattle and the other 15. How much rent should each pay?

III. *Express each of the following ratios in simplest form:*

1. $\frac{57}{76}$. 2. $\frac{2}{\frac{3}{4}}$. 3. $\frac{\frac{1}{2}}{\frac{7}{8}}$. 4. $\frac{2\frac{1}{2}}{10}$. 5. 93:124.

*Find the missing term x in each of the following proportions:*

6. $\frac{6}{x} = 2$. 7. $\frac{5}{x} = \frac{1}{4}$. 8. $\frac{7}{x} = \frac{1}{5}$. 9. $\frac{7}{x} = \frac{1}{2}$.

10. If the ratio of a man's income to his expenses is 9 : 8 and his expenses are \$2400 a year, how much is his income?

## REVIEW AND DRILL

### I. REVIEW SECTION

*Add the following:*

| 1. | 2. | 3. | 4. |
|---|---|---|---|
| \$20.07 | 20 ft. 7 in. | 20 lb. 7 oz. | 20 yd. 7 in. |
| 8.09 | 8 9 | 8 9 | 8 9 |
| 7.11 | 7 11 | 7 11 | 7 11 |
| 16.10 | 16 10 | 16 10 | 16 10 |

The pupil should see that the figures in each set of examples are the same, but that the results differ. The pupil should explain the reason for this difference.

*Subtract the following:*

| 5. | 6. | 7. | 8. |
|---|---|---|---|
| \$30.01 | 30 ft. 1 in. | 30 lb. 1 oz. | 30 yd. 1 in. |
| 16.09 | 16 9 | 16 9 | 16 9 |

*Multiply the following:*

| 9. | 10. | 11. | 12. |
|---|---|---|---|
| \$24.08 | 24 ft. 8 in. | 24 lb. 8 oz. | 24 yd. 8 in. |
| $23\frac{1}{2}$ | $23\frac{1}{2}$ | $23\frac{1}{2}$ | $23\frac{1}{2}$ |

*Divide the following, carrying each quotient to two decimal places:*

13. \$26.07 ÷ 9.
14. 26 lb. 7 oz. ÷ 9.
15. 26 ft. 7 in. ÷ 9.
16. 26 pk. 7 qt. ÷ 9.
17. 26 yd. 7 in. ÷ 9.
18. 26 mi. 7 rd. ÷ 9.
19. 26 sq. yd. 7 sq. in. ÷ 9.
20. 26 cu. ft. 7 cu. in. ÷ 9.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

**1.** If you know the width of a sidewalk in inches, how do you find the width in feet? in yards?

**2.** If you know the area of a table top in square inches, how do you find the area in square feet? in square yards?

**3.** If you know the number of feet in the length, width, and depth of a cellar, how do you find the number of cubic yards?

In computing excavations the cubic yard is the common unit.

**4.** If you know the number of rods in the length and width of a rectangular field, how do you find the number of acres?

**5.** How would you figure the cost of the lumber used in building a barn in your vicinity?

**6.** How would you figure the cost of a concrete walk in front of the schoolhouse?

**7.** Suppose that your school has a playground in the form of a rectangle, how would you find the area in acres?

**8.** If you know the weight of two volumes of a substance and one of the volumes, how do you state the proportion?

**9.** Knowing the number of men required to do a piece of work in a given number of days, how can you find how many men are required to do it in another given number of days?

**10.** How can you find the height of your school building by means of shadows and the height of a pole?

**11.** How can you find the height of a tree by sighting over the top of a pole of known length?

**12.** If you know the railway fare for a certain number of miles, how do you find the fare for a certain other number of miles?

## III. INDUSTRIAL PROBLEMS

### THE CONCRETE INDUSTRY

**1.** For ordinary work about the farm, as in silos, tanks, cisterns, and fence posts, concrete consists of half as much Portland cement as sand and half as much sand as crushed stone, all measured by bulk. This is called a 1 : 2 : 4 mixture. If 70 cu. ft. of ingredients are used, how many cubic feet are there of each?

**2.** In estimating the material needed for concrete, the contractors allow 0.058 bbl. of cement, 0.0163 cu. yd. of sand, and 0.0326 cu. yd. of stone or gravel to 1 cu. ft. of the finished concrete. Estimate the amount of each material for a concrete wall 1 ft. 3 in. thick, 6 ft. high, and 48 ft. long.

**3.** A farmer is building a silo, and he estimates that he needs 935 cu. ft. of concrete. Taking the amounts given in Ex. 2, how much of each kind of material does he need?

**4.** A man is connecting two barns by a concrete driveway. He finds that he needs 5 bags of Portland cement, $\frac{1}{2}$ cu. yd. of sand, and 1 cu. yd. of crushed stone or screened gravel for 60 sq. ft. of surface, and that these materials will cost \$3.60. How much of each material will he need for a driveway 90 ft. long and 6 ft. wide, and how much will these materials cost?

**5.** A farmer wishes to put down a 6-inch concrete floor 24 ft. by 36 ft. He finds that he needs 20 cu. yd. of screened gravel at \$1.10, 10 cu. yd. of sand at \$1, and 28 bbl. of Portland cement at \$2.50. How much does the material for the floor cost?

**6.** A contractor estimates the cost of a 5-inch concrete floor 32 ft. 6 in. by 48 ft. 6 in. as follows: 30 cu. yd. of screened gravel at \$1.15, 15 cu. yd. of sand at \$1.10, and 42 bbl. of Portland cement at \$2.50. How much is his estimate for materials?

### A LITTLE ABOUT MECHANICS

**1.** Tom bought a copy of a magazine treating of mechanics. He found some things that he could not understand. He asked his father what a foot pound was, and found that it was the work done in lifting a weight of one pound to a height of one foot. His father said: "Your weight is 108 lb. If you step up on the chair, which we will say is 2 ft. from the floor, you lift a weight of 108 lb. to a height of 2 ft., which takes as much work as to lift a weight of 1 lb. to a height of $2 \times 108$ ft., or 216 ft. Then you have done 216 foot pounds of work. How many foot pounds of work will you do in going upstairs?" It is 10 ft. upstairs. What is the answer?

**2.** Tom's father weighs 180 lb. How many foot pounds of work does he do when he goes up the stairs mentioned in Ex. 1?

**3.** "To find the work done in foot pounds, we do not always think of something as being lifted," said Tom's father; "the force may be used in pushing or pulling in any direction. When you pull your sled with a force of 1 lb., you use a foot pound of force when you pull it a foot." "Well," said Tom, "suppose I pull with a force of 8 lb. and draw my sled a quarter of a mile, how many foot pounds do I use?" "That is for you to find out," said his father. What is the answer?

**4.** Tom found something in his magazine about horse power and asked his father what that meant. His father told him that it was the force that equals 33,000 foot pounds per minute; that is, the force necessary to lift 33,000 lb. through a distance of 1 ft. in 1 min. Tom found something about the horse power necessary to raise a block of stone weighing 750 lb. to a height of 30 ft. in 5 min. What is the horse power necessary to do this?

Divide $30 \times 750$ by $5 \times 33{,}000$, always using cancellation in such cases.

## CHAPTER III

### PERCENTAGE AND ITS APPLICATIONS

**39. Per Cent.** As you already know, another name for hundredths is *per cent.*

Thus 0.01, or $\frac{1}{100}$, is the same as 1 per cent; 0.06 is the same as 6 per cent; $0.00\frac{1}{2}$, or $\frac{1}{2}$ of $\frac{1}{100}$, is the same as $\frac{1}{2}$ per cent.

That part of arithmetic which treats of per cent is called *percentage.* The word "percentage" is often used for per cent, as in speaking of the percentage of base hits in a ball game.

**40. Symbol for Per Cent.** We found that the symbol for per cent is written thus: %.

We may read 0.06 either "6 hundredths" or "6 per cent." In the same way, we may think of 6% either as "6 per cent" or as "6 hundredths," although it is read "6 per cent."

The expression 800% means $\frac{800}{100}$ and equals the whole number 8; 225% means the mixed number 2.25, or $2\frac{1}{4}$; $\frac{1}{2}$% means $\frac{1}{2}$ of $\frac{1}{100}$, or $\frac{1}{200}$, and is read either "$\frac{1}{2}$ per cent" or, quite commonly, "$\frac{1}{2}$ of 1%."

**41. Relation to Fractions.** Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

**42. Per Cents as Common Fractions.** We have learned that $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$. Therefore

*To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.*

### ORAL EXERCISE

1. A man said that he gained 100% on some property that he sold. What does this mean?

2. A dealer said that he would sell some goods at 25% less than the marked price. Express this as a common fraction.

3. A boy beginning to play tennis won only 20% of the games played the first week. Express this as a common fraction.

*Express as common fractions, whole numbers, or mixed numbers:*

| | | | |
|---|---|---|---|
| 4. 50%. | 7. 40%. | 10. 75%. | 13. 200%. |
| 5. 10%. | 8. 60%. | 11. 125%. | 14. 300%. |
| 6. 30%. | 9. 80%. | 12. 150%. | 15. 120%. |

### WRITTEN EXERCISE

1. A man said that he marked some goods $33\frac{1}{3}\%$ more than they cost him, and another said that he marked his goods $\frac{1}{3}$ more than they cost him. Was there any difference?

2. How much is $\frac{1}{2}\frac{1}{5}$ of 75? 4% of 75? 4% of 80?

3. How much is $\frac{1}{4}$ of 640? 25% of 640? 25% of 888?

4. How much is $\frac{1}{2}$ of 1728? 50% of 1728? 50% of 5280?

5. How much is $\frac{3}{4}$ of 8440? 75% of 8440? 75% of 1672?

6. How much is $1\frac{1}{4} \times 864$? 125% of 864? 125% of 996?

7. Instead of multiplying 64 by $12\frac{1}{2}\%$, what fractional part of 64 may we take? Do this.

8. Instead of multiplying 720 by $37\frac{1}{2}\%$, what fractional part of 720 may we take? Do this.

9. Find $87\frac{1}{2}\%$ of 80 and 1600 in the shortest way.

10. Find $33\frac{1}{3}\%$ of 66 and 3672 in the shortest way.

**43. Per Cents as Decimals.** Since 25.5% and 0.255 have the same value, therefore,

*To express as a decimal a number written with the per cent sign, omit the sign and move the decimal point two places to the left.*

When we omit the per cent sign we must indicate the hundredths in some other way, as by moving the decimal point two places to the left.

Thus $1\frac{1}{2}\% = 0.01\frac{1}{2}$, or 0.015; $125\% = 1.25$; $0.6\% = 0.006$.

### ORAL EXERCISE

1. If 2% of the pupils of this school are absent to-day, how many are absent out of every 100? out of every 50?
2. How much is $\frac{1}{100}$ of \$200? 1% of \$200? 1% of \$1700?
3. How much is 0.06 of \$100? 6% of \$100? 6% of \$300?

*Read as decimal fractions:*

4. 25%.
5. 32%.
6. 17%.
7. 45%.
8. 150%.
9. 175%.
10. 225%.
11. 375%.

### WRITTEN EXERCISE

1. Express $\frac{1}{8}\%$ as a decimal; as a common fraction.
2. Express $\frac{7}{8}\%$ as a decimal; as a common fraction.
3. Express $333\frac{1}{3}\%$ as a decimal; as an improper fraction.
4. How much is 0.15 of \$300? 15% of \$300?
5. How much is $1\frac{1}{4} \times$ \$650? $1.25 \times$ \$650? 125% of \$650?
6. How much is $2.50 \times$ \$460? 250% of \$460?

*Express as decimal fractions, whole numbers, or mixed decimals:*

7. 50%.
8. 75%.
9. $12\frac{1}{2}\%$.
10. $66\frac{2}{3}\%$.
11. 0.5%.
12. 0.7%.
13. 200%.
14. 150%.
15. 125%.
16. 1000%.
17. 0.005%.
18. 0.065%.

**44. Decimals as Per Cents.** Since per cent means hundredths, to express a decimal as per cent we have to consider only how many hundredths the decimal represents.

**1.** Express 0.3 as per cent.

Since $0.3 = 0.30$, we see that 0.3 is the same as 30 hundredths, or 30%.

**2.** Express 0.375 as per cent.

Here $0.375 = 37.5$ hundredths, or $37\frac{1}{2}$ hundredths. Hence 0.375 is the same as 37.5%, or $37\frac{1}{2}$%.

**3.** Express 0.00125 as per cent.

Here $0.00125 = 0.00\frac{125}{1000} = 0.00\frac{1}{8}$, and therefore $0.00125 = \frac{1}{8}$%.

**4.** Express $6.5\frac{1}{2}$ as per cent.

Here $6.5\frac{1}{2} = \frac{655}{100}$, and therefore $6.5\frac{1}{2} = 655$%.

Therefore, *to express a decimal as per cent, write the per cent sign after the number of hundredths.*

### ORAL EXERCISE

**1.** A pint, being 0.5 qt., is what per cent of a quart?

**2.** A peck, being 0.25 bu., is what per cent of a bushel?

**3.** A quart, being 0.125 pk., is what per cent of a bushel?

**4.** Express 0.007 of a mile as per cent of a mile.

**5.** What per cent of a dollar is 37¢? is 37 dimes?

### WRITTEN EXERCISE

**1.** Express 1 gill as per cent of a quart; of a gallon.

**2.** Express 1 quart as per cent of a peck; of a bushel.

*Express the following as per cents:*

| | | | |
|---|---|---|---|
| **3.** 0.8. | **5.** 0.37. | **7.** 0.225. | **9.** $3.333\frac{1}{3}$. |
| **4.** 5.2. | **6.** 0.62. | **8.** 0.450. | **10.** 0.00125. |

**45. Common Fractions as Per Cents.** Since per cent means hundredths, to express a common fraction as per cent we have only to reduce it to hundredths.

For example, $\frac{4}{5} = \frac{80}{100}$, and this is the same as 0.80, or 80%. Likewise $\frac{2}{25} = \frac{8}{100} = 8\%$, and $2\frac{1}{2} = \frac{5}{2} = \frac{250}{100} = 250\%$.

Therefore, *to express a common fraction as a per cent, reduce it to hundredths, omit the denominator, and write the numerator followed by the per cent sign.*

### ORAL EXERCISE

1. A cent is what per cent of \$1? of a dime?
2. 5¢ is what per cent of 10¢? of 20¢? of \$1? of 50¢?
3. 25¢ is what per cent of 50¢? of \$1? of 75¢? of \$1.25?
4. A foot is what per cent of a yard?

*Express the following as per cents:*

5. $\frac{1}{2}$.
6. $\frac{1}{4}$.
7. $\frac{1}{5}$.
8. $\frac{1}{10}$.
9. $\frac{3}{4}$.
10. $\frac{2}{5}$.
11. $\frac{3}{5}$.
12. $\frac{1}{20}$.
13. $\frac{1}{25}$.
14. $\frac{3}{50}$.

### WRITTEN EXERCISE

1. An inch is what per cent of a foot? of a yard?
2. A pint is what per cent of a quart? of a gallon?
3. What per cent of a gallon is 3 qt.? 2 qt.? $2\frac{1}{2}$ qt.?
4. What per cent of a pound is 1 oz.? 2 oz.? $3\frac{1}{5}$ oz.?
5. What per cent of a pound is 7 oz.? $7\frac{1}{2}$ oz.? 11 oz.?
6. What per cent of a rod is 1 ft.? 1 yd.? $5\frac{1}{2}$ yd.?
7. If a man spends $\frac{5}{6}$ of his income, what per cent of his income does he spend?
8. If you have increased $\frac{1}{9}$ in weight during the past year, what per cent have you increased in weight?

**46. Terms used in Percentage.** The number of which a required per cent is to be taken is often called the *base.*

The number of hundredths of the base is called the *rate.*

For example, in 25% of \$300, \$300 is the base and 25% is the rate.

Sometimes 25 is called the *rate per cent,* 25% being called the *rate,* but these two terms are commonly used to mean the same thing.

The result found by taking a certain per cent of the base is called the *percentage.* Therefore,

*The percentage is the product of the base and the rate.*

The terms "base," "rate," and "percentage" are introduced in this review work because they are occasionally used. They are not needed in this book and may be omitted unless demanded by the course of study.

**47. Important Per Cents.** The following equivalents should be memorized:

| | | | |
|---|---|---|---|
| $50\% = \frac{1}{2}$ | $37\frac{1}{2}\% = \frac{3}{8}$ | $16\frac{2}{3}\% = \frac{1}{6}$ | $20\% = \frac{1}{5}$ |
| $25\% = \frac{1}{4}$ | $62\frac{1}{2}\% = \frac{5}{8}$ | $33\frac{1}{3}\% = \frac{1}{3}$ | $40\% = \frac{2}{5}$ |
| $12\frac{1}{2}\% = \frac{1}{8}$ | $87\frac{1}{2}\% = \frac{7}{8}$ | $66\frac{2}{3}\% = \frac{2}{3}$ | $60\% = \frac{3}{5}$ |
| $6\frac{1}{4}\% = \frac{1}{16}$ | $3\frac{1}{8}\% = \frac{1}{32}$ | $83\frac{1}{3}\% = \frac{5}{6}$ | $80\% = \frac{4}{5}$ |

To take $87\frac{1}{2}\%$ of 648 is, therefore, the same as to take $\frac{7}{8}$ of 648.

WRITTEN EXERCISE

*Find the following:*

1. 50% of \$274.
2. 25% of \$372.
3. $12\frac{1}{2}\%$ of \$512.
4. $6\frac{1}{4}\%$ of \$3376.
5. $37\frac{1}{2}\%$ of \$17.76.
6. $62\frac{1}{2}\%$ of \$11.92.
7. $87\frac{1}{2}\%$ of \$34.48.
8. $33\frac{1}{3}\%$ of \$77.16.
9. $66\frac{2}{3}\%$ of \$80.07.
10. $83\frac{1}{3}\%$ of \$71.10.
11. $112\frac{1}{2}\%$ of 72.
12. $137\frac{1}{2}\%$ of 96.
13. $133\frac{1}{3}\%$ of 81.
14. $166\frac{2}{3}\%$ of 84.

**48. Finding a Required Per Cent of a Number.** The more general case of finding per cents — that in which common fractions cannot be used to advantage — will now be considered.

For example, suppose a man is digging a canal, and finds that 9600 cu. yd. of earth must be carted away. If it is reported to him that $23\frac{1}{4}\%$ of the earth has been removed, how many cubic yards have been carted away?

Since $23\frac{1}{4}\% = 0.23\frac{1}{4}$, we multiply 9600 by $0.23\frac{1}{4}$.
We might multiply by 0.2325, but this would be longer.
The easiest way is simply to multiply 96 by $23\frac{1}{4}$.

$$\begin{array}{r} 9600 \\ 0.23\frac{1}{4} \\ \hline 2400 \\ 288 \\ 192 \\ \hline 2232. \end{array}$$

Therefore, *to find a required per cent of a number, multiply the number by the given rate.*

WRITTEN EXERCISE

*Find, by using decimals:*

1. 22% of 75.
2. 35% of 86.
3. 27% of 32.
4. 41% of 78.
5. 62% of 93.
6. 26% of 3.4.
7. 43% of 6.7.
8. 39% of 5.3.
9. 28% of 0.7.
10. 35% of 0.63.
11. $2\frac{3}{4}\%$ of 8.4.
12. 0.5% of 5.7.
13. 1.9% of 6.8.
14. 235% of 742.
15. 622% of 0.72.

*Find, by using common fractions:*

16. 75% of 6448 yd.
17. $12\frac{1}{2}\%$ of \$9616.
18. $6\frac{1}{4}\%$ of \$24,328.
19. $33\frac{1}{3}\%$ of 9051 yd.
20. $66\frac{2}{3}\%$ of 7116.
21. $87\frac{1}{2}\%$ of 35,360.
22. $16\frac{2}{3}\%$ of \$13,806.
23. $37\frac{1}{2}\%$ of \$25,040.
24. $83\frac{1}{3}\%$ of \$22,350.
25. 125% of 948 cu. ft.
26. $133\frac{1}{3}\%$ of 729 cu. yd.
27. $166\frac{2}{3}\%$ of 816 sq. ft.

**28.** A man borrowed \$175 and had to pay 6% of that sum for the use of it for a year. How much did he pay at the end of the year, in addition to the sum borrowed?

**29.** If a manufacturer sells shoes at a profit of 15% and it costs him \$2.33$\frac{1}{3}$ a pair to make them, how much is his profit on 1000 pairs?

**30.** If a shop manufactures 276 locomotives and sells 75% of them for \$11,125 each and the rest for \$9825 each, how much is received for all?

**31.** A farmer can have his land poorly plowed for \$3.48 an acre, but it will cost him 66$\frac{2}{3}$% more to have a good job done. How much will it cost to have 60 acres well plowed?

**32.** How much butter fat is there in 375 lb. of milk brought to a creamery, the creamery test showing that 3.9% of the weight of the milk is butter fat?

**33.** A farmer takes 340 lb. of milk to a creamery and is paid 26¢ a pound for the butter fat. The tests show that the butter fat is 3.8% of the weight of the milk. Find the weight of the butter fat and the amount of money received.

**34.** One cow in a dairy gives 28 lb. of milk a day, which tests 3.1% of butter fat; another gives 24 lb., testing 3.7%. Butter fat being worth 23¢ a pound, which cow is the more profitable, and how much more?

**35.** A farmer has 75 trees on an acre of woodland, of which he decides to cut 60%. If wood is worth \$5.75 a cord and he can cut 3 cords from 5 trees, how much will he receive for the wood?

**36.** A dressmaker bought 37$\frac{1}{2}$ yd. of chiffon velvet at \$4.10 a yard, and received a reduction of 8% by paying promptly. She sold the velvet at \$4.25 a yard. How much did she make on the transaction?

### ORAL REVIEW

1. In a school of 300 pupils 9% were in the seventh grade. How many pupils were in that grade?

2. In a school of 200 pupils 11% were in the third grade and 14% were in the first grade. How many pupils were in each of these grades?

3. Of 300 pupils 5% have been tardy or absent this month. How many pupils have not been tardy or absent?

4. In a school of 300 pupils, if there should be an increase of $3\frac{1}{3}\%$ next year, how many pupils would there then be?

5. In a school of 250 pupils there were 10% less pupils four years ago. How many pupils were there then?

6. If $16\frac{2}{3}\%$ of the pupils in a school of 240 pupils were not in the school last year, how many new ones are there this year?

7. If 20% of the pupils in a school of 250 pupils are in the seventh and eighth grades, how many are in those grades?

8. If 25% of the pupils of a school of 300 pupils are in the first and second grades, how many are in those grades?

9. If 55% of the pupils in a school of 300 pupils are girls, how many girls are there in the school? how many boys?

10. How much must be added to 250 ft. to increase it 50%? to increase it 100%?

11. How much must be added to 240 lb. to make it 125% of the present weight?

12. A column of mercury in a thermometer is 6 in. high. If it increases $16\frac{2}{3}\%$ in height, how high is it then?

13. A rectangle 4 in. long and 3 in. wide is increased 50% in length and $33\frac{1}{3}\%$ in width. What was the original area, and what is the area now?

**49. Three Important Problems of Percentage.** There are three important problems in percentage. The first, and by far the most important one, is that of finding a required per cent of a number. This was considered in § 48. The second problem of importance is to find what per cent one number is of another, and this will be considered in § 50. The third problem of importance is to find the number of which a given number is a given per cent, and this will be considered in § 52.

The problems in §§ 50 and 52 depend upon this principle:

*Given the product of two factors and one of the factors, the other factor may be found by dividing the product by the given factor.*

That is, if we know the product of 2 and 5, which is 10, we can find the factor 2 by dividing 10 by 5, and we can find the factor 5 by dividing 10 by 2.

Teachers should not require the pupils to learn the three problems mentioned in § 49, for they will meet the two new ones in §§ 50 and 52. The principle should be clearly understood, however. Teachers who desire to introduce the equation in connection with §§ 50 and 52 may do so, consulting pages 287 and 288.

### ORAL EXERCISE

*The first number in each of the following examples is the product of two factors, and the second number is one of the factors. Find the other factor:*

| | | | |
|---|---|---|---|
| **1.** 48, 4. | **6.** 90, 9. | **11.** 8, 4. | **16.** 6, $\frac{1}{4}$. |
| **2.** 48, 12. | **7.** 90, 10. | **12.** $\frac{8}{10}$, 4. | **17.** 7, $\frac{1}{2}$. |
| **3.** 72, 8. | **8.** 90, 3. | **13.** 0.8, 4. | **18.** 7, 0.5. |
| **4.** 72, 9. | **9.** 200, 10. | **14.** 0.80, 4. | **19.** 9, 0.5. |
| **5.** 70, 10. | **10.** 200, 20. | **15.** 0.80, 8. | **20.** 8, 50%. |

**21.** I am thinking of the number which, multiplied by 7, gives the product 56. What is the number?

**50. Finding what Per Cent One Number is of Another.** The second problem of importance in percentage mentioned in § 49 is to find what per cent one number is of another.

For example, if Fred was at bat 20 times in the baseball games of last June and made 6 base hits, what per cent of base hits did he make?

Here we have a certain per cent $\times$ 20 = 6; that is, we have the product (6) and one factor (20) to find the other factor (a certain per cent). Therefore

$6 \div 20 = 0.3 = 30\%$, the per cent of hits.

### ORAL EXERCISE

1. If Fred was at bat 30 times and made 15 base hits, what per cent of base hits did he make?

2. If a baseball team wins 12 games out of 20, what per cent of games does it win?

3. If there are 40 pupils in a class and 4 of them are absent to-day, what per cent are absent?

4. On an automobile trip of 80 mi., what per cent has a man traveled when he has traveled 20 mi.?

*Find what per cent the second number is of the first:*

| | | | |
|---|---|---|---|
| **5.** 2, 1. | **10.** 60, 30. | **15.** 64, 32. | **20.** 25, 5. |
| **6.** 4, 1. | **11.** 70, 35. | **16.** 64, 16. | **21.** 25, 2.5. |
| **7.** 4, 2. | **12.** 90, 45. | **17.** 64, 48. | **22.** 25, $12\frac{1}{2}$. |
| **8.** 4, 3. | **13.** 60, 15. | **18.** 72, 54. | **23.** 25, 25. |
| **9.** 8, 6. | **14.** 32, 8. | **19.** 36, 27. | **24.** 25, 50. |

**25.** If a book has 350 pages, what per cent of the number of pages have you read when you have read it through page 35? through page 70? through page 50? through page 175?

**51. Application to Written Exercises.** You now understand the second important kind of problem in percentage. We shall consider once more the first kind of problem, and then the application of the second kind to written exercises.

1. An agent bought an automobile for \$600 and sold it so as to gain 20% of the cost. How much did he gain?

Here we have to find 20% of \$600, or 0.20 of \$600. That is, we have two factors given, 0.20 and \$600, to find the product. Therefore we have

$$0.20 \times \$600 = \$120.$$

This is the first kind of problem. We shall now consider the second kind.

2. An agent bought an automobile for \$600 and sold it at a profit of \$120. What per cent of the cost did he gain?

Here we have \$120 equal to some per cent of \$600.

That is, we have the product (120) of two factors, and one of the factors (600) given, to find the other factor. Therefore we have

$$\$120 \div \$600 = 0.20 = 20\%.$$

That is, the agent gained 20% of the cost.

### WRITTEN EXERCISE

1. \$130 is what per cent of \$2600? of \$3900?
2. \$11.25 is what per cent of \$375? of \$56.25?
3. \$10.12 is what per cent of \$253? of \$202.40?
4. \$29.10 is what per cent of \$465.60? of \$174.60?
5. A foot is what per cent of a yard? of a rod?
6. A quart is what per cent of a gallon? of 25 gal.?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. $33\frac{1}{3}\%$ is what per cent of $66\frac{2}{3}\%$? of 1? of $133\frac{1}{3}\%$?
10. $16\frac{2}{3}\%$ is what per cent of $33\frac{1}{3}\%$? of $66\frac{2}{3}\%$? of 1?

**11.** If Ralph Jenkins is at bat 40 times and makes 12 base hits, what is his per cent of base hits?

**12.** If the seventh grade devotes 45 min. a day to arithmetic, and the sixth grade 42 min., what is the per cent of increase?

**13.** If a class had 24 examples to solve on Monday and 29 on Tuesday, what was the per cent of increase?

**14.** If you have read 78 pages in a book of 300 pages, what per cent of the pages have you read?

**15.** If the last chapter of a book is numbered XXXII, and you have finished reading Chapter XXIV, what per cent of the chapters have you read?

**16.** A man's income is \$1650 a year, and he spends \$693. What per cent of his income does he save?

**17.** In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? What per cent are girls?

**18.** A man invests \$6750 and gains \$810. His gain is what per cent of the amount invested?

**19.** The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

**20.** What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine?

**21.** If 2 members of a class of 32 have not been either tardy or absent during the year, and 4 members have not been absent, what per cent have not been either tardy or absent? What per cent have not been absent?

**22.** A man's income is \$2175 a year, and he saves \$652.50. What per cent of his income does he save? What per cent does he spend? What is the sum of these two per cents?

**52. Finding the Number of which a Given Number is a Given Per Cent.** The third important problem in percentage mentioned in § 49 is to find the number of which a given number is a given per cent.

For example, if a baseball team lost 18 games, which is 45% of the number of games played, how many games were played?

Here we have 45% of the number of games = 18.

That is, we have the product (18) of two factors, and one of the factors (0.45) given, to find the other factor.

Therefore we have $18 \div 0.45 = 40$.

That is, there were 40 games played.

*Check.* 45% of 40 = 18.

### WRITTEN EXERCISE

**1.** If 20% of the inhabitants of a certain city are school children and there are 18,275 school children in the city, what is the total population?

**2.** If 23% of the inhabitants of a certain city voted at a certain election and there were 3956 ballots cast, what was the total population?

**3.** A player reached first base 29 times, or $33\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

**4.** A player reached home base 24 times, or $8\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

**5.** If you pay $37\frac{1}{2}\%$ of the expenses of a camping trip, your contribution being \$6.24, what are the expenses of the trip?

**6.** In a certain school there are 119 boys, which is 85% of the number of girls. How many girls are there in the school?

**7.** If a school is in session 50% of the days of a certain year, and is in session 183 da., is that year a leap year or not?

*Find the numbers of which the following are the given per cents:*

| | | |
|---|---|---|
| **8.** 64, 8%. | **15.** 111, 3%. | **22.** 147, $87\frac{1}{2}$%. |
| **9.** 72, 9%. | **16.** 112, 2%. | **23.** 42.06, $66\frac{2}{3}$%. |
| **10.** 88, 11%. | **17.** 97, $12\frac{1}{2}$%. | **24.** 12.48, $16\frac{2}{3}$%. |
| **11.** 85, 5%. | **18.** 89, $3\frac{1}{3}$%. | **25.** 57.8, 34%. |
| **12.** 84, 6%. | **19.** 62, $33\frac{1}{3}$%. | **26.** 90.2, 110%. |
| **13.** 91, 7%. | **20.** 156, $37\frac{1}{2}$%. | **27.** 65.72, 106%. |
| **14.** 64, 4%. | **21.** 125, $62\frac{1}{2}$%. | **28.** 622.6, 5.5%. |

**29.** Of what number is 287 equal to 5%?

**30.** \$3.15 is 15% of what amount? 20% of what amount?

**31.** \$6.24 is 16% of what amount? 10% of what amount?

**32.** 12 ft. 5.6 in. is 22% of what length? 5% of what length?

**33.** 63 lb. 7 oz. is 35% of what weight? 2% of what weight?

**34.** If a farmer sold a horse for 80% of its cost and received \$152, how much did the horse cost?

**35.** A poultry raiser set some eggs, and 15% failed to hatch. The number failing to hatch being 33, how many eggs did the poultry raiser set?

**36.** A man sells some lumber for \$180, thereby gaining 20% on the cost. What per cent of the cost is the selling price? How much is the cost?

**37.** A man saves \$675.20 a year, which is 32% of his income. How much is his income?

**38.** A school has 20% of its pupils in the sixth grade, which numbers 29. How many pupils are there in the school?

From the examples on pages 88 and 89 it will be seen that the third important problem of percentage is not nearly as practical as the first and second problems mentioned in § 49.

### ORAL REVIEW

**1.** If a man lends \$250 and receives \$10 interest for a year, what is the rate?

**2.** If a man borrows \$300 and pays \$18 interest for a year, what is the rate?

**3.** If a salesman receives a commission of \$15 on a sale amounting to \$750, what is his rate of commission?

**4.** If a dairyman keeps 60 cattle this year and kept 48 last year, what is the increase per cent?

**5.** If the taxes of a certain village were \$8000 last year and \$7000 this year, what is the decrease per cent?

**6.** If the rainfall in a certain place was 7.2 in. in April and 6 in. in March, what was the per cent of increase?

**7.** If the seventh grade devotes 45 min. a day to arithmetic and the sixth grade 40 min., what is the per cent of increase?

**8.** If a dealer sold daily 150 qt. of milk last month and 180 qt. this month, what is the rate of increase?

**9.** If you have read 75 pages in a book of 300 pages, what per cent have you read?

**10.** If the last chapter of a book is numbered XXXV and you have finished reading Chapter XXVI, what per cent of the chapters have you read?

**11.** If you have read 60 pages of a book, and this is 25% of the book, how many pages are there in all? How many pages have you still to read?

**12.** A cork is pushed below the surface of a glass full of water, and the water that runs over is weighed. The cork is found to weigh 1.2 oz. and the water to weigh 5 oz. Cork is what per cent as heavy as an equal volume of water?

**53. Miscellaneous Problems.** We are now prepared to consider miscellaneous problems, and here you will find it necessary to decide upon the method of solution. You will find it best to ask yourself exactly what is given, and then to consider carefully whether you have to find a required per cent of a number (two factors given), or whether you have a product and one factor given, to find the other factor.

It will be noticed that only two principles are involved in the following exercise. The examples after Ex. 21 involve a variation of the principle mentioned on page 84, and may be omitted if desired. They are inserted for the use of those who may wish to carry the work into more practical fields.

### WRITTEN EXERCISE

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that weighs 125 lb. What per cent of the water is drawn off?

2. To a class of 32 pupils 8 more are admitted. What per cent of the original number are admitted? What per cent of the present number are admitted? If 10 pupils now leave the class, what per cent leave the class?

3. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 problems have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

4. If 45% of the 240 pupils that attend a certain school are boys, how many are girls?

5. A man purchased 530 head of cattle and sold 30% of them. How many head of cattle had he left?

6. If an automobile going at the rate of 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

**7.** A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than gold?

**8.** A bank averaged $1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

**9.** A clerk's salary is $12 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in a year of 50 weeks?

**10.** If a baker uses 639 lb. of flour in making bread and adds to the flour 213 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture?

**11.** If $3\frac{1}{2}$ teaspoons of sugar make a tablespoon, a teaspoon of sugar is what per cent of a tablespoon?

**12.** If there are 16 tablespoons to a cup, a tablespoon is what per cent of a cup?

**13.** A teaspoon of butter is $\frac{1}{4}$ of a tablespoon. Express a teaspoon of butter as per cent of a tablespoon.

**14.** The record of the champion team in baseball in a certain year was 52 games won, 14 lost. What per cent of the games did the champion team win that year?

**15.** One of the best records in baseball of recent years was when Brooklyn won the pennant with a total of 82 games won and 54 lost. What was the per cent of games won?

**16.** Two of the best averages for runs in baseball were 597 times at bat, 139 runs, and 543 times at bat, 145 runs. What was the per cent of runs in each case?

**17.** If the expenses of a certain village were $8250 last year and $8002.50 this year, what is the per cent of decrease?

**18.** If the rainfall in a certain place was 7.2 in. in April and $6\frac{1}{4}$ in. in March, what was the per cent of increase?

**19.** In 1801, of the 161,800,000 people who spoke European languages, 31,450,000 spoke French. Of the 585,000,000, in 1911, those who spoke French numbered 70,000,000. What per cent spoke French in each of these years?

**20.** A dressmaker bought a silk dress pattern for \$45 less 20%. She sold it for \$45. What per cent did she gain?

Notice that the gain is reckoned on the price she paid for the pattern.

**21.** Allowing 8 oz. of Paris green to 50 gal. of water in preparing a spray to destroy leaf-eating insects, and considering a gallon of water to weigh 8.4 lb., the weight of the Paris green is what per cent of the weight of the water?

**22.** A dealer sold pencils at 5¢ each, which gave him a profit of 25%. How much did they cost him a dozen?

**23.** A farmer bought some sheep and sold them for \$252, which gave him a profit of 20%. How much did they cost him?

**24.** A dealer sells some goods for \$31.05, thus making a profit of 15% on the cost. How much did the goods cost him?

**25.** A manufacturer sells a suit of clothes to a merchant at a profit of $12\frac{1}{2}$%. The merchant sells it to you for \$12 and makes a profit of $33\frac{1}{3}$%. How much does the suit cost the merchant? How much does the suit cost the manufacturer?

It should be noticed that $133\frac{1}{3}$% of the cost to the merchant is \$12.

**26.** An agent bought 100 doz. eggs, which he sold to a grocer at a profit of 20%. The grocer sold the eggs at retail for \$22.44 and made a profit of 10%. How much did the eggs cost the grocer? How much did the eggs cost the agent?

It should be noticed that 110% of the cost to the grocer is \$22.44.

**27.** A certain number when increased by 30% of itself becomes 650. What is the number?

Since once the number and 30% of the number are 130% of the number, 130% of the number is 650. Then how is the number found?

**28.** A certain number when decreased by 20% of itself becomes 400. What is the number?

Since once the number less 20% of the number is 80% of the number, 80% of the number is 400. Then how is the number found?

*The following numbers are the given per cents more than what numbers, and also the given per cents less than what other numbers?*

**29.** 144, 20%.
**30.** 150, 25%.
**31.** 91, 30%.
**32.** 99, $12\frac{1}{2}$%.
**33.** 350, $16\frac{2}{3}$%.
**34.** 550, $37\frac{1}{2}$%.
**35.** 7.28, $33\frac{1}{3}$%.
**36.** 15.84, 50%.
**37.** 390, $62\frac{1}{2}$%.
**38.** 49.5, $87\frac{1}{2}$%.
**39.** 36.9, 80%.
**40.** 27.3, $16\frac{2}{3}$%.

**41.** If an army has been increased 21% and now numbers 73,810, how many did it number before the increase?

**42.** A man sold some goods for \$192.50, making a profit of 10%. How much did the goods cost him?

**43.** A dealer sold a horse for \$181.50, making a profit of 10%. How much did the horse cost him?

**44.** A village has a population of 1853, an increase of 9% over ten years ago. What was the population ten years ago?

**45.** A village has a population of 2664, an increase of 11% over ten years ago. What was the population ten years ago?

**46.** If you weigh 78.1 lb. and have increased 42% in weight in the last five years, how much did you weigh five years ago?

**47.** If a library has increased in size $16\frac{2}{3}$% in the past five years and now has 8323 volumes, how many volumes had it before the increase?

**54. Discount.** If you have a catcher's mask that you have used during the season and wish to dispose of it so you can use the money toward buying a football, you will probably be glad to sell it for 25% less than it cost you. In that case you would sell the mask at a *discount* of 25%. You might speak of this as "a quarter off," or as "75¢ on a dollar," but the operation of finding the selling price would be to deduct 25%, or $\frac{1}{4}$, of the cost.

The price of goods as given in a printed list is called the *list price.* Manufacturers usually have price lists of this kind. In stores the goods are marked, and the marked price is what is given in the dealer's price list if he has one.

A reduction from the list price of an article, from the amount of a bill of goods, or from the amount of a debt is called *commercial discount.*

A regular reduction from the list price is called *trade discount.*

A reduction for payment within a fixed time is called a *time discount.*

A reduction for payment when the goods are delivered is called a *cash discount.*

Discounts are usually reckoned at some common fraction of the list price, or at some rate per cent of it.

The amount of the bill after the discount has been made is called the *net amount*, or the *net price.*

Read and study the solution of this example:

Find the net amount of a bill of \$275 with a discount of 20%.

| | |
|---|---|
| The amount of the bill | = \$275 |
| 20%, or $\frac{1}{5}$, of \$275 | = 55 |
| The net amount | = \$220 |

Instead of speaking of "20% off," merchants often speak of "$\frac{1}{5}$ off." So they speak of "$\frac{1}{4}$ off," and "$\frac{1}{3}$ off," but it is not so convenient to use common fractions for such discounts as 12%.

### ORAL EXERCISE

1. If some goods are marked \$20, and 10% discount is allowed, what is the selling price?

2. If a dealer offers me an 80¢ book at a discount of 25%, this is how many cents below the regular price?

3. If a merchant buys \$800 worth of goods and is allowed 10% discount in case he pays for them at once, how much does he gain by prompt payment?

*Find the discounts on the following amounts at the rates specified:*

4. \$80, 10%.
5. \$25, 10%.
6. \$50, 20%.
7. \$25, 20%.
8. \$40, 25%.
9. \$88, 25%.
10. \$60, 50%.
11. \$90, 50%.
12. \$120, 10%.
13. \$150, 20%.
14. \$160, 25%.
15. \$250, 50%.

### WRITTEN EXERCISE

1. If goods marked \$475 are sold to a dealer at a discount of 20%, how much do they cost him?

2. If a merchant marks a lot of suits at \$24.75 each, and sells them at $\frac{1}{3}$ off, what is the net price of each?

Pupils should be asked to watch advertisements in the newspapers to see what discounts are offered. They should state the probable reasons for these discounts, and should bring to class the problems which they make.

*Given the marked prices and rates of discount, find the net prices:*

3. \$17.50, 10%.
4. \$16.50, 20%.
5. \$27.75, 20%.
6. \$64.40, 25%.
7. \$86.60, 25%.
8. \$48.60, 25%.
9. \$72.30, $33\frac{1}{3}$%.
10. \$38.70, $33\frac{1}{3}$%.
11. \$43.50, $16\frac{2}{3}$%.
12. \$86.40, $16\frac{2}{3}$%.
13. \$73.60, $12\frac{1}{2}$%.
14. \$27.70, 50%.
15. \$35.00, 10%.
16. \$34.75, 20%.
17. \$65.25, 20%.

**55. Several Discounts.** In some kinds of business two or more discounts are allowed. For example, a dealer may buy \$200 worth of hardware with discounts of 20%, 10% (20% and 10%). This means that 20% is first deducted from the list price, and then 10% is deducted from the remainder.

The list price is \$200.
The list price less 20% is \$160.
The \$160 less 10% is \$144, the *net price.*

### ORAL EXERCISE

1. From \$50 take 10%, and 10% from the remainder.
2. From \$200 take 20%, and 1% from the remainder.
3. From \$100 take 25%, and 10% from the remainder.
4. From \$500 take 20%, and 25% from the remainder.
5. A man bought some paper at a list price of \$80, with discounts of 25%, 10%. How much did he pay?

### WRITTEN EXERCISE

*Find the net prices of goods marked and discounted as follows:*

1. \$200, 30%, 20%.
2. \$700, 35%, 5%.
3. \$500, 25%, 15%.
4. \$500, 12%, 2%.
5. \$650, 10%, 10%.
6. \$400, 30%, 10%.
7. \$300, 15%, 10%.
8. \$275, 10%, 10%.
9. \$240, $12\frac{1}{2}$%, $33\frac{1}{3}$%.
10. \$800, 25%, 25%.

11. What is the difference between a discount of 50% on \$800, and the two discounts of 25%, 25%?

12. Is there any difference between a discount of 10%, 5%, and one of 5%, 10%, on \$900? How is it on \$400? on \$200?

**56. Sample Price List.** The following is a price list of certain school supplies, with the discounts allowed to schools and dealers when the prices are not net:

| | |
|---|---|
| Rulers, | \$0.35 per doz., net |
| Composition books, | 4.50 per gross, less 5% |
| Thumb tacks, | 0.40 per 100, less 40% |
| Drawing pencils, | 4.71 per gross, less 20% |
| Drawing paper, $9 \times 12$ | 1.10 per package, less $\frac{1}{10}$ |
| Pens, | 0.61 per gross, less $\frac{1}{4}$, 10% |
| Tubes of paste, | 4.05 per gross, less 10%, 5% |
| Penholders, | 3.20 per gross, less 12%, 5% |
| Drawing compasses, | 1.65 per doz., less 10%, 5% |

**WRITTEN EXERCISE**

1. How much must a school pay for $\frac{1}{2}$ gross of penholders?

2. A school wishes to buy 5 packages of drawing paper and 300 thumb tacks. How much will they cost?

3. How much will 8 gross of pens and $\frac{1}{2}$ gross of composition books cost? 2 gross of pens and 1 doz. rulers?

4. There are 40 pupils in a class, and each needs drawing compasses and a ruler. How much will they cost the school? How much would they cost if there were 60 pupils?

5. If a dealer sells pens at a cent apiece, how much does he gain per gross? If he sells penholders at 2¢ each, how much does he gain per gross?

6. A dealer buys a gross of rulers and a gross of drawing pencils. He sells both at 5¢ each. How much does he gain in all?

7. If a dealer buys a gross of tubes of paste for mounting pictures and sells the tubes at 5¢ each, how much does he gain on the purchase? how much on 3 gross?

**57. Bill with Several Discounts.** The following is a common form of a bill of goods with several discounts:

*Burlington, Ia., May 13, 19.....*

*Mr. Geo. S. Miller, Cedar Rapids, Ia.*

*Bought of* ROBERTS & COMPANY, Jewelers
1072 Passaic Avenue

Terms: *20%, 10%*

| | | | | | | |
|---|---|---|---|---|---|---|
| *Feb.* | *10* | *8 doz. spoons @ $14.75* | *118* | *00* | | |
| | | *7 doz. plated forks @ $4.20* | *29* | *40* | | |
| | | | | | *147* | *40* |
| | | *Less 20%, 10%* | | | *41* | *27* |
| | | | | | *106* | *13* |

**WRITTEN EXERCISE**

*Make out bills for the following:*

**1.** 24 doz. files at $6.30. Discounts 30%, 20%.

**2.** 840 yd. silk at $1.50; 640 yd. velvet at $1.75. Discounts 25%, 10%.

**3.** 8 doz. pairs hinges at $4.70; 36 doz. table knives at $8.60. Discounts 20%, 10%.

**4.** 24 doz. locks at $4.30; 8 doz. mortise locks at $4.75. Discounts 25%, 8%.

**5.** 460 yd. taffeta at 98¢; 8 gross pompons at $144; 4 doz. pieces braid at $21.60. Discounts 10%, 5%.

**6.** 680 yd. silk at $1.65; 840 yd. lawn at 23¢; 460 yd. taffeta at 96¢. Discounts 10%, 5%, 5%.

**58. Receipted Bills.** The following is a receipted bill for some goods purchased by a retail merchant from a wholesale dealer:

*Chicago, Ill., Dec. 17, 19*.....

*Mr. A. M. Nourse, Joliet, Ill.,*

*Bought of* STARR & TIFFANY, Jewelers
8378 Burlington Ave.

Terms: *2%*

| | | | | | | |
|---|---|---|---|---|---|---|
| *Dec.* | *2* | *3 doz. silver forks @ $22* | *66* | | | |
| | | *1/6 doz. salad forks @ $24* | *4* | | | |
| | | | *70* | | | |
| | | *Less 2%* | *1* | *40* | | |
| | | | | | *68* | *60* |
| | | *Received payment*<br>*Starr & Tiffany* | | | | |

In this case Mr. Nourse is the *debtor* (Dr.), since he is in debt for the amount. Starr & Tiffany are the *creditors* (Cr.), since they trust Mr. Nourse, or give him credit.

A receipt may also be written separately instead of appearing on a bill. Such a receipt should be dated, and should be in substantially this form: "Received from —— the sum of —— dollars for ——." The receipt should be signed by the creditor.

The subject of commercial discount is of great value because of its extensive use, not only in wholesale transactions but even in bargain sales. Pupils should understand some of the reasons for allowing discounts, for buying in large quantities, and for paying cash down or within a specified time, and they should become familiar with the ordinary deductions from list prices. It is well, for obvious reasons, to consider bills and receipts in connection with the study of this topic.

**59. Invoice.** A bill giving in detail a list of items and prices of goods sold is called an *invoice.* A sample invoice follows:

*St. Paul, Minn., Dec. 12, 1918*

*Mr. James S. Dunbar, Fargo, N.D.,*

*Bought of* RANDALL BROS.
Importers of Dry Goods

Terms: *10 da.*

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| *236* | *14* | *pc. blue flannel* | | | | | |
| | | *40, 41, 41¼, 41½,* | | | | | |
| | | *40¾, 40, 41, 42,* | | | | | |
| | | *41½, 42¼, 41, 42¾,* | | | | | |
| | | *41, 40* | *576* | *33⅓¢* | *192* | | |
| *427* | *8* | *pc. silk* | | | | | |
| | | *40, 39, 41¾, 40½,* | | | | | |
| | | *39¼, 41, 42¾, 40* | *324* | *$1.25* | *405* | | *597* |

In the above invoice, $41^1$ means $41\frac{1}{4}$; $41^2$ means $41\frac{2}{4}$, or $41\frac{1}{2}$; and $41^3$ means $41\frac{3}{4}$. This is the customary way of indicating the number of yards in pieces of goods. The numbers 236 and 427 refer to the price list in which the goods are described. The 14 and 8 indicate the number of pieces (pc.). The number of yards of the first is 576, and of the second 324.

The expression "Terms: 10 da." means that Mr. Dunbar has 10 da. in which to pay for the goods. Such an invoice may or may not mention the discount allowed.

There is no essential difference between a bill and an invoice. The latter term is commonly used in wholesale transactions.

### WRITTEN EXERCISE

*Make out bills, or invoices, for the following:*

1. $12\frac{1}{2}$ doz. shirts @ \$14.80, $\frac{2}{3}$ doz. ties @ \$9.75.

2. 16 pc. ribbon @ \$1.25, 18 pc. @ \$1.33$\frac{1}{3}$, 33 pc. @ \$1.66$\frac{2}{3}$, 24 pc. @ \$1.12$\frac{1}{2}$, 32 pc. @ \$1.37$\frac{1}{2}$, 48 pc. @ \$1.75.

3. 2 carloads coal, 21,700 lb., 24,200 lb., @ \$4.66$\frac{2}{3}$ per ton; 3 carloads coal, 22,700 lb., 21,900 lb., 20,400 lb., @ \$4.75 per ton. Terms: 3% for cash.

4. 1 gro. cans sardines @ \$3.33$\frac{1}{3}$ per doz., 8 doz. cans shrimps @ \$1.75 per doz., 9 doz. tins herrings @ \$2.33$\frac{1}{3}$ per doz., $6\frac{1}{4}$ doz. cans lobster @ \$2.88 per doz., 32 cans mackerel @ $16\frac{2}{3}$¢ each. Terms: 4% for cash.

5. 8 doz. packages codfish @ \$1.75 per doz., 9 doz. cans salmon @ \$2.33$\frac{1}{3}$ per doz., 15 doz. cans caviar @ \$3.33$\frac{1}{3}$ per doz., 18 doz. cans Yarmouth bloaters @ \$2.33$\frac{1}{3}$ per doz., 24 doz. cans tongue @ \$8.75 per doz. Terms: $3\frac{1}{2}$% for cash.

6. 8 doz. jars meat extract @ \$3.66$\frac{2}{3}$ per doz., 16 doz. cans chicken @ \$3.12$\frac{1}{2}$ per doz., 24 doz. cans beef @ \$2.33$\frac{1}{3}$ per doz., 18 doz. cans soup @ \$3.33$\frac{1}{3}$ per doz., 9 doz. cans clam chowder @ \$3.66$\frac{2}{3}$ per doz., 16 doz. cans clam juice @ \$1.06$\frac{1}{4}$ per doz. Terms: 3%, 2%.

7. 5 armchairs @ \$6.40 each, 37 kitchen chairs @ \$1.12$\frac{1}{2}$ each, 8 kitchen tables @ \$2.25 each, 3 bedroom sets @ \$33.33$\frac{1}{3}$ per set, 16 dining tables @ \$12.50 each, 9 sideboards @ \$16.66$\frac{2}{3}$ each, 16 rockers @ \$4.87$\frac{1}{2}$ each. Terms: 8%, 4%.

8. 20 pc. linen containing 39, $40^2$, $38^3$, 42, $41^1$, $40^2$, 40, 39, $40^1$, $41^2$, 43, $39^3$, $40^1$, $42^1$, $40^2$, $40^1$, $41^3$, 42, $39^1$, 40 yd., @ $66\frac{2}{3}$¢; 16 pc. silk containing $41^1$, $42^1$, 40, $39^2$, 42, $40^3$, 38, $41^1$, 43, $41^3$, 39, $40^2$, $40^1$, $41^1$, 40, $39^2$ yd., @ \$1.33$\frac{1}{3}$. Terms: 4%, 3%.

**60. Interest.** We have already learned that men often have to borrow money to carry on their business. For example, a merchant wishes to buy a lot of holiday goods. He feels sure that he can sell these for much more than he pays for them. It is therefore good business for him to borrow the money in the early part of the winter, buy the goods, sell them during the holidays, and pay back the money in January. But the bank from which he borrows should not only be paid the money borrowed, but be paid for the use of the money.

This payment for the use of the money is called *interest.*

If the merchant borrows \$1000 and pays at the rate of 6% a year, he would pay \$60 a year, or \$5 a month.

### ORAL EXERCISE

*Find the interest on the given sums for 1 yr. at the given rates:*

1. \$100, 4%.
2. \$100, 3%.
3. \$300, 3%.
4. \$250, 3%.
5. \$250, 4%.
6. \$600, 4%.
7. \$400, 5%.
8. \$600, 5%.
9. \$800, 5%.
10. \$400, 6%.
11. \$700, 6%.
12. \$900, 6%.
13. \$100, $2\frac{1}{2}$%.
14. \$400, $2\frac{1}{2}$%.
15. \$400, $4\frac{1}{2}$%.

16. What is the interest for 1 yr. on \$200 at 6% ? at 5% ? at 4% ? at $4\frac{1}{2}$% ? at 2% ? at $2\frac{1}{2}$% ?

17. What is the interest for 1 yr. on \$400 at 3% ? on \$300 at 4% ? on \$600 at 2% ? on \$200 at 6% ?

18. What is the interest for 1 yr. on \$2000 at 5% ? at 6% ? at 4% ? at $4\frac{1}{2}$% ? at 3% ? at $3\frac{1}{2}$% ? at $2\frac{1}{2}$% ?

19. What is the interest on \$1000 at 6% for 1 yr. ? for $\frac{1}{2}$ yr. ? for 6 mo. ? for $\frac{1}{4}$ yr. ? for 3 mo. ? for $\frac{1}{3}$ yr. ? for 4 mo. ?

20. What is the interest on \$4000 at 6% for 1 yr. 6 mo. ?

### WRITTEN EXERCISE

*Find the interest on the following at 6% :*

1. \$250 for 1 yr. 2 mo.
2. \$375 for 1 yr. 3 mo.
3. \$450 for 1 yr. 8 mo.
4. \$575 for 1 yr. 6 mo.
5. \$475 for 2 yr. 1 mo.
6. \$650 for 2 yr. 6 mo.
7. \$850 for 3 yr. 3 mo.
8. \$750 for 3 yr. 6 mo.

9. A man borrowed \$8000 in Boston at $4\frac{1}{2}\%$, and loaned it in Arkansas at 8%. How much did he gain in 3 yr.?

10. The day a certain boy is 14 yr. old his father invests \$300 for him at 5%, the \$300 and interest being due the day he is 21 yr. old. What is the total amount due at that time?

11. A real estate dealer buys 360 acres of farm land at \$30 an acre, and after keeping it 7 mo. sells it at an advance of \$3.75 an acre. The money being worth to him 5% a year, how much does he gain by the transaction?

12. A man having \$26,750 invested in business has found that his profits average 18% a year. He is offered \$35,000 for the business and he could invest the money at $4\frac{1}{4}\%$. If he sells out and retires, what is his annual loss in income?

13. In April a coal dealer borrowed \$33,250 at 5%. With this he purchased his summer's supply of coal at \$4.75 a ton. He sold the coal at \$5.65 a ton, the buyers paying for the unloading and delivery, and he paid his debt in October after keeping the money 6 mo. How much did he gain?

14. I own a house which I rent at \$25 a month. My taxes are \$50 a year, my repairs \$50, and my insurance \$10. Not considering any change in value, would it be better for me to sell it for \$5000 and invest the money at 4%, if my taxes would then be reduced to \$10? Find the difference, if any, in income.

**61. Interest for Years, Months, and Days.** Suppose a man borrows from another man $400 on September 10, 1916, at 6%. What will the interest amount to August 7, 1919? What would the interest amount to August 7, 1917?

| yr. | mo. | da. | |
|---|---|---|---|
| 1919 | 8 | 7 | = second date |
| 1916 | 9 | 10 | = first date |
| 2 | 10 | 27 | = difference in time |

Since 2 yr. = 720 da., and 10 mo. = 300 da., 2 yr. 10 mo. 27 da. = 1047 da. We therefore have

$$\frac{1047}{360} \text{ of } 6\% \text{ of } \$400 = \frac{\overset{349}{\cancel{1047}} \times \cancel{6} \times \overset{4}{\cancel{\$400}}}{\underset{15}{\underset{90}{\cancel{360} \times \cancel{100}}}} = \$\tfrac{349}{5} = \$69.80.$$

Hence the interest due August 7, 1919, is $69.80.

To find the amount of interest August 7, 1917, we have

$$\frac{327}{360} \text{ of } 6\% \text{ of } \$400 = \frac{\overset{109}{\cancel{327}} \times \cancel{6} \times \overset{4}{\cancel{\$400}}}{\underset{15}{\underset{90}{\cancel{360} \times \cancel{100}}}} = \$\tfrac{109}{5} = \$21.80.$$

Hence the interest due August 7, 1917, is $21.80.

### WRITTEN EXERCISE

*Find the interest on the following:*

1. $325 for 1 yr. 8 mo. 5 da. at 5%.
2. $300 for 1 yr. 4 mo. 15 da. at 6%; at 5%.
3. $150 for 1 yr. 3 mo. 20 da. at 5%; at $5\frac{1}{2}$%.
4. $475 for 3 yr. 6 mo. 15 da. at 6%; at 5%; at 4%.
5. $850 for 2 yr. 7 mo. 6 da. at 4%; at $4\frac{1}{2}$%; at $5\frac{1}{2}$%.
6. $75, at 4%, from June 9, 1916, to June 9, 1918.

*Find the interest on the following:*

7. $172.60 for 3 yr. 2 mo. 10 da. at 4%.
8. $391.75 for 3 yr. 8 mo. 10 da. at 4%.
9. $235.50 for 3 yr. 1 mo. 15 da. at 6%.
10. $175.50 for 2 yr. 8 mo. 10 da. at 6%.
11. $142.80 for 1 yr. 3 mo. 15 da. at 5%.
12. $273.40 for 2 yr. 8 mo. 20 da. at 5%.
13. $172.60 for 2 yr. 5 mo. 6 da. at $4\frac{1}{2}$%.
14. $182.60 for 4 yr. 8 mo. 15 da. at $5\frac{1}{4}$%.
15. $163.40 for 3 yr. 9 mo. 15 da. at $5\frac{1}{2}$%.
16. $625.50 for 2 yr. 11 mo. 15 da. at $5\frac{3}{4}$%.
17. $475.50 for 3 yr. 10 mo. 15 da. at $4\frac{1}{4}$%.
18. $265.50 for 2 yr. 10 mo. 20 da. at $4\frac{3}{4}$%.
19. $3562.50 for 3 yr. 2 mo. 5 da. at $5\frac{1}{2}$%.
20. $730, from August 17 to April 17, at 4%.
21. $1460, from May 23 to March 23, at 4%.
22. $1533, from February 1 to December 16, at $5\frac{1}{2}$%.
23. $7300, from March 3 to October 3, at $5\frac{1}{2}$%.
24. $3650, from May 23 to January 23, at $4\frac{1}{4}$%.
25. $4250, from April 7 to October 7, at 6%.
26. $2500, from May 9 to October 24, at 6%.
27. $1750, from January 21 to November 21, at 6%.
28. $1800, from February 17 to August 17, at 6%.
29. $3300, from March 5 to November 5, at 6%.
30. $2250, from May 4 to December 4, at 6%.
31. $4800, from March 2 to December 2, at 6%.
32. $1500, from May 6 to November 6, at 5%.
33. $3400, from June 5 to December 5, at 5%.

*Find the interest on \$275 at 5%, and also at 6%, for:*

**34.** 3 yr. 6 mo. 3 yr. 9 mo. 4 yr. 7 mo.

**35.** 3 yr. 6 mo. 10 da. 3 yr. 9 mo. 15 da. 4 yr. 7 mo. 20 da.

*Find the interest at 5%, and also at 6%, for 2 yr. 5 mo. 10 da. on the following sums:*

| | | | | | |
|---|---|---|---|---|---|
| **36.** | \$200 | \$2000 | \$20 | \$250 | \$2500 |
| **37.** | \$400 | \$4000 | \$40 | \$475 | \$4750 |

**38.** Find the interest on \$450 for 1 yr. 8 mo. 15 da. at:

4% 5% 6% $3\frac{1}{2}\%$ $4\frac{1}{2}\%$ $5\frac{1}{2}\%$

**39.** A man borrowed \$250 on January 15, at 6%. How much was the interest on October 15?

**40.** A man borrowed \$350 on January 27, at 6%. He paid the principal and interest on February 27 of the following year. How much did he pay?

**41.** A man bought 12 head of cattle on February 1, at \$45 a head, promising to pay for them later, with interest at 6%. He paid the bill on the 16th of the following April. What was the amount of principal and interest?

**42.** A man borrowed \$450 on March 10, at 6%, and \$1200 on April 10, at 5%. He paid the principal and interest in each case on December 30 of the same year. How much did he pay?

**43.** A man borrowed \$650 on May 1, at 5%, and \$1500 on July 5, at $4\frac{1}{2}\%$. He paid the principal and interest in each case on December 16 of the same year. How much did he pay?

**44.** What is the amount of principal and interest on \$750 borrowed February 10, at 6%, and \$1800 borrowed April 15, at 5%, the payment in both cases being made on September 20 of the same year?

**62. Partial Payments.** If a dealer holds a *note*, or a written promise to pay money, against one of his customers, and it bears interest, and partial payments are made, the amount due on the day of settlement is found in the following way:

*Add the interest to the principal whenever the payment, or the sum of the payments, equals or exceeds the interest.*

*Then deduct the payment or payments and continue as before.*

This is the United States Rule for partial payments, the legal one in most of our states. Where other rules are legal, teachers should explain the law and require that the problems be solved in accordance with that law.

For example, a note for \$1000, at 5%, is dated January 1, 1916. The following payments are indorsed (written across the back, as is the custom): July 1, 1916, \$10; January 1, 1917, \$40; July 1, 1917, \$20; January 1, 1918, \$130. How much is due July 1, 1918?

The interest due July 1, 1916, is \$25, and the payment is \$10. Hence the payment cannot be deducted. The reason is easily seen, for if we should take \$1025 − \$10 = \$1015 as the new principal, we should be drawing interest on more than the \$1000.

| | | |
|---|---|---|
| First principal | \$1000 | |
| Interest for 1 yr. | 50 | |
| First amount, January 1, 1917 | \$1050 | |
| 1st and 2d payments | 50 | |
| Second principal, January 1, 1917 | \$1000 | |
| Interest for 1 yr. | 50 | |
| Second amount, January 1, 1918 | \$1050 | |
| 3d and 4th payments | 150 | |
| Third principal | \$900 | |
| Interest for 6 mo. | 22.50 | |
| Due July 1, 1918 | \$922.50 | |

Therefore the amount due on the day of settlement is \$922.50.

### WRITTEN EXERCISE

**1.** A note for \$375, at 5%, has a payment of \$18.75 indorsed annually at the close of each year from its date, for three years. How much is due at the end of the fourth year?

**2.** A note for \$500, at 6%, is dated January 1, 1916, and has the following payments indorsed: July 2, 1916, \$100; December 1, 1916, \$2; January 1, 1917, \$98. How much is due July 1, 1917?

**3.** A note for \$300, at 6%, is dated April 1, 1916, and has the following payments indorsed: September 1, 1916, \$50; February 16, 1917, \$75; January 1, 1918, \$98. How much is due July 1, 1918?

**4.** A note for \$1800, at 5%, is dated April 16, 1916, and has the following payments indorsed: July 2, 1916, \$500; October 1, 1916, \$250; February 1, 1917, \$100; May 1, 1917, \$375. How much is due September 5, 1917?

**5.** A note for \$750, at 4%, is dated June 15, 1916, and has the following payments indorsed: March 6, 1917, \$200; September 6, 1917, \$150; January 17, 1918, \$250. How much is due May 14, 1918?

**6.** A note for \$600, at 5%, is dated July 2, 1916, and has the following payments indorsed: January 2, 1917, \$50; February 11, 1917, \$75; July 8, 1917, \$200; September 10, 1917, \$80; November 2, 1917, \$50. How much is due January 2, 1918?

Teachers should remember that the subject of partial payments has lost most of its practical value in the last few years. Money is now usually borrowed at a bank for 30 da., 60 da., or 90 da., and if a payment is made, this is done at the end of these periods and a new note is then given. The subject is not obsolete however, and it affords such excellent drill in computing interest that we have included it in these Essentials. In communities where the so-called Merchants' Rule is still used although commonly illegal, the teacher may find it desirable to mention it at least.

## LITTLE EXAMINATIONS

I. 1. Express 0.1% as a decimal fraction.

2. Express 1.2 as per cent.

3. Express $2\frac{1}{2}$ qt. as per cent of a gallon.

4. Express an inch as per cent of a yard.

5. Express $62\frac{1}{2}\%$, $87\frac{1}{2}\%$, and $83\frac{1}{3}\%$ as common fractions.

II. *Find the values of the following:*

1. 50% of 725.
2. 75% of $62\frac{1}{2}$.
3. $37\frac{1}{2}\%$ of \$7.36.
4. $33\frac{1}{3}\%$ of \$19.56.
5. $87\frac{1}{2}\%$ of \$19.28.
6. 0.3% of 174 ft.
7. 0.06% of 48,000.
8. 125% of 2756.
9. 1.25% of 2756.
10. 300% of $42\frac{7}{8}$.

III. 1. \$30 is what per cent of \$360? of \$3600? of \$36,000?

2. 8 in. is what per cent of 12 in.? of 12 ft.? of 12 yd.?

3. A pint is what per cent of 1 qt.? of 1 gal.? of 6 gal.?

4. 56 ft. is 8% of what distance? 4% of what distance?

5. 7 ft. 6 in. is 10% of what distance? 8% of what distance?

IV. 1. \$60.80 is 5% less than what amount?

2. \$102 is 15% less than what amount?

3. Find the interest on \$275 for 6 mo. 8 da. at 6%.

4. List price, \$450; discount 8%; find the net amount.

5. A bill of goods amounting to \$725 is allowed a discount of 15%. Find the net amount.

## REVIEW AND DRILL

### I. DRILL SECTION

#### WRITTEN EXERCISE

1. Write in words the numbers 400.075 and 0.475.
2. Write in Roman numerals the numbers 79 and 64.
3. Write in common figures the number seventeen million.

*Add, without writing in columns:*

4. \$2.20 + \$3.50 + \$1.75 + \$6.75 + \$9.30 + \$15.
5. 6 ft. 4 in. + 3 ft. 7 in. + 5 ft. 8 in. + 9 ft. 3 in. + 8 ft. 4 in.

*Subtract the following:*

6. \$497.63 − \$289.98.
7. \$601.71 − \$324.85.
8. \$510.00 − \$273.21.
9. \$812.27 − \$533.89.

*Multiply or divide as indicated, carrying the quotients to four figures only:*

10. 382 × \$296.50.
11. 427 × \$837.75.
12. 428.3 ÷ 29.4.
13. 59.68 ÷ 3.74.
14. 2.73 × 42.6.
15. 74.6 ÷ 2.13.

*Find the results of the following:*

16. 16% of \$325.
17. $5\frac{1}{4}$% of \$650.
18. $3\frac{1}{2}$% of \$785.

*Find the values of $x$ in the following:*

19. $x : 5 = 7 : 75$.
20. $5 : x = 75 : 7$.
21. $3 : 7\frac{1}{2} = x : 60$.
22. $4 : 9\frac{1}{4} = 16 : x$.
23. $1 : x = x : 1$.
24. $1 : x = x : 4$.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

**1.** If you buy a baseball bat and sell it at a certain rate below cost, how do you find the selling price?

**2.** If you buy a dog and sell him at a certain rate above cost, and know how much it has cost you to keep him while you owned him, how do you find whether you gained or lost, and the amount?

**3.** If you know the number of times a baseball player has been at bat during a season and the number of hits he has made, how do you find the percentage of hits?

**4.** If you know the number of times a player has been at bat during a season and the percentage of runs he has made, how do you find the number of runs?

**5.** If you know the number of pupils in a school and the number in Grade VII, how do you find the per cent that are in Grade VII?

**6.** If you know a man's income and the amount which he pays for house rent, how do you find the per cent of his income that is paid for house rent?

**7.** If you know the list price of some goods and the several rates of discount, what are you then able to find, and how do you find it?

**8.** A man borrowed a certain sum of money at a certain rate and for a certain length of time. The money was not paid when due, but some payments were made from time to time, each greater than the interest then due. Finally the money was paid. How do you find the amount due at the time of payment?

## III. INDUSTRIAL PROBLEMS

### BAKING BREAD

**1.** Harriet wishes to earn some money. She has learned to bake bread, and her mother suggests that Harriet make their bread and receive what is saved. If flour is worth \$6 per barrel (196 lb.), and $\frac{2}{3}$ lb. of flour makes a loaf of bread, and we allow \$4.29 per barrel of flour for the cost of the other ingredients and of the fuel, how much will it cost per loaf to make the bread?

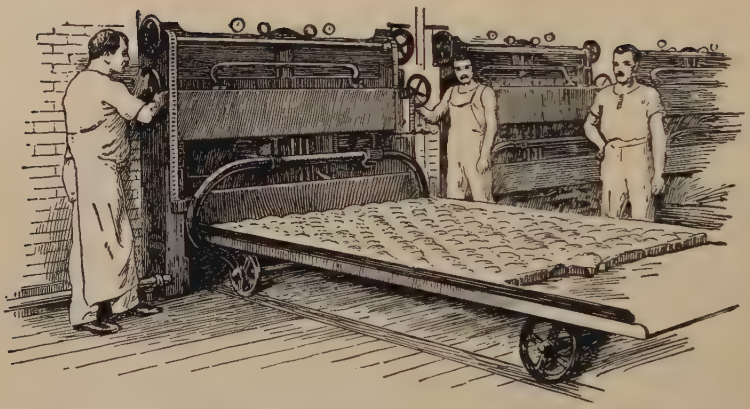
**2.** The family uses 3 loaves a day, and the baker's price is 5¢ a loaf. If Harriet bakes the bread, how much is saved every day, and how much does she earn in a year?

**3.** After a few weeks Harriet makes much better bread than the baker, and Mrs. Cate, a neighbor, wishes to buy 12 loaves a week. The bread is so good that she is willing to pay 90¢ each week for the 12 loaves. How much are Harriet's profits per year on these sales?

**4.** Another neighbor wishes to buy 2 loaves a day, at the same rate per loaf as Mrs. Cate pays. If Harriet agrees to this, how much is her yearly income from this source? How much is now her total yearly income from baking bread for the three families?

**5.** Harriet wants to earn \$2200 to pay her college expenses when she is old enough to go. At an average profit of 2.9¢, how many loaves would she have to bake to earn this amount?

**6.** Harriet's savings during 6 yr. are increased by $87\frac{1}{2}\%$, which her father gives her. The total amount is then invested so as to increase 15% by the time she goes to college. How much money does Harriet then have toward her college expenses? How much more money must she secure to complete the \$2200 necessary to pay her college expenses?



**7.** This picture shows the interior of a large bakery. The bakery averages 9000 loaves of bread a day, six days a week, and sells its bread to dealers at 4¢ a loaf. The dealers, on an average, sell 93% of the bread they get from the bakery, at 5¢ a loaf, and send back the balance. How much do the dealers receive for the bread they sell?

**8.** Of the bread sent back, the bakery is able to sell 60% at an average price of 2¢ a loaf, the rest being ground up daily and sold for $2.50 for the lot. What are the weekly receipts of the bakery from the bread? How much is this per loaf baked?

**9.** The bakery spends every week $1250 for flour, and $240 for various other materials; it pays its workmen $140; and the other charges are $25. What is the average cost per loaf of bread?

**10.** If the baker could make bread at 3.1¢ a loaf and sell it at 4¢ a loaf, what would be his per cent of profit?

**11.** In making bread the baker uses one third as much liquid as flour. If 1 pt. of liquid weighs 1 lb., how many pints of liquid will he use with 20 bbl. of flour?

# CHAPTER IV

## INDUSTRIAL APPLICATIONS

**63. Industrial Applications of Arithmetic.** You have now studied the most important operations of arithmetic, and are able to do any of the computation that you will meet in the ordinary problems of daily life. These important operations are the writing of numbers, addition, subtraction, multiplication, and division. These operations you have studied with whole numbers, with common fractions including ratios, with decimal fractions including per cents, and with compound numbers involving various measures.

You have also applied these operations to many problems, so that you know how each operation is used in a practical way. But up to this time you knew what operations were needed in most of the problems that you had to solve, because these problems came in chapters on certain subjects, such as addition, division, common fractions, or decimal fractions.

You are now ready to consider the simpler applications of arithmetic and to apply to each problem whatever operation may be needed for its solution. The leading applications are industrial; commercial, including banking, insurance, and a study of corporations; civic, including taxes and the expenses of the government; and scientific, including more difficult measurements. Problems in these various lines will be considered in the subsequent pages, particularly such problems as relate to the lives and interests of the people generally.

WOODWORKING OUTFIT

This price list may be used in solving the problems below:

| | | | |
|---|---|---|---|
| School workbench . | $8.50 | Plane . . . . . | $1.70 |
| Chisel, $\frac{1}{4}''$ . . . . | .20 | Vise for woodwork . | 8.25 |
| Chisel, $\frac{1}{2}''$ . . . . | .25 | Miter square . . . | .65 |
| Try-square . . . . | .25 | Steel square . . . | .75 |

**1.** What is the amount of a 10% discount on a workbench?

**2.** What is the cost of 5 quarter-inch chisels?

**3.** Find the cost of 3 half-inch chisels and a plane.

**4.** Find the cost of a vise, allowing 8% discount.

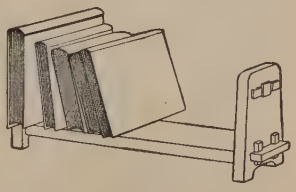
**5.** Find the cost of 2 miter squares and 3 steel squares.

**6.** Find the cost of 6 try-squares, allowing 6% discount.

**7.** A school buys 4 workbenches, 3 planes, and a vise for woodwork, and is allowed 12% discount. Find the net price.

### INDUSTRY IN THE SCHOOL

**1.** The class made this bookrack. If the rack will hold 12 books $1\frac{1}{2}$ in. thick, it will hold how many books $\frac{3}{4}$ in. thick?



**2.** The inside length of the shelf is 18 in., and each end piece is $1\frac{1}{4}$ in. thick, and the shelf projects $1\frac{1}{4}$ in. at each end. What is the total length?

**3.** The end pieces are $6\frac{1}{2}$ in. high. The shelf is $4\frac{1}{2}$ in. wide, and the greatest width of the end pieces is $5\frac{7}{8}$ in. Taking the total length of the shelf as found in Ex. 2, what length of board 6 in. wide will be needed for the entire shelf, allowing $\frac{3}{8}$ in. for waste in sawing and dressing?

**4.** The price of inch boards of this kind is \$80 per M. This means, as we have already learned, that 1000 sq. ft. of boards 1 in. thick cost \$80. At this rate, how much will 1000 sq. ft. of boards $1\frac{1}{4}$ in. thick cost?

**5.** If a shelf requires a board 32 in. long and 6 in. wide, how many square feet will such a board cover?

**6.** If a shelf requires a board 32 in. long and 6 in. wide, what will be the cost of the lumber at \$100 per M, if the board is 1 in. thick? if it is $1\frac{1}{4}$ in. thick?

**7.** If a shelf is $4\frac{1}{2}$ in. wide and 18 in. long, the width is what per cent of the length?

**8.** If an end piece is $6\frac{1}{2}$ in. high and $5\frac{7}{8}$ in. wide, the width is what per cent of the height?

**9.** If an end piece is $1\frac{1}{4}$ in. thick and $6\frac{1}{2}$ in. high, the thickness is what per cent of the height? The height is what per cent of the thickness?

**HOUSEHOLD INDUSTRY**

1. These girls are making 12 cups of cocoa in a saucepan. How large a pan must they use, estimating 2 cups to a pint and allowing an extra quart in the capacity of the pan?

2. If 12 cups of flour and 4 cups of liquid are used in making some bread, the liquid is what per cent of the mixture? If it should be a quarter of the mixture, is this mixture right?

3. If 1 cup of rice is cooked in 4 qt. of water and we allow 2 cups to a pint, the rice is what per cent of the total amount?

4. French salad dressing may be made with 6 tablespoons of oil and half as much vinegar, or a third as much vinegar. In each case the vinegar is what per cent of the dressing?

5. How large a kettle is needed for cooking 1 cup of rice in 4 qt. of boiling water, allowing 2 cups to a pint, and allowing $1\frac{1}{2}$ pt. extra in the capacity of the kettle?

6. In making ice cream the freezer should not be more than 75% full before freezing. How large a freezer will you need for $1\frac{1}{2}$ gal. of the liquid?

### HOUSEHOLD ECONOMICS

1. If a woman can buy corn at 12¢ a can or $1.30 per dozen, how much does she save on 24 cans in buying by the dozen?

2. If a woman can buy soup at 16¢ a can or $1.80 per dozen, how much does she save on 36 cans in buying by the dozen?

3. A dealer sells coffee in half-pound packages at 17¢ a package or in 5-pound cans at $1.64 a can. If a woman wishes 5 lb., how much does she save in purchasing by the can?

4. A woman can buy Dutch cocoa in $\frac{1}{4}$-pound boxes at 22¢ a box or in 5-pound cans at $2.78 a can. If she wishes 5 lb., how much does she save in purchasing by the can?

5. A woman can buy maple sirup at 44¢ a quart or in gallon cans at $1.38 a can. If she wishes 4 qt., how much does she save in purchasing by the can?

6. A woman buys a pound each of mustard seed, allspice, and cayenne pepper, paying 60¢ for the three. The mustard seed costs 12¢ a pound, and the allspice costs the same as the pepper. How much does the allspice cost?

7. A woman bought a can each of cinnamon, nutmeg, and pepper, paying 39¢ for the three. The pepper cost 9¢, and the nutmeg cost twice as much as the cinnamon. How much did the cinnamon cost?

8. If flour costs $6.70 a barrel (196 lb.) or 4¢ a pound, how much does a family save in purchasing flour by the barrel if it requires 196 lb.?

9. If a family's ice bill is $1.50 a month, and ice costs 30¢ per 100 lb., how many pounds does the family use? If, by having a better ice box, only 0.9 as much ice is used, how many pounds are used?

HELPING AT THE MARKET

1. Henry helps his father at the market. They have been buying grapefruit, 64 to a box, at \$4.50 a box, and selling them at 10¢ each, averaging two boxes a day. Henry notices that another dealer sells two boxes a day of larger grapefruit, 48 to a box, at 15¢ each. They can buy the larger fruit at \$5.50 a box. Henry thinks it would be better to deal in the larger fruit. Is he right? What is the difference in income?

2. They find that they can buy a box of 360 lemons for \$5.20, selling them at 22¢ a dozen. How much do they gain on a box? What per cent do they gain on the money invested? Is this more profitable, in per cent on the investment, than to buy a box of 196 oranges for \$5.30, selling them at 40¢ a dozen?

3. Henry's father pays \$8 a month for the rent of his stall in the market, \$3.25 a month for light, and \$2.25 a day for help, besides allowing Henry 10¢ an hour. Henry works 12 hr. a week. There being 313 market days in a year, how much are the expenses for a year?

### SEWING AT HOME

**1.** If a sheet is $2\frac{1}{2}$ yd. long and $5\frac{1}{2}$ in. is allowed for hemming the ends, how many sheets can be cut from 54 yd. of the right width? Will there be any material left over?

**2.** In buying sheeting, the width of the material is given in quarters of a yard. If sheeting is $\frac{5}{4}$ (5 quarters) wide, what is the width in yards and a fraction? in inches?

**3.** How many sheets for single beds can be made from a piece of sheeting $\frac{6}{4}$ wide and 64 yd. long, each sheet being $2\frac{1}{2}$ yd. long, with a top hem $3\frac{1}{2}$ in. and a bottom hem 2 in., an additional allowance of $\frac{1}{4}$ in. being made for each hem?

**4.** A piece of sheeting of $\frac{8}{4}$ width contains 56 yd. How many sheets for three-quarter beds can be made from it, allowing the same measurements as in Ex. 3?

**5.** A piece of sheeting of $\frac{10}{4}$ width contains 48 yd. How many sheets for double beds can be made from it, allowing the same measurements as in Ex. 3?

**6.** Dish toweling can be bought for \$5 per roll of 50 yd., or at the rate of 12¢ a yard. What per cent is saved in purchasing 50 yd. by the roll instead of by the yard?

**7.** Allowing 27 in. for each towel, how many dish towels can be made from a piece of toweling 39 yd. long?

**8.** A housekeeper wishes to make two curtains for each of four windows. The windows are 6 ft. 9 in. high, and the curtains are to be the same length. She wishes to put a hem 3 in. wide at the bottom and another $1\frac{1}{2}$ in. wide at the top, allowing $\frac{1}{4}$ in. for finishing each hem. She can buy a 20-yard piece of curtain material for \$3, losing what she does not need, or she can purchase exactly the amount she needs at 16¢ a yard. Which is the more economical plan, and how much more economical?

### PROBLEMS OF THE DRESSMAKER

1. A dressmaker bought 32 yd. of chiffon velvet at \$4.10 a yard. The dealer allowed her a discount of 8%, 3%. She sold the velvet to a customer at \$4.25 a yard. How much did she gain?

2. A dressmaker bought 50 yd. of silk at 95¢ a yard, less a discount of 10%, 2%. She sold half of it at \$1.20 a yard and the remainder at \$1.10 a yard. How much did she gain?

3. If it takes $2\frac{1}{2}$ yd. of thread to stitch 1 yd. of cloth, how many spools of 50 yd. each will it take to stitch 40 yd. of cloth?

4. At $3\frac{5}{8}$ yd. to a skirt, how many yards of ruffling will be needed for a dozen skirts?

5. How many quarter-inch tucks can be made in a piece of cloth $6\frac{1}{2}$ in. long, allowing $\frac{1}{4}$ in. between the tucks and $\frac{3}{8}$ in. at each end of the cloth?

6. If a skirt requires $4\frac{3}{8}$ yd. of lawn, what is the cost of the material for two skirts, the lawn costing 23¢ a yard, and the extras costing 11¢ per skirt?

7. A dressmaker used for a shirt waist for one of her customers $2\frac{1}{2}$ yd. of Persian lawn costing 28¢ a yard, and $3\frac{3}{4}$ yd. of embroidery costing 20¢ a yard. What fraction of the cost of the embroidery was the cost of the lawn? Express this as a common fraction and also as a decimal.

8. If $3\frac{1}{2}$ yd. of material 32 in. wide are required for a garment, how much material 42 in. wide will be required, if it cuts as economically?

9. If material 32 in. wide costs \$1.50 a yard, what will be the cost of material of the same quality 42 in. wide, at the same rate per square yard?

### PROBLEMS OF THE FARMER

**1.** A farmer asks his son to measure a field for him. The boy finds the field to be 20 rd. by 80 rd. How many acres does it contain?

**2.** In measuring the field the boy finds that on an average a strip 2 ft. wide, next to the fence inclosing the field mentioned in Ex. 1, is not cultivated, and that 1 ft. 6 in. of it could be cultivated as easily as the rest of the field. How many square feet of land are needlessly wasted?

**3.** The boy has a garden 60 ft. by 100 ft. which he must water on account of the dry weather. If a cubic foot of damp clay loam contains on an average 18 lb. of water, what is the weight in tons of water in the garden to the depth of a foot, when the soil contains as much water as damp clay loam?

**4.** The farmer wishes to fence in the garden mentioned in Ex. 3, using four strands of barbed wire. How many pounds of barbed wire will he need, allowing 1 lb. of wire to a rod?

**5.** On the farm is an artesian well producing $3\frac{1}{4}$ cu. ft. of water per minute. If a cubic foot of water weighs $62\frac{1}{2}$ lb., how long will it take the well to produce half enough water required for the garden as found in Ex. 3?

**6.** A farm wagon costing \$75 depreciates in value about $6\frac{2}{3}\%$ a year if handled with reasonable care and kept in a shed when not in use. If left out in the yard it depreciates about twice as much. In three years what would be the loss by leaving it out, over the depreciation if it were carefully kept?

**7.** A farmer allows 350 cu. ft. of solidly stacked hay to the ton. How many tons in a mow containing 4375 cu. ft.?

**8.** If corn fodder loses 60% of its weight in curing, how much will 28 T. of green fodder weigh after it is cured?

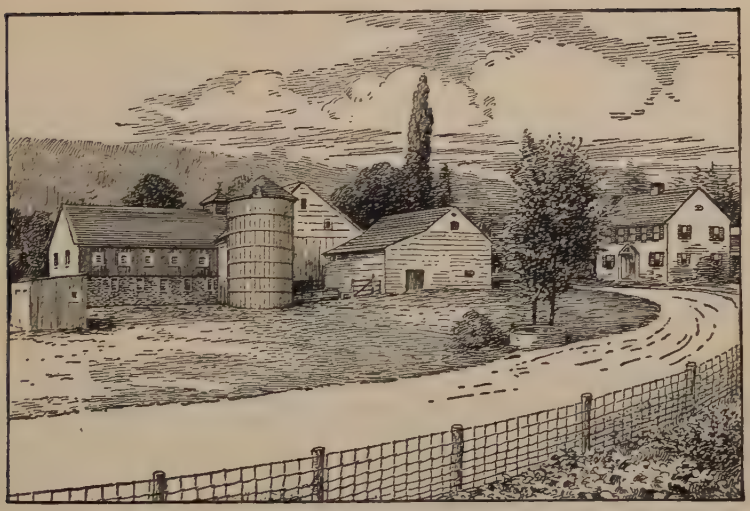
### CORN AS BEEF AND PORK

1. On a farm where corn was selling at 30¢ a bushel it was decided to feed the corn to cattle. It was estimated that the increase in the value of the cattle, from the corn alone, was 40¢ for each bushel used for feed. What was the per cent of increase in the value of the corn by using it as feed?

2. On another farm corn was selling at 31¢ a bushel. Estimating that it was worth $62\frac{1}{2}$¢ when fed to cattle, find the per cent of increase in the value of the corn by using it as feed.

3. A record of the result of feeding corn to hogs was kept on several farms. On one farm, where corn was selling at 35¢ a bushel, it was found that the increase in the value of the hogs was equivalent to 79¢ per bushel of corn fed. What was the per cent of increase in the value of the corn by using it as feed?

4. On another farm the figures of Ex. 3 were 32¢ a bushel for corn when sold, and 64¢ a bushel when used as feed. What was the per cent of increase in the value of the corn?



**PROBLEMS ABOUT A SILO**

**1.** A farmer thinks of building a silo with inside measurements 12 ft. long, 10 ft. wide, 28 ft. high. Allowing 40 cu. ft. to a ton of ensilage, how many tons will this silo contain?

**2.** He decides, however, to build the silo in the more modern form, with a circular base. The rule for finding the volume of such a silo is to multiply the height by $\frac{22}{7}$ times the square of the radius of the base. If the radius of the base is 7 ft. and the height is 28 ft., how many cubic feet will the silo contain?

**3.** Allowing 40 cu. ft. to a ton of ensilage, how many tons can be placed in the silo mentioned in Ex. 2?

**4.** The concrete floor of the silo in Ex. 2 is 22 in. thick. By the rule given in Ex. 2, find the amount of concrete needed.

In places where silos are not used this page may be omitted. The teacher should explain that the allowance in Exs. 1 and 3 is only approximate. The more ensilage is put in the more it is pressed down.

### THE FARMER'S INCOME

1. James's father owns two farms of the same size and value. One he runs himself and the other he lets on shares. James asked his father if it was worth while to learn any more, and his father said: "Let us see what brains do on our property. Here is the total income of the farm we let on shares, and there are my books for last year. Now see the result." This is what James found:

| | *Home farm* | *Farm rented* |
|---|---|---|
| Dairy products | $248.60 | $93.75 |
| Wool | 21.75 | 10.43 |
| Eggs and poultry | 232.60 | 80.04 |
| Domestic animals | 595.75 | 288.17 |
| Crops | 975.30 | 507.22 |

Find the total receipts on each farm.

2. James's father told him that the tenant had very little schooling as a boy and that he did not read the agricultural papers, and he asked James to find the per cent of increase of each of their items over the tenant's. How much was this?

3. He told James that the expenses of the two places showed poor management by the tenant, and showed him these figures:

| | *Home farm* | *Farm rented* |
|---|---|---|
| Labor | $175.45 | $102.43 |
| Fertilizers | 86.75 | 10.00 |
| Feed | 52.60 | 47.13 |
| Maintaining buildings | 22.60 | 49.72 |
| Maintaining equipment | 18.75 | 39.78 |
| Taxes and miscellaneous | 85.70 | 83.03 |

Find the total expenses, the difference in each pair of items, the difference in the total expenses, and the net gain of each farm.

### PROBLEMS OF THE PAY ROLL

**1.** A mason in a certain city receives 70¢ an hour, and a plasterer $62\frac{1}{2}$¢ an hour. How much will 8 masons and 4 plasterers receive for 8 hr. of work?

**2.** The following is last week's pay roll of a manufacturer:

PAY ROLL *For the week ending* * 19 *

| No. | Name | No. of Hours per Day | | | | | | Total Time | Wages per Hour | Total Wages | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | M. | T. | W. | T. | F. | S. | | | | |
| 1 | J. P. Driscoll | 8 | 8 | 8 | $6\frac{1}{2}$ | 8 | 4 | $42\frac{1}{2}$ | 50¢ | $21 | 25 |
| 2 | R. L. Bennett | 8 | $7\frac{1}{2}$ | 8 | 8 | 8 | 4 | * | 50¢ | * | * |
| 3 | P. F. Purvis | 7 | $7\frac{1}{4}$ | 8 | $7\frac{1}{2}$ | 8 | 4 | * | 50¢ | * | * |
| 4 | B. J. Mead | 8 | 7 | 6 | 8 | $7\frac{1}{2}$ | 3 | * | 55¢ | * | * |
| 5 | R. K. King | 8 | 8 | 7 | $7\frac{3}{4}$ | 8 | $3\frac{1}{2}$ | * | 55¢ | * | * |
| 6 | M. L. Drake | 8 | 6 | 8 | 0 | 7 | $3\frac{3}{4}$ | * | $27\frac{1}{2}$¢ | * | * |
| | | * | * | * | * | * | * | * | | * | * |

Fill out the spaces marked with an asterisk (*).

*Make out pay rolls (inserting names), when the numbers, the hours per day, and the wages per hour are as follows:*

**3.** No. 1: 8, 8, 8, 8, $7\frac{1}{2}$, 6, $62\frac{1}{2}$¢; No. 2: 8, 8, 8, 7, 6, 8, $57\frac{1}{2}$¢; No. 3: 8, $7\frac{1}{2}$, 8, 8, 8, 6, $48\frac{1}{4}$¢; No. 4: 8, 8, 8, 8, $7\frac{1}{2}$, 8, $66\frac{2}{3}$¢; No. 5: 8, 7, 8, 8, 8, $6\frac{1}{2}$, 72¢.

**4.** No. 1: 8, 7, 6, 8, 8, 8, $67\frac{1}{2}$¢; No. 2: 8, 7, 6, $8\frac{3}{4}$, $8\frac{3}{4}$, $8\frac{3}{4}$, 60¢; No. 3: 8, 6, 6, 8, 8, $7\frac{3}{4}$, 62¢; No. 4: 7, 8, $7\frac{3}{4}$, $6\frac{1}{2}$, 8, 8, 57¢; No. 5: 8, 6, 7, $7\frac{1}{2}$, $6\frac{1}{4}$, 8, 48¢.

**5.** No. 1: 8, 8, 8, 8, 8, 8, $62\frac{1}{2}$¢; No. 2: 8, 8, $7\frac{3}{4}$, $7\frac{1}{2}$, $7\frac{3}{4}$, $7\frac{3}{4}$, 60¢; No. 3: 8, 8, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, $7\frac{3}{4}$, 58¢; No. 4: $7\frac{1}{2}$, $7\frac{1}{4}$, $7\frac{1}{2}$, 8, $7\frac{3}{4}$, $7\frac{3}{4}$, 62¢; No. 5: 8, 8, 8, 8, 8, 8, $63\frac{1}{3}$¢.

### OWNING A HOUSE

**1.** John's and Helen's father rents the house in which they live. He pays as rent \$22 a month. How much does this amount to in a year?

**2.** Their father has saved \$3000, which he has invested in 5% bonds. How much would he lose in interest on his money if he bought the house and lot for this amount?

**3.** He asks John and Helen to figure out whether it would pay to own a house instead of renting one. First they put down as loss the interest on \$3000 at 5%. Then they put down 2% of \$3000 for yearly repairs, \$48.50 for taxes, and \$6.50 for fire insurance. So far, what is the total annual expense of owning their home? How does this compare with their annual rent?

**4.** John and Helen conclude that there is no great advantage in owning a house, except for the pleasure, and they cannot measure this in money. But they know that a neighbor bought a house for \$2400 and sold it for \$3120. He gained what per cent on his investment? If their father could gain the same per cent, how much would he gain on the cost of the house?

**5.** But people do not always gain on such a transaction. Their father might have to sell again for only \$2700. In this case what would be his per cent of loss?

**6.** John's and Helen's father finally decides to risk the purchase. But he finds that he has underestimated the repairs, which average \$75 a year for the next four years. His taxes, however, come only to \$35 a year, and his insurance costs \$15 for a four-year policy. At the end of four years he sells the property for \$3600. Meantime he has lost the interest on his \$3000 at 5%, but has saved his rent of \$22 a month. What per cent on \$3000 does his investment net him per year?

### HEATING THE HOUSE

**1.** Maude's father put a steam-heating plant in his house last winter. In a season it consumes 15 tons of chestnut coal at $7 per ton. How much does the coal cost?

**2.** Of this coal, 15% is lost in ashes, the rest being turned into "heat units," as the dealers say; that is, 85% is really used for heating. How many pounds of coal are lost in ashes? How many pounds are turned into heat units?

**3.** The house has 10 rooms and was heated for 212 da. What was the average cost per day of heating the house? What was the average cost per room per day?

**4.** Maude's father paid $325 for putting in the steam-heating plant. If it consumes, on an average, 15 tons of coal at $7 per ton each year, and lasts 10 yr., we have to consider the loss of interest at 6% on $325, the final loss of the $325 through the loss of the heating plant, the cost of the coal, and the loss of interest on the money paid for the coal. Taking this last item as $20, in round numbers, find the average annual cost of heating the house.

**5.** Maude's uncle, Mr. Preston, thought it would be better for him to put in a hot-water heater costing $500. This consumes 10 tons of coal per year at $7 per ton. Considering only the first cost of the two heating plants, and the cost of the coal consumed, which, if either, is the cheaper plant at the end of 5 yr.? How much cheaper?

Problems concerning the cost of heating and lighting the home and the school building should be made up from actual data. If possible, some boy in the class should measure the amount of coal put into a furnace during the day, and the amount of ashes taken out, so as to give the class a practical knowledge of the amount of coal used in heating a house, and the per cent of coal that actually goes into heat.

## LITTLE EXAMINATIONS

I. 1. A dealer sold some fruit for $12\frac{1}{2}\%$ less than it cost, thereby losing \$140. What was his selling price?

2. How much will 360 ft. of iron pipe cost if listed at \$1.50 a foot, a discount of 40% and 10% being allowed?

3. How much will 4 gross of jars of mixed pickles cost at \$6 a dozen, a discount of 20% and 10% being allowed?

4. A man's salary of \$2100 a year was increased 10% and then it was decreased 10%. How much was it then?

5. If the ratio of cream to the rest of the milk of a certain quality is 1:5, how many pounds of cream in 750 lb. of milk?

II. 1. A triangular gable to a house has a base 36 ft. and an altitude 12 ft. 8 in. Find the area of the gable.

2. If 72 reams of paper are used in printing 3800 books, how many reams of paper will be needed for printing 9800 books, each $\frac{5}{8}\frac{7}{4}$ as large?

3. If the ratio of butter fat to the rest of the milk of a certain quality is 1:24, how many pounds of butter fat in 1050 lb. of milk of that quality?

4. If the ratio of solid matter to the rest of the milk of a certain quality is 33:217, how many pounds of solid matter in 375 lb. of milk of that quality?

5. A hardware dealer bought a shipment of stoves. After deducting a discount of $33\frac{1}{3}\%$ and 25%, and paying \$5 cartage and \$19.50 freight, the stoves cost him \$504.50. What was the list price of the stoves?

# REVIEW AND DRILL

## I. DRILL SECTION

### WRITTEN EXERCISE

1. Write in ordinary numerals the numbers five hundred five thousandths, and five hundred and five thousandths.

2. How many years from MDCCCLXXVI to MDCCCCXX, or MCMXX, as it is also written?

*Add, and also subtract:*

| 3. | 4. | 5. | 6. |
|---|---|---|---|
| \$427.32 | \$932.01 | \$870.23 | \$900.01 |
| 198.56 | 687.56 | 592.68 | 48.75 |

*Multiply the following:*

| 7. | 8. | 9. | 10. |
|---|---|---|---|
| \$423.78 | \$573.81 | \$609.98 | 23 ft. 8 in. |
| 65 | 74 | 127 | 15 |

*Divide the following, in turn:*

11. 864 by 16, 32, and $\frac{4}{5}$.

12. 9.72 by 3.6, 0.36, and 36.

13. 58 ft. 8 in. by 8.

14. 64 lb. 2 oz. by 9.

*Express as decimal fractions and also as per cents:*

15. $\frac{1}{2}$. 17. $\frac{3}{4}$. 19. $\frac{5}{8}$. 21. $\frac{2}{3}$. 23. $\frac{5}{6}$.

16. $\frac{1}{4}$. 18. $\frac{3}{8}$. 20. $\frac{7}{8}$. 22. $\frac{3}{5}$. 24. $\frac{7}{16}$.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

**1.** A man buys a certain number of pecks of chestnuts. How do you find how many bushels he buys?

**2.** Given the length of a sheet and the amount to be added for hemming, how do you find the number of sheets that can be made from a given number of yards of sheeting of the right width?

**3.** If you know the number of yards of lawn needed to make a skirt and the price per yard, how do you find the cost of the lawn for a given number of skirts?

**4.** If you know the number of cups of flour needed for a certain number of loaves of bread, how do you find the number of cups of flour needed for a certain other number of loaves?

**5.** If you know how much water flows through a pipe in a minute, how do you find the amount of water that will flow through the pipe in a whole day, at the same rate?

**6.** If a boy wishes a piece of board to make a bookshelf, how do you find the cost of the board, knowing the size of the shelf and the cost of the lumber per M?

**7.** If you know how much per foot an iron rod expands in length when heated from the ordinary temperature to red heat, how do you find the amount of expansion of an iron rod a given number of inches in length, when heated to red heat?

**8.** If a salesman sells on a certain salary plus a certain commission, how do you find the amount of his income for a year?

**9.** If you know the number of hours per day that a man works, for each day of the week, and his rate of wages per hour, how do you find the amount due him at the end of the week?

## III. INDUSTRIAL APPLICATIONS

THE DAIRY INDUSTRY

**1.** If the cream from this model dairy contains 25.9% butter fat, what is the weight of butter fat in 970 lb. of cream?

**2.** If a cow's average daily ration is 28 lb. of ensilage, 12 lb. of clover hay, $6\frac{1}{2}$ lb. of bran, and $3\frac{1}{2}$ lb. of corn meal, what per cent of the total is each kind of food?

**3.** If each cow averages 4400 lb. of milk per year and the milk contains 4.3% butter fat, how many pounds of butter fat will 32 cows produce in a year?

**4.** If a cow in a year eats $3\frac{1}{4}$ T. of hay at \$12 a ton, and 900 lb. of ground feed at 85¢ per C, and if \$6 per cow is allowed for pasturage, how much will it cost to keep 32 cows for 1 yr.?

**5.** Allowing 4 ft. 3 in. by 9 ft. 6 in. of floor space for a stall and 4 ft. 3 in. by 3 ft. extra for a manger, how much floor space will be required for 32 cows?

In rural schools there should be special computations suited to farm practice, on such subjects as rations, the cost of labor, and the income from cows.

### THE COTTON INDUSTRY

**1.** A mill bought 10 bales of cotton weighing respectively 489 lb., 513 lb., 478 lb., 492 lb., 521 lb., 498 lb., 503 lb., 496 lb., 503 lb., and 517 lb. If the mill bought on the basis of 500 lb. of cotton to the bale and paid 14¢ a pound, did it gain or lose by the variation in weight, and how much?

**2.** A mill bought 34 bales of Egyptian raw cotton, averaging 800 lb. each. At 30¢ a pound, how much did the shipment cost?

**3.** A New Jersey manufacturer agreed to take $12\frac{1}{2}\%$ of a cargo of cotton shipped from New Orleans to New York. The cargo consisted of 4864 bales averaging 500 lb. each, and the manufacturer paid 14¢ per pound for the cotton. How many bales did he buy, and how much did they cost him?

**4.** A certain cotton mill used in one day 4840 lb. of Egyptian cotton valued at 31¢ a pound, 12,674 lb. of upland cotton valued at 13¢ a pound, and 1247 lb. of sea-island cotton valued at 62¢ a pound. Find the total value of the cotton used that day.

**5.** A pound skein of No. 1 cotton yarn contains 840 yd. How many yards are there in 840 lb. of cotton yarn?

**6.** A pound skein of No. 3 cotton yarn contains 2520 yd. How many yards are there in 2520 such skeins?

**7.** A certain manufacturer finds that in cleaning baled cotton he has to allow 30% for waste. If he uses 48 bales of 500 lb. each, how many pounds of waste are there?

**8.** The teacher took the class to a mill one day to see cotton thread made. The foreman told them that the frames for spinning a certain size of cotton thread consist of three sides with 104 spindles to each side, and that the room they were visiting had 72 such frames. He then asked them to find the number of spindles in the room. How many were there?

# CHAPTER V

## BANKS AND BANKING

**64. Bank Account Essential.** One thing that is essential at some time to any one who hopes to succeed is a bank account. A reliable person may "open an account," as it is called, as soon as he begins to save even a small amount.

**65. Savings Bank.** To deposit money a person goes to a bank, says that he wishes to open an account, and leaves his money with the officer in charge. The officer gives him a book in which the amount is written, and the depositor writes his name in a book or on a card, for identification. When he wishes to draw out money, he takes his book to the bank, signs a receipt or a check for the amount he desires, and receives the money.

### WRITTEN EXERCISE

1. How much does 25¢ saved each day amount to in a year?

2. How much will 15¢ saved each day, 310 days to the year, amount to in 10 years, not counting interest?

3. If a boy, beginning when he is 16 yr. old, saves 25¢ a day for 300 da. a year, and deposits it in a savings bank, how much will he have when he is 21 yr. old, not counting interest?

4. If a father gives his daughter $1 when she is 1 yr. old, $2 on her next birthday, and so on until she is 21 yr. old, depositing it for her in a savings bank, how much will she have when she is 21 yr. old, not counting interest?

**5.** A man saves on an average 25¢ a day the first year he works on a certain farm, and 20% more the second year, depositing it in a savings bank. If he works 306 days each year, how much does he save in the two years?

**6.** A farmer's savings-bank deposits average $25 a month during a certain year, and 97% as much during the next year. The year following he deposited 110% as much as he did the second year. How much did he deposit in the savings bank during the three years?

**7.** If a man saved $1 a week during the year in which he was 21 years old, and increased his savings each year by 10% on the amount of the preceding year, and worked 50 weeks a year, how much would he save during the year in which he was 29 years old?

**8.** A merchant saves $375 the first year he is in business. The second year he increases his savings one third. The third year they are only 85% as much as the second year. The fourth year they increase 30% over the third year. How much does he save in the four years?

**9.** A man works on a salary of $15 a week for 50 weeks in a year. His expenses are $15 a month for house rent, 60% as much for clothing, 300% as much for food as for clothing, and 20% as much for other necessary expenses as for food. How much of his salary can he reserve for the savings bank?

**10.** A clerk in a store had a salary of $12 a week two years ago, and a commission of 2% on the goods he sold. That year he worked 48 weeks and sold $2350 worth of goods. Last year his salary was increased 25%, his commission remaining the same. He worked 49 weeks and sold $2750 worth of goods. How much was his income increased?

**66. Interest at Savings Banks.** Savings banks usually pay interest every six months or every three months. This interest is added to the principal, and the total amount draws interest.

**67. Compound Interest.** When interest as it becomes due is added to the principal and the total amount then draws interest, the investor is said to receive *compound interest* on his money.

Compound interest is no longer allowed on notes; but if one collects his interest when it is due and at once reinvests it, of course he practically has the advantage of compound interest. The method of finding compound interest is substantially the same as that used in simple interest.

For example, required the amount of \$2000 in 2 yr., deposited in a savings bank that pays 4% annually, the interest being compounded semiannually. How much is the compound interest?

| | |
|---|---|
| \$2000. | = first principal |
| .02 | |
| \$40. | = int. first 6 mo. |
| 2000. | |
| \$2040. | = amt. after 6 mo. |
| .02 | |
| \$40.80 | = int. second 6 mo. |
| 2040. | |
| \$2080.80 | = amt. after 1 yr. |
| .02 | |
| \$41.62 | = int. third 6 mo. |
| 2080.80 | |
| \$2122.42 | = amt. after $1\frac{1}{2}$ yr. |
| .02 | |
| \$42.45 | = int. fourth 6 mo. |
| 2122.42 | |
| \$2164.87 | = amt. after 2 yr. |
| 2000. | |
| \$164.87 | = int. for 2 yr. |

Simple interest for the same time is \$160, or \$4.87 less than the compound interest.

In this example the compound interest has been found exactly, but savings banks pay interest only on the dollars and not on the cents. In this example the compound interest would be \$164.84 instead of \$164.87. The saving to the bank in the work of computation is considerable.

**68. Savings Bank Account.** Usually savings banks pay interest semiannually, adding this to the principal on deposit. This is, therefore, a case of compound interest. The following is a specimen account at a bank paying 4% interest, payable semiannually, on the smallest balance on deposit in an interest period.

| Date | | Deposits | | Interest | | Payments | | Balance | |
|---|---|---|---|---|---|---|---|---|---|
| 1917 | | | | | | | | | |
| July | 1 | $600 | 50 | | | | | $600 | 50 |
| July | 20 | 75 | | | | | | 675 | 50 |
| Sept. | 6 | | | | | $120 | | 555 | 50 |
| Dec. | 7 | 60 | | | | | | 615 | 50 |
| Dec. | 20 | | | | | 65 | | 550 | 50 |
| 1918 | | | | | | | | | |
| Jan. | 1 | | | $11 | | | | 561 | 50 |
| May | 9 | 200 | | | | | | 761 | 50 |
| July | 1 | | | 11 | 22 | | | 772 | 72 |

Here the smallest balance during the first interest period was $550.50. Interest is computed on the dollars only, the cents being neglected. At 2% the interest on $550 is $11. In the second period the smallest balance is $561.50, the interest being 2% of $561, or $11.22.

Some banks allow interest from the first of each month; others from the first of each quarter; others, as above, from the first of each half year. The interest is computed on the smallest balance on hand between this day and the next interest day, and is usually added every half year.

In the above example, if the interest period had been 3 mo. and the interest had been credited every 6 mo., the interest would have been:

First quarter, 1% of $555 = $5.55.
Second quarter, 1% of $550 = 5.50. Credit $11.05.
Third quarter, 1% of $561 = 5.61.
Fourth quarter, 1% of $561 = 5.61. Credit $11.22.

In this case the difference in interest would have been only five cents.

### WRITTEN EXERCISE

*At simple interest, and also at interest compounded annually, find the amount of principal and interest:*

1. \$2500, 4 yr., 5%.
2. \$4000, 3 yr., 6%.
3. \$3500, 3 yr., 4%.
4. \$2250, 4 yr., 3%.
5. \$2750, 4 yr., 3%.
6. \$1750, 3 yr., $3\frac{1}{2}$%.
7. \$625.50, 4 yr., $3\frac{1}{2}$%.
8. \$10,000, 6 yr., 4%.
9. \$425.50, 4 yr., $4\frac{1}{2}$%.
10. \$275.50, 3 yr., $4\frac{1}{2}$%.

*Find the amount of principal and interest, the interest being compounded semiannually:*

11. \$200, 3 yr., 4%.
12. \$300, 2 yr., 4%.
13. \$650, 2 yr., 6%.
14. \$700, 3 yr., 3%.
15. \$800, 3 yr., 4%.
16. \$500, 2 yr., 4%.
17. \$1000, 2 yr., $4\frac{1}{2}$%.
18. \$1000, 3 yr., 4%.
19. \$2000, 2 yr., $3\frac{1}{2}$%.
20. \$3000, 2 yr., $2\frac{1}{2}$%.

21. Which brings the more interest in 2 yr. on \$1250, 4% in a savings bank, compounded semiannually (the money being deposited at the beginning of the year), or $4\frac{1}{2}$% simple interest?

22. If a man on January 1 deposited \$2000 in a savings bank, and left it for 2 yr., at 4%, the interest being compounded semiannually, how much less interest would he receive than by lending it at 5% simple interest?

23. If a man deposits \$200 in a savings bank on January 1, \$300 on February 1, \$100 on May 1, \$400 on August 1, and \$350 on November 1, and the rules of the bank allow 4% interest on all these sums, compounding it on July 1 and on January 1, how much will the man have to his credit on the January 1 following these deposits?

**69. Postal Savings Bank.** The United States government conducts a savings bank in connection with the post office. Although all savings banks are very carefully regulated and inspected by the state governments, there are many persons who prefer a smaller rate of income than ordinary savings banks pay, so long as they know that our government guarantees the payment of their money when they wish to use it.

Postal savings banks allow any person of the age of 10 yr. or over to deposit money in amounts of not less than \$1, but no fractions of a dollar are accepted for deposit. No one can have a total balance of more than \$2500, exclusive of accumulated interest. Deposits are made at the larger post offices, and postal savings certificates are given to the depositor. Interest is paid by the government at the rate of 2% a year, beginning on the first day of the month following the one in which the deposit is made. Interest is not paid for any fraction of a year. A person may exchange his deposits in sums of \$20 or multiples of \$20 for government bonds bearing $2\frac{1}{2}\%$.

The following exercise refers to postal savings banks.

ORAL EXERCISE

*Find the interest for 1 yr. on the following deposits:*

**1.** \$20. **2.** \$50. **3.** \$100. **4.** \$200. **5.** \$400.

*Find the interest for 2 yr. on the following deposits:*

**6.** \$10. **7.** \$40. **8.** \$200. **9.** \$300. **10.** \$500.

*Find the interest received for 1 yr. by a holder of $2\frac{1}{2}\%$ bonds of:*

**11.** \$100. **12.** \$200. **13.** \$300. **14.** \$400. **15.** \$500.

**16.** How much does a depositor increase his annual income by transferring \$400 of deposits into a government bond?

**70. Bank of Deposit.** When a man has money enough ahead to pay his bills by checks, he will find it convenient to have an account with a bank such as merchants commonly use, sometimes called a *bank of deposit.*

Such banks do not pay interest on small accounts, the deposit being a matter of convenience and safety. If a man wishes to open an account he sometimes has to give references, for banks do not wish to do business with unreliable persons.

SECOND NATIONAL BANK
KANSAS CITY, MO.

Deposited for the account of

.....

Date .....19.....

| | | |
|---|---|---|
| Bills . . . . . | | |
| Gold . . . . . | | |
| Silver and small coin . | | |
| Check on .....Bank | | |
| Check on .....Bank | | |
| Total | | |

**71. Deposit Slip.** When a man makes a deposit of money or checks in a bank, he fills out a *deposit slip*, as shown above, and leaves it at the bank.

### ORAL EXERCISE

*Give the sum of each of the following lists of deposits:*

| 1. | 2. | 3. | 4. | 5. |
|---|---|---|---|---|
| $15.50 | $12.75 | $23.80 | $12. | $62.50 |
| 3.75 | 2.25 | 12.70 | 13.75 | 2.50 |
| 1. | 5. | 10. | 1.25 | 3. |

| 6. | 7. | 8. | 9. | 10. |
|---|---|---|---|---|
| $10. | $22.50 | $45. | $16.75 | $75.50 |
| 5.75 | 2.50 | 5.75 | 10. | 4.50 |
| 1.25 | 5. | 1.25 | 3.25 | 20. |
| 2. | 3. | 5. | 7. | 10. |

### WRITTEN EXERCISE

*Fill out deposit slips for the following deposits, inserting the name of the depositor and of some bank in your vicinity:*

1. Bills, $254; silver, $40; checks on First National Bank, $87.50; Traders Bank, $127.50.

2. Bills, etc., $423.57; gold, $275; silver, $135.75; check on Cotton Exchange Bank, $342.60.

3. Bills, etc., $135.50; checks on Chemical National Bank of New York, $325; Third National Bank, $63.73.

4. Bills, etc., $1726.45; gold, $125; silver, $100; checks on Traders Bank, $335.50; Second National Bank, $175.40.

5. Bills, etc., $262.75; checks on Garfield National Bank, $96.50, $200; Jefferson and Lee National Bank, $325, $46.50.

6. Gold, $50; checks on First National Bank, $27.62, $41.75, $32.80; Lincoln Trust Company, $37.42, $21.85.

7. Bills, etc., $146.73; silver, $275; checks on Miners National Bank, $43.50; Rocky Mountain National Bank, $250.

8. Bills, $145; silver, $350; gold, $480; checks on Merchants National Bank, $255; Farmers Trust Company, $162.50; Second National Bank, $275.50.

9. A merchant deposited $175.80 in cash to-day, a check for 25% of a debt of $176 due him, and a check in payment for $18\frac{3}{4}$ yd. of velvet at $2.16 a yard less $33\frac{1}{3}\%$ on account of a bargain sale. Make out a deposit slip.

10. A jeweler received cash for the following: 3 doz. forks @ $16.50, $4\frac{1}{2}$ doz. teaspoons @ $15, a watch @ $32.75, 2 clocks @ $6.75. He also received a check on the First National Bank for the following: 3 doz. dessert spoons @ $17.75, 9 nutcrackers @ $9 a dozen. Make out a deposit slip.

**72. Check Book.** A *check book* containing *checks* and *stubs* is given the depositor when he opens an account at the bank.

| *Stub* | *Check* |
|---|---|
| No. *275* | No. *275* CHICAGO, ILL., *May 2, 1921* |
| Date *May 2, 1921* | FIRST NATIONAL BANK OF CHICAGO |
| To *J. H. Smith* | Pay to the order of |
| For *Bal. acct.* | *John H. Smith* *$15.50* |
| Amt. *$15.50* | *Fifteen and 50/100* ..... Dollars |
| | *Robert J. Brown* |

**73. Payee.** The person to whom a check is payable is called the *payee*. A check is usually made payable to the order of the payee, in which case the payee must *indorse* it, that is, he must write his name across the back; or to "Self," in which case the drawer alone can collect it; or to the payee or bearer, when any one can collect it; or to "Cash," when any one can collect it.

### ORAL EXERCISE

*Given the following deposits and checks, tell the balances:*

| 1. | | 2. | | 3. | |
|---|---|---|---|---|---|
| *Deposits* | *Checks* | *Deposits* | *Checks* | *Deposits* | *Checks* |
| $25 | $15 | $37 | $19 | $48 | $20 |
| 70 | 27 | 41 | 32 | 27 | 15 |

| 4. | | 5. | | 6. | |
|---|---|---|---|---|---|
| *Deposits* | *Checks* | *Deposits* | *Checks* | *Deposits* | *Checks* |
| $62 | $37 | $75 | $62 | $50 | $65 |
| 48 | 41 | 25 | 17 | 40 | 20 |

### WRITTEN EXERCISE

**1.** If your deposits in a bank have been \$14.75, \$32, \$15, \$25, \$50, \$75, and \$1.75, and you have drawn checks for \$9.50, \$18.75, and \$30, what is your balance?

**2.** A man earning \$15 a week deposits \$12 every Saturday, and each Monday gives a check for \$6 for his board. What will be his balance in 6 months (26 weeks)?

**3.** If your deposits in a bank have been \$32.75, \$49.63, \$28, \$14.90, \$25, \$10, \$30, \$42.75, and \$71.25, and you have drawn checks for \$5.75, \$13.50, \$32, \$4.50, \$36.50, and \$52.80, what is your balance?

**4.** A farmer having \$275.60 in the bank deposits during the next six months \$23.50, \$17.42, \$75, \$32.40, and the proceeds of the sale of 7 cows @ \$37.50 less 5%. He gives checks for \$17.92, \$8.50, \$72.80, \$9.50, \$6.75, \$13.90, and \$26.50. What is his balance?

**5.** A merchant's deposits this week have been \$127.42, \$135, \$72.50, \$265.75, \$327.40, and \$182.50, and he has drawn checks for \$236.50, \$15.75, \$27.90, \$241.60, and \$29.50. He has also paid by check a bill of \$176.70 less 10%, another of \$12 less 4%, and another of \$75.25 less 10%, 3%. At the beginning of the week he had \$475.80 in the bank. What is his balance at the end of the week?

**6.** A merchant's deposits for this week have been \$261.50, \$392.75, \$62.40, \$112.30, \$98.76, and \$115, and he has drawn checks for \$42.80, \$174.95, \$162.30, \$81.50, \$15, \$27.40, and \$37.50. He has also given a check for \$125 plus 6 months' interest at 5%, and one in payment of a bill for \$248.50 less 8%, 5%. At the beginning of the week he had \$692.80 in the bank. What is his balance at the end of the week?

**74. Promissory Note.** A paper signed by a borrower, agreeing to repay a specified sum of money on demand or at a specified time, is called a *promissory note*, or simply a *note*.

**75. Face of Note.** The sum borrowed is called the *principal*, or, if a note is given, the *face of the note*.

**76. Amount of Note.** The sum of the principal (or face) and the interest is called the *amount* of the note.

A note should state the date, face, rate, person to whom payable, and date of maturity (date when it is due to be paid), and should state the fact that it has been given for value received.

The following is a common form of a time note:

$75.\frac{40}{100}$ TOPEKA, KANSAS, *February 7*, 19*17*

*Six months* after date, for value received, I promise to pay to *John Williamson* or order,

*Seventy-five and* $\frac{40}{100}$ Dollars,

with interest at 5%.

*Frank Francis*

The following is a common form of a demand note:

$50.\frac{00}{100}$ AUSTIN, TEXAS, *May 2*, 19*19*

On demand, for value received, I promise to pay to

*Robert J. Jones* or order,

*Fifty and* $\frac{00}{100}$ Dollars,

with interest.

*James P. Poole*

**77. Indorsing a Note.** If the payee wishes to sell the note, he must, when it is payable to himself or *order*, indorse it.

A note is *indorsed* by the payee's writing his name across the back. The indorser must pay the note if the maker does not.

If the payee wishes to make the note payable to the order of Frank R. Roberts, he should write across the back, "Pay to the order of Frank R. Roberts," and sign his name.

A note payable to John Williamson or *bearer* may be sold without indorsement. Such notes are not common.

### WRITTEN EXERCISE

1. Compute the amount of the first note on page 145.

2. Write a promissory note, signed by X, payable to Y, for $25, due in 1 yr., at 6%. Find the amount.

3. Thomas H. Holmes borrows $350, at 6%, for 1 yr., from A. B. Seel. He gives a note payable to Mr. Seel or order. Mr. Seel sells the note to B. C. Dean. Make out the note, indorse it, and find the amount due at the end of the year.

*Make out and indorse, payable to the order of the buyer, the following notes, and find the amount due at the end of the time stated:*

| | *Maker* | *Payee* | *Buyer* | *Face* | *Rate* | *Time* |
|---|---|---|---|---|---|---|
| 4. | J. R. Jones | A. N. Cole | P. R. Carr | $375 | 6% | 6 mo. |
| 5. | M. L. King | A. R. Doe | E. F. Dun | $750 | 5% | 2 yr. |
| 6. | A. K. Iles | S. M. Roe | E. L. Cree | $75 | 6% | 10 mo. |
| 7. | B. S. Hill | O. N. Olds | B. R. Hall | $250 | 5% | 6 mo. |
| 8. | O. M. Coe | D. E. Bell | C. N. King | $125 | 6% | 1 yr. |
| 9. | A. R. Carr | A. J. Burr | G. F. Ray | $250 | $5\frac{1}{2}$% | 4 mo. |
| 10. | F. E. True | A. B. Bain | G. F. Coe | $350 | $4\frac{1}{2}$% | 3 mo. |
| 11. | J. B. Kaye | E. P. Fell | P. R. Call | $275 | 6% | 8 mo. |

**78. Loan.** When a person deposits money in a bank he really lends to the bank. The bank lends the money to some one else at a higher rate of interest than it pays the depositor. When a person takes a note for money, this note represents a *loan.* When a lender wishes excellent security for a loan of some size, he may take a *bond* instead of a note. A bond is a longer document than a note and is more formal, being acknowledged before an officer of the law (usually before a *notary public*), and being made more secure by a *mortgage.*

It should be remembered that all the forms of investment we have studied are in reality different ways of lending money, the income being the amount received for the use of the money lent. Show how this is true of interest. In addition to these indirect methods of lending money, people frequently lend money as a matter of accommodation, or for interest received, usually at a higher rate than can be obtained at banks.

### WRITTEN EXERCISE

*Find the income on the following, whether in the form of real estate, bonds and mortgages, or notes:*

1. \$750, 2 yr., 9%.
2. \$875, 3 yr., 8%.
3. \$4750, 2 yr., $8\frac{1}{2}$%.
4. \$5500, 3 yr., $9\frac{1}{2}$%.
5. \$6000, 4 yr., 7%.
6. \$7500, 3 yr., 9%.
7. \$8000, 5 yr., $8\frac{1}{2}$%.
8. \$6000, 3 yr., $2\frac{1}{2}$%.

The above examples represent investments in real estate. In most states bonds and notes do not in general pay more than 6%.

9. \$1250, 4 yr., 6%.
10. \$2250, 2 yr., 5%.
11. \$1875, 2 yr., 4%.
12. \$6200, 3 yr., 5%.
13. \$3300, 2 yr., 3%.
14. \$4500, 2 yr., $3\frac{1}{2}$ %.
15. \$750, 2 yr. 6 mo., 3%.
16. \$500, 1 yr. 6 mo., 3%.
17. \$1500, 1 yr. 3 mo., 6%.
18. \$2500, 2 yr. 6 mo., 5%.

**79. Borrowing Money.** If Fred lends a baseball bat to Frank, it is evident that Frank borrows the bat from Fred. Lending by one person means borrowing by another.

Business houses usually have need of large sums of money at certain times of the year, and it is better for them to borrow it for two or three months than to keep such an amount on hand for a whole year. Borrowing money is, therefore, often good business policy.

Business houses usually borrow money at banks. The banks get their money from depositors, who find it safer and more convenient to keep their money in a bank than to lock it up in their homes or their offices. The banks get paid for their work by charging borrowers more than they pay depositors.

Ask the pupils to inquire about the rates of interest charged by local banks.

### WRITTEN EXERCISE

**1.** Find a bank's net annual gain if it has \$1,250,000 on deposit, pays its depositors 3%, and lends \$950,000 of its deposits at an average of $5\frac{1}{4}\%$.

**2.** Find a bank's net annual gain if it has \$2,750,000 on deposit, pays its depositors on an average $1\frac{1}{8}\%$, and lends \$2,175,000 at an average of $5\frac{3}{8}\%$.

**3.** If a bank lends \$150,000 at 5% and pays its depositors 2%, how much does it gain in 3 mo.?

**4.** If a bank lends \$275,000 at an average of $5\frac{1}{4}\%$ and pays its depositors $2\frac{1}{2}\%$, how much does it gain in a year?

**5.** A bank has many expenses (*overhead charges*, as they are called). If the bank pays 2% for its money, lends it at $4\frac{1}{2}\%$, and has overhead charges of 0.7% on a certain loan of \$250,000, how much is its gain in 2 mo.?

**80. Note Payable at a Bank.** If John Brown wishes to borrow some money from a bank, and if Richard Roberts will indorse a note for him, and if the bank is willing to lend the money on this security, a note similar to the following is then made out:

Louisville, Ky., *May 15,* 19*18*

*Two months* after date, *I* promise to pay to the order of

*Richard Roberts* $*1000.*$^{00}$

*One thousand* Dollars

at **The First National Bank, Louisville, Ky.**

Value received

Residence *7030 Beacon Street* *John Brown*

Due *July 15, 1918*

The original idea was that Mr. Brown borrowed of Mr. Roberts, and Mr. Roberts sold the note to the bank, thus getting the money to lend to Mr. Brown. But now the note means simply that Mr. Roberts agrees to pay if Mr. Brown does not. Banks often take notes payable directly to the cashier.

Banks often lend money without an indorser if the borrower will deposit property of enough value to make them secure.

**81. Usual Time for Bank Notes.** Notes are usually made for 30 da., 60 da., or 90 da., and are then renewed if the borrower desires to keep the money for a longer time. On account of the great increase in banking facilities, money is more commonly borrowed from banks than was formerly the case. Hence the most common interest periods are now 30 da., 60 da., or 90 da.

A note payable 2 mo. after May 15 is due July 15, but a note payable 60 da. after May 15 is due July 14.

**82. Bank Discount.** When a man borrows from a bank he pays the interest in advance. Interest is not mentioned in the note, because it has already been paid.

Interest paid in advance on a note is called *discount.*

Teachers should call attention to the fact that the same word is used for bank discount and for commercial (trade) discount and should explain that the process is the same in both cases, that is, finding some per cent of a number. The teacher should ascertain the local custom in discounting notes.

In all work at present, call 30 days a month.

**83. Proceeds.** The face of a note less the discount is called the *proceeds.*

What are the discount and the proceeds on a note for \$200 for 6 mo. at 5% ?

The discount on \$200 for 1 yr. at 5% is 5% of \$200, or \$10.
The discount for 6 mo. is $\frac{1}{2}$ of \$10, or \$5.
The proceeds are \$200 − \$5, or \$195.

### WRITTEN EXERCISE

*Find the discounts and the proceeds on the following:*

1. \$75, 1 mo., 6%.
2. \$90, 30 da., 6%.
3. \$250, 3 mo., 5%.
4. \$275, 3 mo., 5%.
5. \$425, 4 mo., 5%.
6. \$300, 90 da., 6%.
7. \$375, 6 mo., 6%.
8. \$400, 90 da., 6%.
9. \$150, 60 da., 6%.
10. \$400, 1 mo., $5\frac{1}{2}$%.
11. \$200, 60 da., 6%.
12. \$175, 2 mo., $4\frac{1}{2}$%.
13. \$300, 4 mo., $5\frac{1}{2}$%.
14. \$500, 2 mo., $3\frac{1}{2}$%.
15. \$2975, 60 da., 5%.
16. \$4250, 90 da., 6%.
17. \$1250, 30 da., 6%.
18. \$2500, 60 da., 6%.
19. \$1500, 90 da., 6%.
20. \$1250, 90 da., 5%.
21. \$1750, 90 da., 6%.
22. \$2475, 30 da., 5%.

*Find the discounts and the proceeds on the following:*

**23.** \$875, 30 da., 6%.
**24.** \$975, 90 da., 5%.
**25.** \$425, 30 da., 6%.
**26.** \$2568, 60 da., 5%.
**27.** \$1250, 60 da., 6%.
**28.** \$1500, 90 da., 5%.
**29.** \$2565, 90 da., 5%.
**30.** \$3250, 90 da., $5\frac{1}{2}$%.
**31.** \$4250, 30 da., $4\frac{1}{2}$%.
**32.** \$325.50, 30 da., 6%.
**33.** \$427.50, 60 da., 5%.
**34.** \$237.50, 30 da., 5%.

**35.** Make out a 60-day note for \$750, dated to-day, payable to John Doe's order at some bank. Discount it at 5%.

**36.** Make out a 30-day note for \$675, dated to-day, payable to Richard Roe's order at some bank. Discount it at 6%.

**37.** Make out a 90-day note for \$1250, dated to-day, payable to James Jameson's order at some bank of which you know. Discount it at 6%.

**38.** A man's bank account shows deposits of \$37.50, \$82.50, \$75, \$100, \$50, \$300, \$40, \$125, and \$500; checks drawn, \$13.75, \$62, \$5, \$125.50, and \$82.75. He needs \$2000 to start him in business, and wishes to keep about \$500 in the bank. How much money should he borrow, to the nearest \$100?

**39.** If he makes out a note for the amount found in Ex. 38 for 90 da. at 6%, how much discount must he pay? What are the proceeds? What are the proceeds for 60 da.?

**40.** A merchant has to pay a debt of \$1275 less 10%. He has in the bank \$672.80, and has \$127.50 in cash in his safe. He wishes to leave about \$300 in the bank and about \$65 in his safe. How much must he borrow, to the nearest \$50? Discount the note for 30 da. at 6%.

Teachers should from time to time call attention to the importance of estimating the answer in advance.

**84. Discounting a Note.** If a dealer buys some goods for the fall trade but does not wish to pay for them until after the holidays, he often buys them on credit, giving his note. The manufacturer may need the money at once, in which case he will indorse the note and sell it to a bank for the amount less the discount.

For example, if you give a manufacturer your note for $500, dated September 1 and due on January 1, with interest at 5%, and he needs the money and discounts the note at a bank on September 16 at 6%, what are the proceeds?

| | |
|---|---|
| Face of the note | $500. |
| Interest for 4 mo. at 5% | 8.33 |
| Amount due at maturity | $508.33 |
| Discount for 107 da. at 6% | 9.07 |
| Proceeds | $499.26 |

Banks usually compute the discount for the exact number of days. In some parts of the country both the first day and the last day are included in the discount period, thus making it 108 da. in the above example.

### WRITTEN EXERCISE

*Find the discounts on the following notes:*

1. $500, 30 da., 6%.
2. $400, 60 da., 6%.
3. $600, 90 da., 6%.
4. $750, 30 da., 6%.
5. $875, 60 da., 6%.
6. $925, 90 da., 6%.
7. $350, 30 da., 6%.
8. $475, 60 da., 6%.
9. $525, 90 da., 6%.
10. $750, 93 da., 6%.
11. $1250, 30 da., 6%.
12. $3750, 60 da., 6%.
13. $7500, 90 da., 6%.
14. $12,500, 30 da., $5\frac{1}{2}$%.
15. $17,500, 60 da., $5\frac{1}{2}$%.
16. $22,500, 90 da., $5\frac{1}{2}$%.
17. $35,000, 30 da., 5%.
18. $75,000, 30 da., 5%.
19. $125,000, 10 da., 6%.
20. $250,000, 10 da., 5%.

*Find the discounts and the proceeds on the following notes:*

| | *Face* | *Date* | *Due* | *Interest* | *Date of Discount* | *Rate of Discount* |
|---|---|---|---|---|---|---|
| **21.** | \$300 | July 1 | July 31 | 4% | July 16 | 6% |
| **22.** | \$800 | Sept. 1 | Nov. 1 | 5% | Sept. 11 | 6% |
| **23.** | \$550 | Oct. 1 | Jan. 1 | 4% | Dec. 2 | 6% |
| **24.** | \$325 | July 1 | Dec. 1 | 6% | Nov. 1 | 5% |
| **25.** | \$1000 | Nov. 1 | Dec. 1 | 6% | Nov. 1 | 6% |
| **26.** | \$1250 | Aug. 15 | Jan. 1 | 5% | Sept. 1 | 5% |

The notes in Exs. 27–52 draw no interest.

**27.** \$3750, 60 da., 5%.

**28.** \$4225, 60 da., 5%.

**29.** \$9500, 60 da., 6%.

**30.** \$7500, 60 da., 5%.

**31.** \$4225, 90 da., 5%.

**32.** \$5575, 90 da., 6%.

**33.** \$7500, 30 da., 5%.

**34.** \$4750, 30 da., 6%.

**35.** \$42.50, 30 da., 6%.

**36.** \$27.60, 30 da., 6%.

**37.** \$35.50, 30 da., 6%.

**38.** \$28.75, 30 da., 6%.

**39.** \$125.50, 60 da., 6%.

**40.** \$275.25, 60 da., 6%.

**41.** \$375.50, 90 da., 6%.

**42.** \$450.75, 90 da., 6%.

**43.** \$457.50, 30 da., 5%.

**44.** \$296.50, 30 da., 5%.

**45.** \$287.60, 90 da., 5%.

**46.** \$375.40, 90 da., 5%.

**47.** \$2750, 93 da., $4\frac{1}{2}$%.

**48.** \$3275, 93 da., $4\frac{1}{2}$%.

**49.** \$4760, 63 da., $5\frac{1}{2}$%.

**50.** \$2745, 63 da., $5\frac{1}{2}$%.

**51.** The discount, at 6%, on a note for 90 da. is \$18.75, and on another note for 30 da. the discount is \$32.50. What is the face of each note?

**52.** A dealer buys \$1750 worth of goods, giving his note on October 4, for 90 da., at 5%. On October 10 the holder of the note discounts it at 6%. Find the proceeds.

**85. Draft.** A written order by which one person directs another to pay a certain sum of money is called a *draft.*

A check is therefore a kind of draft. Usually the word *check* is used for an order on a bank, signed by any person except a representative of another bank; *draft* is used for an order other than a check. A draft may therefore be made by an individual on a company, by one bank on another, by a company on a private individual or firm, and so on.

**86. Commercial Draft.** A draft made to secure the payment of a debt is called a *commercial draft.*

The following is a form of commercial draft payable at sight:

DAYTON, OHIO, *Feb. 15,* 1920

At sight pay to the order of

*The First National Bank of Dayton* $735.80

*Seven hundred thirty-five and* 80/100 ..... Dollars

To *The National Cash Register Co.*

*James X. Wye* *Per J. K. L.*

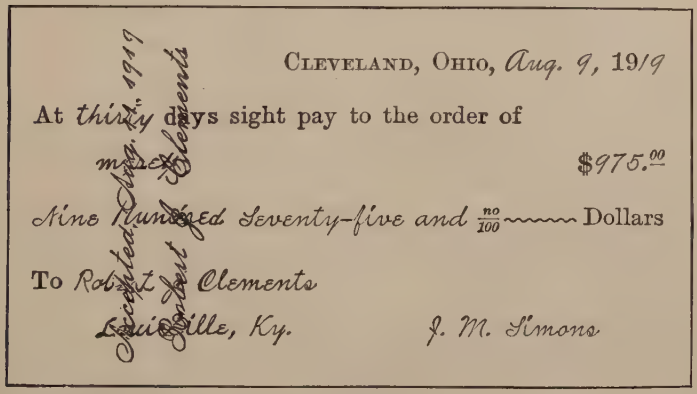
*Cleveland, Ohio*

**87. Parties to a Draft.** Here the National Cash Register Company is the *drawer*; Mr. James X. Wye is the *drawee*; The First National Bank is the *payee.*

The Register Company deposits this draft with the First National Bank. This bank sends it to some bank in Cleveland. The Cleveland bank sends to Mr. Wye for the money, and, having collected it, sends the money, or its equivalent in a check, to the Dayton bank. The Dayton bank then notifies the company that the draft is paid, and the amount, less some slight commission, — the *proceeds* of the draft, — is added to the company's account in the First National Bank.

**88. Time Draft.** A draft that is payable some time after being presented to the payee is called a *time draft.*

The following is a common form of time draft:


Accepted Aug. 11, 1919
Robert J. Clements

Cleveland, Ohio, Aug. 9, 19/9

At thirty days sight pay to the order of

$975.00

Nine Hundred Seventy-five and no/100 Dollars

To Robert J. Clements
Louisville, Ky.

J. M. Simons

In this case J. M. Simons has sold Robert J. Clements $975 worth of goods on 30 days' credit. When he sends the bill he also sends this draft. Mr. Clements writes "Accepted, Aug. 11, 1919, Robert J. Clements" across the face and returns it to Mr. Simons. This becomes thereby a promise to pay $975 "at 30 days sight," that is, on Sept. 10, or 30 days after the acceptance on Aug. 11. Mr. Simons may now discount this at a bank.

This is one way of borrowing money at a bank, replacing the older method by a promissory note. A general name for such evidences of debt as can be sold to banks is *negotiable paper.*

### WRITTEN EXERCISE

1. Brown & Co. draw on J. H. Brownson for $750. The banks charge 0.1% for collection. What are the proceeds?

2. The Electric Company draws on Mr. X for $550. The banks charge 0.1% for collection. What are the proceeds?

3. The Arithmetic Publishing Company draws on R. T. Dow for $150. The banks charge 0.2% for collection. What are the proceeds? Write the draft.

4. Robertson Bros. draw on J. P. Shipley for $37.50. The bank's charges for collection are 10¢. This is what per cent of the face?

5. M. D. St. John collects a $150 debt through the bank, the proceeds being $149.70. What is the bank's rate for collecting this debt?

6. S. L. James of Hoboken draws on L. D. Richards of Cedar Rapids, Iowa, for $75. The bank charges $\frac{1}{5}\%$ for collecting. What are the proceeds?

7. A. B. Stanley owes M. S. Stanton for 2 doz. suits @ $113.50; $1\frac{1}{2}$ doz. suits @ $130; $7\frac{1}{2}$ doz. pairs trousers @ $40. Stanton draws on Stanley for the money, and the bank charges him 50¢ for collecting. What rate is this?

8. Suppose William Bentley of Winnipeg owes you $250, and you wish to draw upon him for this amount. Write the draft, payable to the order of some bank near your home. If the bank charges $\frac{1}{10}\%$ for collecting, what are the proceeds?

9. R. H. Dudley owes Cayley & Co. for 4 dressers @ $14, and 6 washstands @ $3.50, all less 15%, and a bedroom set at $34.55 net. They draw on him for the amount, the bank charging 10¢ for collecting. What is the net amount received by Cayley & Co.?

10. R. J. Doane of Montreal owes A. D. Kane of Pittsburgh for 10 steel girders @ $50, and tells the latter to draw upon him for the amount. Mr. Kane keeps his account at the Iron Exchange Bank. Make out a draft for Mr. Kane. What is the bank's charge for collecting, at $0.1\frac{1}{2}\%$?

**11.** A draft dated Aug. 8, at 30 days sight, for $975, is accepted on Aug. 10, and is discounted on Aug. 16 at 6%. What are the proceeds?

First find when the draft is payable, then discount it at 6% for the number of days between Aug. 16 and the date of payment.

**12.** A draft dated Sept. 1, at 30 days sight, for $350, is accepted on Sept. 4, and is discounted on Sept. 7 at 6%. What are the proceeds?

**13.** A draft dated April 2, at 60 days sight, for $725 is accepted on April 7, and is discounted on April 12 at 6%. What are the proceeds?

**14.** A draft dated Oct. 15, at 60 days sight, for $1225.75, is accepted on Oct. 19, and is discounted on Oct. 25 at 6%. What are the proceeds?

**15.** A draft dated Feb. 7, at 90 days sight, for $1675.50, is accepted on Feb. 14, and is discounted on Feb. 22 at 5%. What are the proceeds?

**16.** A manufacturer sells goods listed at $725.50, discounted at 10% and 5%, and draws on the buyer at 60 days sight. The buyer accepts the draft, and it is discounted 10 days later at 6%. Write the draft and find the proceeds.

**17.** A manufacturer sells goods listed at $1672.25, discounted at 8% and 3%, and draws on the buyer at 60 days sight. The buyer accepts the draft, and it is discounted 5 days later at 5%. Write the draft and find the proceeds.

**18.** A wholesale house sells goods listed at $2737.50, discounted at 15%, 8%, and 3%, and draws on the buyer at 90 days sight. The buyer accepts the draft, and it is discounted 30 days later at 5%. Write the draft and find the proceeds.

**89. Paying Bills at a Distance.** If you owe money to a merchant living in another place, from whom you have ordered a base ball, tennis racket, book, or other article, you may send the money by a registered letter. You will be more likely, however, to pay the bill by a money order. A man who owed a large amount would be apt to pay it by a check or a draft.

**90. Exchange.** The payment of money by means of checks, money orders, or drafts is called *exchange*.

**91. Paying by Check.** If a man sends his check for the amount owed, the creditor will deposit it in the bank where he keeps his account. This bank will send it to the debtor's bank for collection, and may charge a small amount for the trouble.

**92. Paying by Money Order.** A money order may be purchased at the post office, or from an express company, or it may, at considerable expense, be telegraphed. Postal or express orders may be sent to the creditor, who can then obtain the money at his post office or express office. The extra cost of postal money orders is as follows:

For sums not exceeding \$2.50, 3 cents; from \$2.51 to \$5, 5¢; \$5.01 to \$10, 8¢; \$10.01 to \$20, 10¢; \$20.01 to \$30, 12¢; \$30.01 to \$40, 15¢; \$40.01 to \$50, 18¢; \$50.01 to \$60, 20¢; \$60.01 to \$75, 25¢; \$75.01 to \$100, 30¢.

### ORAL EXERCISE

*Referring to the above list, state the cost of money orders for:*

| | | | |
|---|---|---|---|
| 1. \$31.50. | 4. \$86.50. | 7. \$16.30. | 10. \$60. |
| 2. \$19.90. | 5. \$92.30. | 8. \$90.50. | 11. \$80. |
| 3. \$52.75. | 6. \$69.95. | 9. \$75.50. | 12. \$100. |

**93. Paying by Bank Draft.** One of the common methods of paying a debt due in another place is by means of a bank draft.

No. 48104

MERCHANTS NATIONAL BANK OF AUSTIN

AUSTIN, TEXAS, July 6, 1919

Pay to the order of ..... John Roberts ..... $78.$\frac{75}{}$

Seventy-eight and $\frac{75}{100}$ ..... Dollars

*To The Chemical National Bank,*
*New York City*

*A. B. Smith*
*Cashier*

A draft is the same as a check, except that it is made by the cashier of some bank and is drawn on another bank.

Banks usually charge a slight premium on the face of the draft. Thus, a $250 draft at 0.1% premium would cost $250, with the premium at 0.1% of $250, or $250.25 in all.

If John Roberts, who purchased the above draft, owed Robert Jones the money, he could indorse it thus:

*Pay to the order of Robert Jones*

*John Roberts*

It might have been made payable directly to the order of Robert Jones in the first place, but this is not the custom.

Drafts on large money centers are usually cashed for customers of the bank without any discount.

How much would the above draft cost at 0.1% premium?

0.1% of $78.75 = $0.08, to the nearest cent.

The bank, however, would probably charge 10¢ to make a convenient amount. Therefore the draft would cost $78.85.

### WRITTEN EXERCISE

1. How much will a draft for \$300 cost at $\frac{1}{5}\%$ premium?

2. How much will a draft for \$3200 cost at 0.1% premium? a draft for \$2500 at $\frac{1}{20}\%$ premium?

3. How much will a postal money order for \$37.50 cost? for \$62.75? for \$14.30? for \$86.50? for \$75.40?

4. How much will a Camden merchant pay for a draft on Chicago for \$4000 at 40¢ premium per \$1000?

5. When the government charges 30¢ per \$100 for a money order, what per cent premium does it charge?

6. A draft cost a merchant \$2752.75, including 0.1% premium. What was the face of the draft? What was the premium?

7. A draft cost a merchant \$3751.50, including the premium of 40¢ per \$1000. What was the face of the draft?

8. If a man owes bills of \$250, \$150, \$100, and \$350 to different jobbers from whom he buys goods, how much will drafts for these amounts cost at 0.1% premium?

9. Which is cheaper for you if you owe \$75 for some goods, to send a money order, or a draft for which you have to pay 15¢ premium? how much cheaper?

10. It costs 10¢, besides the regular postage, to register a letter. If the letter is not delivered, the government will pay the loss, up to \$50. If you owe \$22, is it cheaper for you to send it by registered letter or by money order? how much cheaper?

11. If you owe \$100 to a manufacturer at a distance, from whom you have bought goods, how will you make the payment? Tell why you will make it in that way, and how much it will probably cost. The premium on a draft may be taken at the common rate of 0.1%.

**94. Rate of Exchange.** A money order is always sold at a slight advance over its face, and usually a bank draft costs more than its face. In each case the rate of the variation from the face is called the *rate of exchange.*

**95. Premium.** If the exchange is added to the face, exchange is *at a premium*, as we have already seen.

**96. Par.** If there is no exchange, exchange is *at par.*

**97. Discount.** If the exchange is deducted from the face, exchange is *at a discount.*

For small sums, say for $500 or less, New York, Chicago, or Philadelphia exchange usually sells at a premium of about 0.1%. This is to pay the bank for its trouble and for the expense of shipping the money when its balance at these cities gets low. Banks usually buy such drafts at their face value, thus making no charge for cashing them.

For large sums the rate of exchange varies. If the Chicago banks owe the New York banks $2,000,000, they must send that amount by express, an expensive proceeding. If a man in Chicago at that time wished to buy a draft on New York for $30,000, they would charge him more than usual because they would have to express that much more to New York; but if a man in New York wished to buy a draft on Chicago, he might buy it for less than $30,000 because the bank would get its money at once, and the risk and expense of transmitting it would be saved.

The premium or discount is usually quoted as a certain per cent of the face of the draft, but sometimes as so much on $1000, a fraction of $1000 counting as $1000. In the latter case the quotation of $\frac{1}{4}\%$ premium is the same as that of $2.50 premium.

The explanation of the *Clearing House*, a place where bank officials of a city meet daily to exchange drafts and checks and to pay balances due one another, is too technical for most classes, and if given at all, this should be done orally.

### WRITTEN EXERCISE

*Find the cost of the following drafts:*

1. \$3756.70, at par.
2. \$750, 0.2% premium.
3. \$2450, 0.1% premium.
4. \$3500, 0.1% discount.
5. \$6750, $\frac{1}{20}$% premium.
6. \$17,500, 0.2% discount.

7. What is the cost of a draft on San Francisco for \$5200 at $\frac{3}{4}$% discount? Write the draft.

8. What must be paid in Rahway for a draft on Chicago for \$3400, exchange being at $\frac{1}{4}$% premium?

9. A draft for \$4800 was bought for \$4794. Was exchange at a premium or a discount? What was the rate?

10. When a Boston draft for \$35,000 can be bought in New Orleans for \$34,930, is exchange at a premium, at par, or at a discount? What is the rate?

11. If J. R. Glover draws on J. B. Thornton for \$250, and the banks charge 0.1% for collection, what are the net proceeds that Glover will receive? Write the draft.

12. My agent in Toronto sells a house for me for \$2500. He charges 2% commission, and the bank charges \$2.50 premium for a draft for any amount between \$2000 and \$2500. What sum does he remit to me?

13. A telegraphic money order costs twice the rate for a ten-word message, plus 1% premium on the face. A ten-word message from Trenton to Kansas City costs 50¢. How much will a telegraphic money order for \$375 cost?

14. Mr. Edgcomb of Denver owes \$3243.24 to Mr. Nourse in that city. Mr. Nourse has gone to New York on business and asks that the money be sent, less the cost of exchange. Exchange being at 0.1% premium, what is the face of the draft?

### REVIEW PROBLEMS

1. Find the amount of principal and interest of \$500, compounded semiannually, at the rate of 4%, for 5 yr.

2. A man deposits \$750 in a savings bank on July 1, 1918. His further deposits are as follows: Aug. 1, 1918, \$30; Sept. 15, 1918, \$25; Dec. 31, 1918, \$100; June 15, 1919, \$50. He draws out money as follows: Oct. 12, 1918, \$40; June 20, 1919, \$75; Aug. 2, 1919, \$20; Aug. 15, 1919, \$30; Sept. 12, 1919, \$25. The savings bank pays 1% every three months on the smallest balance for the preceding quarter, on Jan. 1, April 1, July 1, and Oct. 1. Find his balance on Jan. 1, 1920.

3. Make out a deposit slip for the following deposits, naming some bank in your vicinity: bills, \$575; gold, \$35; silver, \$72.25; checks, \$48.25, \$72.50, \$61.30, \$227.52, and \$86.95. If your balance was \$326.42 before, how much was it after the deposit?

4. A merchant's balance on the first of the month was \$1725.60. He has since deposited \$37.50, \$296.75, \$21.30, \$572.32, and \$217.65. He has given checks for \$128.92, \$27.36, \$18.92, \$78.76, and \$273.96. How much is his present balance?

5. Make out a 90-day note for \$1250, dated to-day, payable to R. P. Foster, or order, at some bank in your vicinity. Discount it at 6%. If with the proceeds you buy 10 shares of stock and pay \$108.50 a share for them, how much money will you have left?

6. Which is the greater, the proceeds of a note for \$750 for one year, drawing interest at 6%, and discounted on the day of its date at 5%, or the amount of a note for \$700 for one year at 6%? How much greater?

In this review, or earlier in this chapter, pupils should be encouraged to tell what plans they have for saving money, and in what form of investment they would put their money if it amounted to a considerable sum. This may be tactfully done so as to encourage no boasting or rivalry.

## LITTLE EXAMINATIONS

I. *Find the cost of postal money orders for:*

1. \$2.75. 2. \$5.50. 3. \$10.75. 4. \$35.

Refer to the rates given on page 158. Each answer should include the amount sent and the charge made by the post office.

*Find the cost of these drafts at the premiums stated:*

5. \$3750, 0.1%. 6. \$4800, 0.2%. 7. \$725, $\frac{1}{4}$%.

*Find the cost of these drafts at the premiums per \$1000 stated:*

8. \$1500, \$1.50. 9. \$3750, \$1.60. 10. \$25,750, \$1.80.

II. *Find the cost of these drafts at the discounts stated:*

1. \$7500, 0.1%. 2. \$4500, $\frac{1}{5}$%. 3. \$25,000, $\frac{1}{20}$%.

4. Find the discount on a note for \$725 for 60 da. @ 6%.

5. Make out a check for \$2.75 and a bank draft for \$50.

6. Make out a promissory note for \$17.50.

7. Make out a deposit slip for four items deposited.

8. Make out a commercial draft for \$75.

9. What is the cost of a draft for \$1750 at par?

10. Find the cost of a draft for \$1800 at 0.1% premium.

III. *Find the discounts and the proceeds on the following:*

1. \$350, 30 days, 6%.
2. \$625, 60 days, 5%.
3. \$800, 60 days, 6%.
4. \$375, 60 days, 6%.
5. \$4275, 60 days, 5%.
6. \$3450, 90 days, 5%.

# REVIEW AND DRILL

## I. DRILL SECTION

### WRITTEN EXERCISE

| | I | II | III | IV |
|---|---|---|---|---|
| *a* | \$275 | 0.1% premium | \$217.50 | 0.3% premium |
| *b* | \$450 | 0.2% discount | \$316.75 | $\frac{1}{20}$% premium |
| *c* | \$1250 | 0.1% discount | \$425.25 | $\frac{1}{20}$% discount |
| *d* | \$1500 | 0.2% premium | \$675.75 | 0.4% discount |
| *e* | \$1875 | 0.3% discount | \$842.36 | 0.4% premium |

**1.** For each sum in columns I and III above find, in turn, the premium or the discount at each of the rates in II and IV.

This gives 100 examples. The teacher should select as many as necessary.

| | I | II | III | IV | V | VI |
|---|---|---|---|---|---|---|
| *a* | $\frac{1}{32}$ | 0.23 | 41.3 | 1.276 | $6\frac{1}{4}$ | 12,250 |
| *b* | $\frac{1}{16}$ | 0.32 | 62.1 | 2.007 | $7\frac{3}{5}$ | 13,175 |
| *c* | $\frac{1}{8}$ | 0.41 | 67.9 | 3.982 | $8\frac{2}{3}$ | 16,022 |
| *d* | $\frac{5}{32}$ | 0.68 | 71.8 | 4.871 | $9\frac{1}{6}$ | 19,685 |

*In each of the six columns above perform the following operations, carrying each decimal fraction to two decimal places:*

**2.** $a + b$.
**3.** $b - a$.
**4.** $a + d$.
**5.** $a + c$.
**6.** $d - a$.
**7.** $c \div a$.
**8.** $c \times a$.
**9.** $b \times d$.
**10.** $b + c$.
**11.** $c - b$.
**12.** $c + d$.
**13.** $d - c$.
**14.** $a \times b$.
**15.** $b \div a$.
**16.** $a \times d$.
**17.** $d \div a$.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

**1.** How do you ascertain the interest due on the first day of the year on an account in a savings bank?

**2.** Which pays the better interest at the same rate, if the money is left undisturbed for five years, a promissory note or a deposit in a savings bank? Why?

**3.** How do you fill out a deposit slip? After entering the items, what operation do you perform? How do you make sure that the result is correct?

**4.** How do you find the discount on a promissory note?

**5.** If a note drawing a certain rate of interest is discounted on the day it is made, at the same rate, are the proceeds greater than the face, or equal to it, or less? Why is this?

**6.** How can a manufacturer discount a claim against a purchaser, the claim not being yet due? How is the discount found?

**7.** If you know the proceeds and the rate of discount, how do you find the face of a note?

**8.** If you know the face of a note and the proceeds, how do you find the discount?

**9.** If you know the face of a note and the proceeds, how do you find the rate of discount?

**10.** If you change money from a postal savings-bank to a government bond, how much higher rate of interest do you get?

**11.** If a dealer buys some produce and agrees to pay for it in 90 days, with interest at 6%, and the seller takes a note and at once discounts it at 5%, will he receive more than the selling price, or the same amount, or less? Why?

## III. INDUSTRIAL PROBLEMS

### MONEY IN FERTILIZERS

**1.** George's big brother has come home from the agricultural college and his father decides that on their farm they ought to try more modern methods of feeding their plants. His brother tells him that they need nitrogen, phosphoric acid, and potash, and that if these are put in the soil by means of fertilizers the plants grow better. George's brother tried applying \$2.16 worth of fertilizer to one acre of a field, and the increase in the yield of corn was 8.6 bu. Corn being worth 40¢ a bushel, what was the value of the increase? The value of the increase was how much more than the fertilizer cost?

**2.** George's brother tried another kind of fertilizer on another acre of corn, paying \$3.46 for this kind. He increased the yield 7.2 bu. If corn was worth 40¢ a bushel at that time, was this experiment profitable or not? Find the gain or loss.

**3.** George's brother knew of another fertilizer, said to be excellent for wheat, and put \$5.19 worth on an acre. The increase in yield was 8.4 bu., and wheat was then selling on the farm at 80¢ a bushel. What was the gain or loss in this case?

**4.** George's brother explained to him that a complete fertilizer is one containing all three of the foods that a plant needs. These are nitrogen, phosphoric acid, and potash. A formula written 2–8–2 means that this fertilizer contains 2% nitrogen, 8% phosphoric acid, and 2% potash. How many pounds are there of each of these foods in a ton of 2–8–2 fertilizer?

In trade, the order of the formula given in Ex. 4 is often stated thus: nitrogen, potash, phosphoric acid instead of nitrogen, phosphoric acid, potash. In this book the order recommended by experiment stations and given in Ex. 4 is followed. Pupils should inquire as to local customs.

5. George's brother showed the picture given below, representing different yields of wheat, and asked him to find how many pounds of nitrogen, phosphoric acid, and potash there are in 300 lb. (the amount used per acre) of the fertilizer that gave the 35-bushel yield. How many pounds are there of each?

| 4–8–4 FORMULA | UNFERTILIZED | 4–0–4 FORMULA |
|---|---|---|
| 35 BU. PER ACRE | 15 BU. PER ACRE | 17 BU. PER ACRE |

6. In the same picture, what is omitted in the 17-bushel yield? How many pounds of each of the other important plant foods are there in 300 lb. of this fertilizer? How many are there in a ton?

| 4–8–0 FORMULA | UNFERTILIZED | 0–8–4 FORMULA |
|---|---|---|
| 33 BU. PER ACRE | 15 BU. PER ACRE | 30 BU. PER ACRE |

7. In the above picture, which also represents different yields of wheat, how many pounds of each of the three plant foods are there in 300 lb. of fertilizer?

Teachers in rural schools should talk with practical farmers on this subject and get the pupils to do so. Different localities have different soils, and hence the question of fertilizers is to some extent a local one. Agricultural colleges are glad to give farmers expert advice as to the best fertilizers to use for various crops in various kinds of soil. In rural schools the pupils should be encouraged to make inquiries as to the success of commercial fertilizers.

# CHAPTER VI

## CIVIC AND BUSINESS RELATIONS

**98. Meaning of Taxes.** Some boys in school wanted to organize a baseball team. They found that they could rent a piece of land for a ball field for \$4 a month, and decided that they needed \$5.50 at once for balls and bats. Each boy bought his own suit, but they had to raise \$9.50 for the first month. They decided that ten boys should be allowed to join, and that each should pay his share, which was 95¢ for the first month. This 95¢ was a *tax* upon each member of the club.

A class wished to buy a picture for the schoolroom. There were 30 pupils in the class, and the picture cost \$3.90. Each pupil agreed to pay his share. A committee was appointed, and each member of the class was taxed 13¢. Here the *total tax* on the class was \$3.90, the tax of each pupil was 13¢, the committee *levied a tax* of 13¢ on each, and a *collector* received the money.

A village needed \$8000 for building a new schoolhouse, and its citizens voted to raise the amount by a tax. The sum was so large that it would not be fair to make each citizen pay the same amount, so each one was required to pay according to the value of his property. In this way those who had more property were taxed more than those who had little property. A man who owned \$10,000 worth of property was required to pay twice as much as one who owned \$5000 worth, and half as much as one who owned \$20,000 worth. This is the rule of taxation usually followed by states, cities, villages, and counties.

**99. Expenses of a City, County, or Village.** It costs a large amount of money to run a city, county, or village. In a city the streets have to be cleaned, lighted, and kept in repair; the policemen have to be paid; the fire department has to be kept up; the city buildings have to be cared for; the city officers have to be paid; the schools have to be supported; and there are many other expenses. In the county the roads have to be kept in good condition, bridges have to be built and kept in repair, and the officers have to be paid.

For example, suppose that all the expenses of a certain city amount to about \$470,000. The city may decide to attempt to raise by tax \$480,000, so as to have a little margin. If the property of the citizens is valued at \$60,000,000, each dollar of property ought to pay \$480,000 ÷ 60,000,000, or \$0.008; that is, every dollar should pay 8 mills. In this case 8 mills is called the *tax rate.*

**100. State and Local Taxes.** State and local taxes are usually a certain per cent levied on the land, money, and other property of individuals, business concerns, and corporations.

**101. Assessors.** The property to be taxed is valued by officers called *assessors.* The value placed by the assessors upon property for taxation is called the *assessed valuation.*

**102. Tax Rate.** Upon the assessed valuation a certain *tax rate* is fixed. The words *tax rate* are often used to designate the *number of mills* of tax on each dollar of valuation. Thus, a tax of $5\frac{1}{4}$ mills means $5\frac{1}{4}$ mills on a dollar.

Male citizens over 21 years of age pay a *poll* (head) *tax* in many places. This tax may be levied by the state, the county, or the local taxing body, or by all three, and the amount of the poll tax varies, but it is usually between \$1 and \$3.

**103. Finding the Tax Rate.** If a village with an assessed valuation of \$3,200,000 must raise \$16,800, what is the tax rate?

$\$16{,}800 \div 3{,}200{,}000 = \$0.005\frac{1}{4}$.

That is, for every dollar of property that a person living in the village is assessed, he must pay $5\frac{1}{4}$ mills of tax. Although, as we have seen, the mill is not coined, it is sometimes used for such purposes as this.

$$
\begin{array}{r}
\$0.005\frac{1}{4} \\
32\text{ØØØØ}\,\overline{)\,\$0.16\text{ØØ}} \\
\underline{160} \\
8
\end{array}
$$

$\frac{8}{32} = \frac{1}{4}$

**104. Finding the Tax.** After the tax rate has been determined for the village, city, county, or other political division, the tax of each taxpayer can be found.

If a man's property is assessed at \$12,000, for example, and the tax rate is $5\frac{1}{4}$ mills, he must pay $5\frac{1}{4}$ mills on every dollar; that is, he must pay $12{,}000 \times 5\frac{1}{4}$ mills; or, what is the same thing, he must pay $0.005\frac{1}{4} \times \$12{,}000$, which is \$63.

The method of collecting the tax varies so much in different parts of the country that the teacher should have the pupils inquire at home and bring the information to school.

### WRITTEN EXERCISE

1. If the tax rate is 8 mills on \$1, what are the taxes of a man whose property is assessed \$5000?

2. If the tax rate is \$2.18 on \$100, what are the taxes of a man whose property is assessed \$8000?

*Find the taxes paid on the following, at the rates specified:*

3. \$8500, $7\frac{1}{2}$ mills on \$1.
4. \$9250, 7.4 mills on \$1.
5. \$16,500, \$1.46 on \$100.
6. \$13,800, \$0.45 on \$100.
7. The assessed valuation of a certain city is \$75,000,000 and the total tax levied is \$675,000. What is the tax rate?

EA

### WRITTEN EXERCISE

*Find the taxes at $5\frac{1}{2}$ mills on the following amounts:*

| | | | |
|---|---|---|---|
| 1. $6500. | 4. $7675. | 7. $24,400. | 10. $23,250. |
| 2. $8750. | 5. $12,000. | 8. $23,500. | 11. $18,875. |
| 3. $4875. | 6. $16,500. | 9. $31,100. | 12. $45,250. |

*Find the taxes at 8 mills on the following amounts:*

| | | | |
|---|---|---|---|
| 13. $4500. | 17. $7000. | 21. $6750. | 25. $32,500. |
| 14. $6500. | 18. $4550. | 22. $8500. | 26. $38,500. |
| 15. $8000. | 19. $8750. | 23. $12,000. | 27. $45,250. |
| 16. $4600. | 20. $9750. | 24. $25,000. | 28. $75,750. |

29. The tax rate is 9.6 mills. What is the tax on property assessed at $6800?

30. The tax rate is 12 mills. What is the tax on property assessed at $22,800?

31. The tax rate is $1.23 on $100. What is the tax on property assessed at $12,500?

32. If a man's tax is $195 and the rate is 13 mills on a dollar, find the assessed valuation of his property. What is the assessed valuation of the property of a man whose tax is $234?

33. If the state tax is 1.1 mills, the county tax 3.6 mills, and the town tax 4.2 mills, what tax must a man pay on $8000?

Local problems of this kind should be brought to class.

34. A man died, leaving an estate valued at $23,750, of which $22,000 was willed to his children and the rest to his nephew. Under the laws of his state there is no inheritance tax to be paid by children, but 4% must be paid by the nephew. What inheritance tax must be paid by the nephew?

The details of the inheritance tax vary in different states.

**105. Expenses of the United States Government.** The expenses of the United States government vary from year to year, and the enormous increase due to the war has made it impossible to tell what the expenses will be in the next few years. Before the war the expenses of our government averaged about $2,000,000 a day, or about $800,000,000 a year, and some items of our income and expenditures were as follows:

*Income*

| | |
|---|---|
| Customs (duties on imported goods) | $320,000,000 |
| Internal revenue (tobacco, etc.) | 400,000,000 |
| Income tax and miscellaneous | 120,000,000 |

*Expenditures*

| | |
|---|---|
| War Department | $225,000,000 |
| Navy Department | 200,000,000 |
| Pensions | 175,000,000 |
| Indians | 20,000,000 |
| Salaries, diplomatic service, etc. | 200,000,000 |

**WRITTEN EXERCISE**

1. When our income was $724,000,000, our customs receipts were $319,000,000. Customs were what per cent of the income?

2. Taking our expenditures as $820,000,000 a year, what per cent of this goes to the War Department, as above stated? to the Navy Department? to pensions?

3. In a certain year the internal-revenue receipts of the government amounted to $344,000,000. The largest amount paid by any state was $16\frac{2}{3}\%$ of this total and was paid by the state of Illinois. What was this amount?

**106. Tariff.** The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue*, *tariff*, or *duty*.

**107. Customhouse.** Customs revenue is collected at *customhouses*. These are situated at *ports of entry*.

**108. Classes of Goods.** Goods imported may be on the *free list*, and not subject to duty, as raw silk; subject to *ad valorem* (on the value) duty, a certain per cent on the value at the place of purchase, as books, on which the duty is 15% ad valorém; subject to *specific* duty, a certain amount per bushel etc., as apples, on which the duty is 10¢ per bushel of 50 lb.; subject to both *ad valorem* and *specific* duty, as perfumery, on which the duty is 40¢ per pound and 15% ad valorem.

### ORAL EXERCISE

*State the duty on the following:*

1. 700 lb. of firecrackers, duty 6¢ per pound.
2. 1 T. of Roman candles, duty 10¢ per pound.
3. $100 worth of beads, duty 35%.
4. $800 worth of buttons, duty 25%.
5. $200 worth of dolls, duty 35%.
6. $700 worth of furs, duty 40%.
7. $500 worth of fans, duty 50%.
8. $2000 worth of jewelry, duty 50%.
9. $1250 worth of pearls, duty 20%.
10. 250 doz. pairs of gloves, duty $1 per doz.
11. 7000 pineapples, duty $5 per thousand.
12. 6000 lb. of cork bark, duty 4¢ per pound.

### WRITTEN EXERCISE

1. How much is the duty on $4350 worth of needles at 20%? Is this specific duty or ad valorem duty?

2. How much is the duty on 3500 bu. of barley at 15¢ per bushel? Is this specific duty or ad valorem duty?

3. Find the duty on 2750 lb. of hops at 16¢ a pound.

4. How much is the duty on $7565 worth of horses at 10%?

5. How much is the duty on 9750 bu. of oats at 6¢ a bushel?

6. A dealer imports 375 lb. of maple sugar from Canada. How much is the duty at 3¢ per pound?

7. A grocery house imports $1250 worth of cheese from Canada. How much is the duty at 20%?

8. A dealer imports 2750 lb. of butter. How much is the duty at $2\frac{1}{2}$¢ per pound?

9. The duty on honey is 10¢ per gallon. If a dealer imports 975 gal., how much is the duty?

10. A dealer imported 475 T. of straw, paying duty at the rate of 50¢ per ton. How much duty was paid?

11. How much is the duty on 15 sets of Thackeray's works at £7 (7 English pounds) a set, allowing $4.87 to the pound, the duty being 15%?

12. How much is the duty on 2 doz. sets of Scott's works at £4 10s. (4 pounds 10 shillings, or £$4\frac{1}{2}$) a set, allowing $4.87 to the pound, the duty being 15%?

13. How much is the duty on 2000 yd. of carpet, invoiced at $1.50 a yard, at 50%?

14. The duty on cigars is $4.50 per pound, plus 25% ad valorem. If the consignment weighs 275 lb. and is worth $825, how much is the total duty?

**109. Income Tax.** The United States secures part of the money needed to run the government by a tax on incomes. All citizens of the United States, whether residing at home or abroad, and all persons residing in the United States, and all business concerns in the country must pay the government a certain per cent of their income above a certain amount. Owing to the war, the rate of taxation has increased, and for some time to come it will vary more or less from year to year.

The details of the income tax change so rapidly and the rates vary so greatly that the school cannot be expected to give attention to them. All that is essential is that the pupils shall understand why such a tax is raised, and in a general way how it is computed. The rates given below are mere possibilities.

In a year in which the income tax was 8%, how much would a man pay on an income of which \$9872.50 was subject to tax?

His tax is 8% of \$9872.50, or \$789.80.

Teachers need not consider any question of surtaxes or of exemptions. These do not affect the mathematical treatment, and they are details which the pupil would not understand, under present conditions, even if those details were presented.

\$9872.50
0.08
\$789.8000

WRITTEN EXERCISE

*Find the income tax on incomes taxable for the following amounts at the rates given:*

1. \$7500, 8%.
2. \$6857, 6%.
3. \$9723, 4%.
4. \$7486.80, 4%.
5. \$9835.72, 5%.
6. \$8638.96, 6%.
7. \$14,286.93, 8%.
8. \$37,962.84, 2%.
9. \$173,436.84, 4%.

10. A manufacturing establishment reports an income of \$2,758,264 that is subject to taxation and the normal rate of tax is 2%. Find the normal income tax that it must pay.

**110. Taxing the Manufacturer.** Not only do persons pay taxes to the government, but manufacturing establishments have to do the same. These establishments are usually taxed by cities and states on the value of their property, and by the government on their income and through duties on imports.

### WRITTEN EXERCISE

1. A manufacturer needs some crucible plate steel. He decides to buy \$12,650 worth in England. The duty is 12% ad valorem. How much does the shipment cost, including the duty, but not including the freight?

2. A manufacturer needed a supply of galvanized iron wire. He placed the order in Germany for \$5467 worth. How much duty must he pay, the rate being 15%?

3. In a large factory a time detector records, by electricity, the places visited by the watchman in his rounds, and the hours. The factory purchased a detector in Germany for \$286, and paid 15% duty on it and \$7.20 transportation charges. How much did it cost the factory?

4. A clothing manufacturer is taxed on his manufacturing plant. This is assessed at \$75,000, and the tax rate is $6\frac{1}{2}$ mills. How much is his tax?

5. The income tax applies also to the entire net income of every business carried on by a corporation in the United States. If the net taxable income of a certain corporation is \$17,500, what would be the amount of the tax at 1%?

6. A manufacturing establishment wishes to import some lithographic advertising matter from Germany. The quality of material desired is subject to a duty of 7¢ per pound, and the amount desired weighs 748 lb. What is the amount of the duty?

**111. Nature of Insurance.** If your house should burn, the loss would be very great to you and your family. It would come suddenly, without any chance of providing for it, and a long time might be needed to save the money for a new house. A sudden calamity like this means a great loss to those who bear it. On this account people sometimes join together and contribute a small sum each year, and if the house of any one of them burns, they pay him enough to build a new house from the money they have collected. Thus no one of them suffers any great loss, and every one feels sure that if a fire comes he will be protected as far as possible.

This shows the general nature of insurance. Usually, however, instead of a number of people joining together and paying a little each year, they pay annually a small sum (a *premium*) to an insurance company, which agrees to pay for some specified loss. This company goes to the trouble of getting many people to join for protection and collects their contributions, charging a certain amount for its work.

**112. Insurance.** A written agreement to compensate any one for some specified loss is called *insurance.*

**113. Policy.** The written agreement of an insurance company to pay a certain amount in case of loss is called a *policy.*

**114. Face of the Policy.** The amount specified to be paid in case of loss is called the *face of the policy.*

The teacher should encourage the pupils to make inquiry as to the insurance of houses and of various forms of property in the vicinity, and to report to the class, avoiding all matters of a purely personal nature. In particular, the value of life and accident insurance, of fire insurance, and, in some places, of tornado insurance and plate-glass insurance should be mentioned. In states where there is a workmen's compensation act, it should be explained that this forms a kind of insurance, and the leading features should be ascertained, particularly with reference to the industries of the locality.

**115. Premium.** The cost of insurance is called the *premium.* The *rate of premium* in fire insurance is usually stated as a certain sum for each \$100 of the face for a certain period of years, generally from 1 yr. to 5 yr.

What is the premium for insuring a store for \$4000 against loss by fire, at \$1.20 a year?

$40 \times \$1.20 = \$48$, the premium.

### ORAL EXERCISE

1. What is the premium on a policy for \$650 at 80¢?
2. What is the premium on a policy for \$3000 at 60¢?
3. A man paid \$50 for insuring a house, the rate being \$2.50. What was the face of the policy?

*State the premiums on the following policies at the rates specified:*

| | | |
|---|---|---|
| 4. \$1000, 50¢. | 8. \$1750, \$1. | 12. \$3500, \$1.20. |
| 5. \$2000, 80¢. | 9. \$3750, \$2. | 13. \$6250, \$2.25. |
| 6. \$1250, \$1. | 10. \$725, \$2. | 14. \$10,000, \$0.80. |
| 7. \$4250, \$2. | 11. \$825, \$2. | 15. \$50,000, \$1.50. |

*State the faces of the policies, given the following premiums and rates:*

| | | |
|---|---|---|
| 16. \$25, \$2.50. | 18. \$40, \$2. | 20. \$13.75, \$1. |
| 17. \$17.50, \$1. | 19. \$80, \$1. | 21. \$18.50, 50¢. |

*State the rates, given the following faces of policies and premiums:*

| | | |
|---|---|---|
| 22. \$5000, \$25. | 24. \$5000, \$50. | 26. \$7500, \$150. |
| 23. \$6000, \$60. | 25. \$4000, \$80. | 27. \$6500, \$130. |

28. A building worth \$8000 is insured for $\frac{3}{4}$ of its value at \$1. What is the premium?

### WRITTEN EXERCISE

**1.** At \$0.95, what is the premium on a \$2500 policy?

**2.** At \$1.10, what is the premium on a \$2800 policy?

**3.** At \$1.15, what is the premium on a \$3750 policy?

**4.** A building worth \$12,000 is insured for $\frac{2}{3}$ of its value at \$1.10. What is the premium?

**5.** If you insure the contents of your store for \$6000, what is the premium at \$1.25?

**6.** The premium for insuring some property at \$1.50 is \$52.50. What is the face of the policy?

**7.** A man insured his factory, valued at \$135,000, for $\frac{4}{5}$ of its value, at \$1.90. What was the premium?

**8.** A dealer insured his stock of goods, valued at \$14,000, for $\frac{7}{8}$ of its value, at \$1.65. What was the premium?

**9.** If a three-year policy for \$1700 costs \$26.01, what is the rate of premium for the 3 years? What is the rate per year?

**10.** If a three-year policy for \$3000 costs \$36, what is the rate of premium for the 3 years? What is the rate per year?

**11.** A merchant insures his stock for $\frac{3}{4}$ of its value, at \$1.75. The premium is \$131.25. What is the value of the stock?

**12.** A manufacturer insures his factory for $\frac{5}{8}$ of its value, at \$2. The premium is \$210. What is the face of the policy? What is the value of the factory?

**13.** A factory worth \$33,000 is insured for $\frac{5}{6}$ of its value, at \$2. In case of total loss, how much would the owner lose, including his premium paid?

**14.** A man insured his library for its full value, \$2500, at \$1.25. What was the premium? If a fire destroyed $\frac{3}{5}$ of the library, how much could he recover?

**116. Life Insurance.** If a man is careful as to the future and considers the fact that in later years there will probably be one or more persons dependent upon him, he very likely will wish to insure his life. The earlier this is done, the lower will be the premium, and the sooner he will cease to make payments if he takes out a 10-payment, 15-payment, or 20-payment policy.

Premiums in life insurance are stated at so much on $1000. They vary with the age of the person insured, reckoned at the time he takes out his policy.

The best plan is to take a policy in a company of unquestionable standing, and never forfeit it by nonpayment of premiums.

ORAL EXERCISE

*State the premiums on the following policies, the premium on $1000 being given:*

1. $2000, $26.
2. $3500, $25.
3. $2000, $21.
4. $4500, $30.
5. $3000, $22.
6. $10,000, $29.
7. $20,000, $28.
8. $25,000, $32.
9. $30,000, $32.
10. $20,000, $27.
11. $4000, $22.50.
12. $12,000, $25.
13. $5000, $28.40.
14. $2500, $22.20.
15. $1000, $27.65.

*State the faces of the policies, given the following premiums and the premiums on $1000:*

16. $131, $26.20.
17. $175, $25.
18. $55, $27.50.
19. $49, $24.50.
20. $96.60, $32.20.
21. $63.90, $21.30.

*State the premiums on $1000, given the following policies and premiums:*

22. $2000, $80.
23. $3000, $84.
24. $4000, $96.
25. $5000, $150.
26. $5000, $105.50.
27. $6000, $180.50.

**117. Kinds of Policies.** The following are four of the leading kinds of life-insurance policies:

*Ordinary Life*, the insured agreeing to pay during his life a certain premium, usually annually or semiannually.

The rate is always given as the cost of $1000 worth of insurance; that is, the rate $25.50 means that the annual premium on a policy for $1000 is $25.50. Some insurance companies allow *dividends* each year, thus reducing the premium slightly.

*Limited Life*, the premiums being payable for some fixed number of years, as twenty, after which the policy is called *paid up* (no more premiums being due), but the face is not paid until the death of the insured.

Naturally the premiums are higher on this form.

*Endowment*, the premiums being paid for some fixed number of years, as ten, fifteen, or twenty, at the end of which time the face of the policy will be paid to the insured.

Since this may be paid during the lifetime of the insured, the premiums are also higher than on an ordinary life policy.

*Term Insurance*, the premiums being paid for a specified length of time and the face of the policy being payable if the insured dies within the term of insurance.

Thus a man may insure his life or his health for thirty days, as in certain forms of accident insurance. He may also insure his life for a certain number of years only. The premium on this form of policy is low, since the company may not have to pay the face at all.

There are various other forms of policies, but most of them are modifications of the above types. Some are arranged so that the face will not be paid on the death of the insured, but a certain amount will be paid annually (an annuity) during the lifetime of the one for whose benefit the policy was written.

### WRITTEN EXERCISE

1. What is the annual premium that a man must pay on a 10-year endowment policy for \$4000 at \$102.60? How much would he pay in the 10 years?

2. If a young man takes out a 20-payment policy for \$5000 at \$27.39, how much will he have paid when the policy matures; that is, after 20 payments?

3. What is the premium on a 20-payment policy for \$5000, taken out by a man at such an age that the rate is \$34.20? How much are the total premiums for the 20 years?

4. What is the premium on a 20-payment policy for \$7500, taken out by a man at such an age that the rate is \$32.50? How much are the total premiums for the 20 years?

5. If a man takes out a \$2500 policy in one company at \$26.40 and a \$3500 policy in another at \$23.50, how much are his annual premiums?

6. A young man takes out a \$10,000 policy on the 25-payment plan, at \$26.36. How much is the annual premium? How much are the premiums for the 25 years?

7. A man took out a \$5000 policy at \$26.40. He died just before the eighth annual payment was due. How much did his estate receive above what he paid to the company? How much would the man have saved in the 7 yr. if he had put the amount of the premium each year at simple interest at 5%?

8. Twenty years ago a young man took out a 20-payment policy for \$5000, paying \$136.95 a year. How much has he paid in the 20 years? Estimating that the insurance company has had the use of all this amount for the equivalent of 10 years, at 5%, what is the total amount received by the company?

**118. Investments.** After a man has saved some money either he will use it to pay for something he wishes to buy, or he will invest it. A man feels much more independent if he has saved money for use in case of long illness or for the purchase of a home.

The subject of investment in stocks and bonds is presented briefly on pages 189–197, but, if any pupils know about such investments, it is well to discuss the subject. The teacher may speak to the class along these lines: Having saved money by wise economy, it is essential that we should know how to take care of it. Formerly people hoarded their savings in hidden places about their homes. It is customary now to invest money that is not needed for immediate use, so that it will not only be safer than when kept at home but will also bring some income. Unfortunately some of the investments offered to the public are offered by dishonest men and are not desirable, but many others are perfectly safe. It is well to remember that the larger the income offered by any form of investment, the greater is likely to be the risk of losing the money put into it. Would it be better to put money into an investment which promised a large income, but which might not be safe, or into something which promised a very small income, but which was perfectly safe, like United States bonds? We should learn how to distinguish between good investments and bad investments so that we may invest wisely whatever money we may be able to save.

### WRITTEN EXERCISE

1. If you have \$75 to invest and deposit it in a savings bank that pays 2% interest semiannually, how much interest is due you at the end of 6 mo.?

2. If a man has \$1250 to invest and can so invest it as to receive 4% interest, how much will it bring him each year?

3. A man has deposited \$225 in a savings bank that pays 2% interest semiannually. How much interest is due him in 6 mo.?

4. A woman has saved \$1750. She can deposit it in a savings bank at 4%, or invest it in a mining business which she knows nothing about and which the bankers in her town have never heard of but which advertises that it will probably pay 10% a year. How much income would she get from the bank? Which investment should she make?

5. A man has saved \$3500. He can invest it in his city in a well-managed manufacturing business of which he is thoroughly informed, and can get 7% on his investment. Is this a good investment? How much income will he receive a year?

6. A man comes to town and offers investments in a fruit plantation in Mexico. No one has heard of the man or of his plantation. He says the business will pay 15%, and he asks Mr. Doe to invest \$7500. How much income does he promise Mr. Doe? Mr. Doe can invest the money in his own town and get 6%. How much would this bring him per year? Which investment would you advise him to make? Write the reason.

7. How much better income will a man receive on an investment of \$6500 at 6% than on one of the same amount at $4\frac{1}{2}$%?

*Find the incomes from the following investments:*

| | | |
|---|---|---|
| 8. \$400, 5%. | 14. \$925, $5\frac{1}{2}$%. | 20. \$8750, 6%. |
| 9. \$475, 5%. | 15. \$875, $4\frac{1}{2}$%. | 21. \$8225, 6%. |
| 10. \$575, 4%. | 16. \$1250, 4%. | 22. \$6500, 3%. |
| 11. \$725, 4%. | 17. \$1575, 4%. | 23. \$7500, $4\frac{1}{2}$%. |
| 12. \$650, 6%. | 18. \$6250, 5%. | 24. \$8500, 7%. |
| 13. \$875, 6%. | 19. \$7500, 5%. | 25. \$2500, 7%. |

26. Mr. Sinclair has \$7500 invested at 6% and \$9000 at 5%. Which, if either, brings him the greater income, and how much greater? Which investment would you rather have? Why?

27. How does the income from \$9000 at 8% compare with that from \$12,000 at 6%?

28. Mrs. Radcliffe has inherited \$2500, and has a chance to invest it at 5% in a first-class security or at 12% in a banana plantation that her banker tells her is very risky. How much would be the income from each? Which should she take?

**119. Investing in Real Estate.** Besides investing in banks and in promissory notes, by which an income is received in the form of interest, people who have saved some money often buy real estate; that is, land and buildings.

The following suggestions relating to this subject should be read and understood:

Real estate makes a particularly desirable form of investment because of its comparative safety, of the tendency of wisely selected property to increase in value, and of the possibility of using such property for a home for the owner and thus saving the expense of renting.

The disadvantages of this form of investment are first, the expense of maintaining the property, insurance, taxes, repairs, etc., together with the loss of interest on investment; second, the liability to depreciation in value, which is, of course, generally greater with houses than with land; and third, the difficulty of selling the property readily at its full value.

Let each pupil in the class select some piece of property and ascertain the price at which it was held five years ago and its present price. What has been the gain or loss per cent during the five years? If it can be ascertained, the amount expended for insurance, taxes, repairs, etc., and the interest on the original investment and on the annual expenses for the five years at 4% or 5% should be deducted from the gain or added to the loss.

### WRITTEN EXERCISE

1. Robert's father paid \$4000 for a house. He rents it for \$30 a month. He pays \$44 a year for taxes, \$4 for insurance, and \$32 for repairs. What per cent does he receive on his investment?

2. Mr. St. John bought a house for \$4500 and rented it for \$28 a month. He paid annually \$50 for taxes, \$4.50 for insurance, and \$45 for repairs. At the end of 3 yr. from the time when he purchased the property he sold it for \$5200. How much more did he gain than he would have gained by investing the money for 3 yr. at 6%?

3. Mary's father bought a house for $3800 and placed on it a mortgage of $2000. This means that some one else had a legal claim against the property for this amount, with a paper called a mortgage allowing him to sell the property in case his claim was not paid when due. The mortgage drew interest at 6%. Allowing $50 a year for taxes, insurance, and repairs, and supposing that Mary's father could have invested his $1800 in a bank at 4%, how much does the house cost him each year?

4. A man owns a house which he rents for $30 a month, but in 3 yr. the house is vacant for 2 mo. The house cost him $4500, and his annual expenses for taxes, repairs, and insurance are $87.50. What rate of income does he receive?

5. Mr. Jewett buys a house for $4500, mortgaging it for $2000 at 6%. His annual expenses for taxes, repairs, and insurance are $75, and he rents the property for $28 a month, paying a collector 5% for collecting the rent. After 3 yr. he sells the property for $5200 and pays the mortgage. Find the difference in his income and the income that he would have had for the 3 yr. if he had invested the money at 6%.

6. Mr. Davids rents a house in St. Louis for $45 a month. He pays $65 a year for taxes, $30 for insurance and repairs, and $130 which the city assesses him for paving the street. How much is his net income? This being 5% on the cost of the property, how much did the property cost?

7. Mr. Carson buys a house and lot for $4200, mortgaging the property for $1800 at 6%, and paying the balance in cash. His annual expenses for taxes, repairs, and insurance are $68, and he rents the property for $25 a month. After 2 yr. he sells it for $3000, the buyer agreeing to pay the mortgage. Find his gain, assuming that he could invest the money at 6%.

**120. Business Messages.** Formerly when one business man wished to communicate with another he had to go to see him or send a letter by a messenger. To-day he can send a letter to any part of our country, including our island possessions, to Panama, Cuba, Mexico, Canada, and certain parts of Europe for 2¢. For 5¢ he can send a letter to almost any part of the world. The telegraph and telephone are also extensively used.

The writing of business letters, including the proper way of addressing envelopes, should form part of the work in writing, but it is not part of arithmetic.

### WRITTEN EXERCISE

**1.** A telegram to a certain place costs 25 and 2, meaning 25¢ for 10 words and 2¢ for each additional word. How much will a 36-word message cost?

**2.** If the rate is 50 and 4, what will a 45-word telegram cost?

**3.** For a night letter sent by telegraph the standard day rate for 10 words is charged for 50 words or less, and one fifth the standard day rate for 10 words is charged for each additional 10 words or less. If the day rate is 75¢, how much will a night letter of 100 words cost?

**4.** For a day letter sent by telegraph one and one half times the standard night-letter rate is charged for 50 words or less, and one fifth the initial cost for 50 words is charged for each additional 10 words or less. If the day rate is 50¢, how much will a day letter of 150 words cost?

**5.** A cable message from New York to Paris costs 25¢ a word. Find the cost of a 38-word message.

**6.** A telegraphic money order costs twice the rate for a 10-word message plus 1% premium on the face of the money order. A 10-word message from Helena to New York costs 75¢. What will a telegraphic money order for $550 cost?

**121. Nature of a Corporation.** Some boys in the eighth grade have organized a baseball club. There are fourteen boys and they pay \$100 for uniforms and \$12 for balls and bats. If each of the boys contributes $\frac{1}{14}$ of \$112, he will contribute \$8, and if they make some money from tickets to the games, each boy will have $\frac{1}{14}$ of the profits after the expenses are paid.

But some of the boys cannot afford to contribute as much as others, so they divide the \$112 into 224 shares of 50¢ each, and sell to each member as many shares as he cares to buy.

James is one of the chief promoters of the club, and he takes 40 shares, thus making his payment \$20. Fred takes 20 shares, and the others take various amounts.

The first three games draw large crowds, and the gate receipts are heavy. The boys divide the profits according to the number of shares they hold. Fred tries to buy some of the shares that James owns, so as to get more of the profits, but James will not sell for less than 60¢ a share. He says his stock is now above par.

The boys really formed a *corporation.* The *capital* was \$112. There were 224 *shares* of *stock*, the *par value* of each being 50¢. The profits they divided were *dividends*, and these dividends were so high that the stock went *above par.*

In practical life men form corporations in this way, only they play the game of business instead of the game of baseball.

**122. Corporation.** The laws of our various states permit a number of persons who wish to go into business together to organize as one body, called a *corporation.*

**123. Capital.** The total amount contributed by the members of a corporation to carry on the business is called *capital.*

The capital varies with different corporations. Some corporations have very large capital.

### ORAL EXERCISE

1. Some boys organize a tennis club. They put in \$150, and issue shares at 50¢ each. How many shares are there?

2. If William buys 30 shares, how much does he pay?

3. If Frank buys 20 shares, how much does he pay?

4. If there are enough profits from games and from renting the courts so that the boys who bought the shares of stock can sell each share for 55¢, what per cent has the stock increased in value?

5. If the stock goes still higher and becomes 20% above par, what is each share worth?

6. If a heavy storm injures the courts, and the boys have to raise more money, probably no one will want to buy the stock, although Frank offers some of his for 45¢ a share. This price is what per cent below par?

7. The boys appoint a few of their number to direct or manage the business. These are called directors, and they decide on a 10% assessment on all stockholders. How much has to be paid on each share of stock?

8. Why is stock sometimes above par and sometimes below?

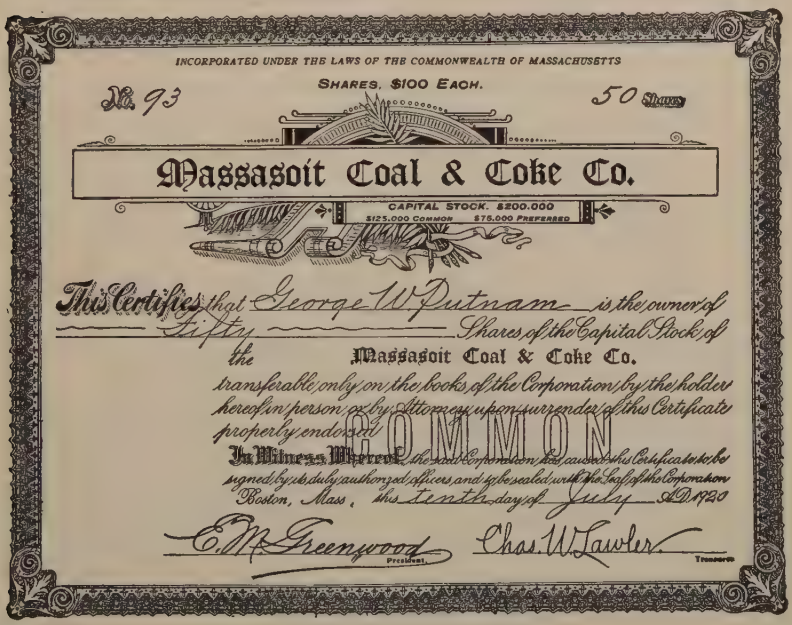
9. Why is it necessary in carrying on business to have stock companies, or corporations, instead of having one man furnish the capital?

10. Why is it necessary for the stockholders to select a few men to direct the business instead of managing it themselves?

11. If you own \$5 of stock in a baseball club and the stock goes 10% above par, how much is it then worth?

12. If you own \$5 of stock in a tennis club and the stock falls 10% below par, how much is it then worth?

**124. Shares and Dividends.** The capital is divided into *shares*, and a person who owns any shares is a *stockholder* and receives a *certificate of stock*. Railway shares are usually \$100 each, although sometimes \$50. Mining and other shares are often \$25, \$5, or \$1 cach, but the amount varies. In our work the par value of each share is to be taken as \$100 unless otherwise stated.


INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS

SHARES, $100 EACH.

No. 93 — 50 Shares

Massasoit Coal & Coke Co.

CAPITAL STOCK, $200.000
$125.000 COMMON — $75.000 PREFERRED

This Certifies that George W Putnam is the owner of Fifty Shares of the Capital Stock of the Massasoit Coal & Coke Co.

transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof the said Corporation has caused this Certificate to be signed by its duly authorized officers and to be sealed with the Seal of the Corporation

Boston, Mass., this Tenth day of July A.D. 1920

E. M. Greenwood, President — Chas. W. Lawler, Treasurer

A CERTIFICATE OF STOCK

The net earnings of a company are divided among the stockholders in the form of *dividends*. There are often two kinds of stock: *preferred stock*, which pays a fixed rate of dividend if that amount is earned; and *common stock*, which takes whatever dividends may be paid after other claims have been satisfied.

**125. Directors and Officers.** The stockholders elect a few of their number to have general direction of the company. These are called *directors*, and they elect the *officers*. These officers include a president, a vice president, a secretary, and a treasurer.

For example, if the capital is \$100,000 and the shares are \$100 each, there are 1000 shares. If a man has 50 shares, he owns \$5000 worth of stock, or 5% of the whole corporation, and at the election of directors he is entitled to a vote for each share. If this company earns \$6000 beyond all expenses, the directors may declare a 6% dividend. Then a man who owns \$5000 worth of stock will receive a check for \$300.

The custom as to preferred and common stock and the par value of stock varies with different companies. The pupils should study these different forms of securities as issued by some local corporation. Some stock is assessable; that is, in case of financial difficulty of the company the stockholders may be assessed to make up any loss. Other stock is nonassessable.

### WRITTEN EXERCISE

1. A company divides \$50,000 in dividends, and the stockholders receive \$4 per share. What is the capital?

2. A company with a capital of \$3,000,000 declares a 5% dividend. How much does the holder of 100 shares receive?

3. A company with a capital of \$500,000 divides \$45,000 in dividends. How much does the holder of 30 shares receive?

4. How much does the holder of 30 shares of railway stock receive when a $4\frac{1}{2}\%$ dividend is declared?

5. A company with \$50,000,000 capital declares a dividend of $2\frac{3}{4}\%$ every six months. How much money does it distribute annually among the stockholders?

6. A company with a capital of \$250,000 has earned \$15,000 this year above all expenses. It decides to save \$2500 of this for emergencies, and to divide the rest in dividends. What is the rate of dividend?

**126. Above Par and Below Par.** If a stock is paying a high rate of dividend, that is, more than can be received from good ordinary investments, people will be so anxious to buy it that they will pay more than \$100 for a \$100 share. The stock is then said to be *above par*. If a \$100 share can be bought for just \$100, the dividends are about on a par with other investments, and the stock is said to be *at par*. If the dividends are low, the public will not care to buy the stock, and it will be *below par*.

**127. Buying Stock.** A purchaser usually buys and sells stock through a *broker*, generally in a place called a *stock exchange*, a kind of auction room for such business.

**128. Brokerage.** The broker charges *brokerage* or *commission*, usually $\frac{1}{8}\%$ of the par value of the stock; that is, $12\frac{1}{2}¢$ per \$100.

**129. Meaning of Stock Quotations.** A quotation of $118\frac{3}{4}$ means that a share of stock, par value \$100, will cost $\$118.75 + \$\frac{1}{8}$ (or $12\frac{1}{2}¢$) brokerage. Thus the buyer will pay $\$118.87\frac{1}{2}$ per share, while the seller, who must also pay his broker, will receive $\$118.75 - \$0.12\frac{1}{2}$, or $\$118.62\frac{1}{2}$ per share.

In stock quotations fractions are usually expressed in halves, fourths, or eighths. Fractional parts of a share cannot be bought on the stock exchange.

**1.** What is the cost of 10 shares of stock quoted at $137\frac{1}{8}$, allowing the usual brokerage?

One share costs $\$137\frac{1}{8} + \$\frac{1}{8}$ brokerage, or $\$137\frac{1}{4}$.
Therefore 10 shares cost $10 \times \$137\frac{1}{4}$, or \$1372.50.

**2.** What is the amount received from the sale of 100 shares of stock quoted at $96\frac{5}{8}$, allowing the usual brokerage?

One share brings $\$96\frac{5}{8} - \$\frac{1}{8}$ brokerage, or $\$96\frac{1}{2}$.
Therefore 100 shares bring $100 \times \$96\frac{1}{2}$, or \$9650.

**130. Bond.** A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond.* When corporations wish to borrow any considerable amount of money, they issue bonds. These bonds usually have small notes called *coupons* annexed, agreeing to pay the interest as due. Bonds are usually secured by a *mortgage*, an agreement by which the holders of the bonds may sell the property if the bonds and the interest are not paid when due.

**131. Difference between Bonds and Shares of Stock.** Bonds differ from shares of stock in this way: the stockholders of a corporation are the owners, while the bondholders are owed money by the corporation. Bonds bear a fixed rate of interest, but the income on shares of stock depends on the earnings of the company after the running expenses and the interest on the bonds have been paid. A share of stock is like a deed to a farm; a bond is like a mortgage on a farm.

**132. Computing Dividends.** Stock dividends and bond incomes are always computed on the par value.

**1.** How much is the income on \$5000 worth of 4% bonds?

4% of \$5000 = \$200.

**2.** What is the income on \$5000 of railway stock at 5%?

Since the dividend is 5%, we have 5% of \$5000 = \$250.

**3.** If a 5% bond is bought at $124\frac{7}{8}$, what is the rate of income on the investment?

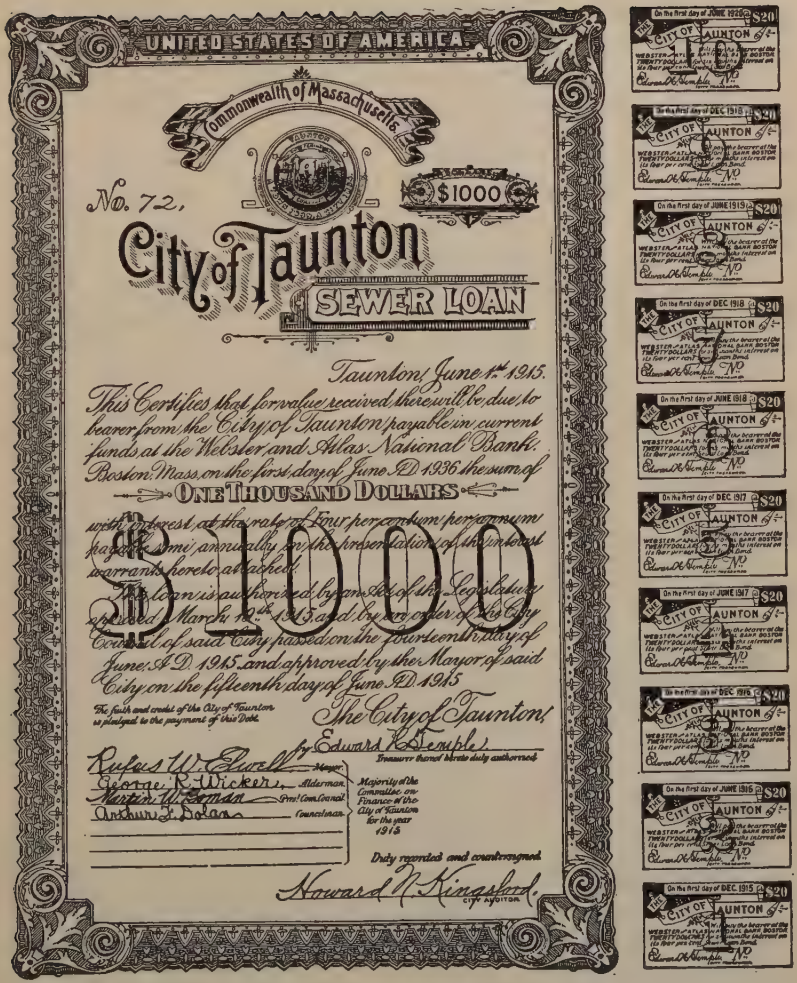
The cost of \$100 worth of bonds = $\$124\frac{7}{8} + \$\frac{1}{8}$ (brokerage) = \$125.

The income on \$100 worth of bonds is 5% of \$100, or \$5.

Since we have a product, \$5, and one factor, \$125, we divide.

Therefore $\$5 \div \$125 = 0.04$.

Hence the income is 4% on the *investment* (but 5% on the *par value*), not considering the date of maturity of the bond.


UNITED STATES OF AMERICA.

Commonwealth of Massachusetts

No. 72.

$1000

City of Taunton

SEWER LOAN

Taunton, June 1st 1915.

This Certifies that for value received there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston Mass. on the first day of June A.D. 1936 the sum of

ONE THOUSAND DOLLARS

with interest at the rate of Four per centum per annum payable semi-annually on the presentation of the interest warrants hereto attached.

$1000

This loan is authorized by an Act of the Legislature approved March 12th 1915, and by an order of the City Council of said City passed on the fourteenth day of June A.D. 1915 and approved by the Mayor of said City on the fifteenth day of June A.D. 1915

The faith and credit of the City of Taunton is pledged to the payment of this Debt.

The City of Taunton,
By Edward K. Temple,
Treasurer thereof duly authorized

Rufus W. Purcell, Mayor
George R. Wickes, Alderman
Martin W. Forman, Pres. Com. Council
Arthur L. Dolan, Councilman

Majority of the Committee on Finance of the City of Taunton for the year 1915

Duly recorded and countersigned.

Howard N. Kingsford
CITY AUDITOR.


On the first day of JUNE 1916
On the first day of DEC 1916
On the first day of JUNE 1917
On the first day of DEC 1917
On the first day of JUNE 1918
On the first day of DEC 1918
On the first day of JUNE 1919
On the first day of DEC 1919
$20
CITY OF TAUNTON
WEBSTER-ATLAS
TWENTY DOLLARS
Edward K. Temple

A COUPON BOND

**133. Newspaper Quotations.** Daily newspapers give the prices of the leading stocks. The examples in the next exercise may be solved by using the newspaper quotations or the following short list, and by taking \$100 as the par value per share:

| | | | |
|---|---|---|---|
| At., Top., & S.F. | $93\frac{1}{2}$ | Penn. R.R. | $104\frac{1}{2}$ |
| Balt. & Ohio | 67 | Union Pac. | $118\frac{1}{2}$ |
| N.Y. Central | $84\frac{1}{2}$ | Western Un. Tel. | $62\frac{1}{2}$ |

The quotations of stocks are often, as in the above list, the same as if the par value were \$100 per share, even though this is not the case. That is, the par value of a share of the Pennsylvania R.R. is \$50, and when it is worth \$52.25 the quotation is often given as \$104.50 ($104\frac{1}{2}$). These are technicalities of the stock market which do not concern the school.

In the following examples add $\frac{1}{8}$ to the quotation if you are buying, and subtract $\frac{1}{8}$ if you are selling, to pay the broker.

If a man should buy 100 shares of Western Union Telegraph stock when it is quoted at $63\frac{1}{4}$ and should sell it when it is quoted at $62\frac{1}{2}$, how much does he lose on the transaction?

He buys the stock at $63\frac{1}{4} + \frac{1}{8}$ (brokerage), or $63\frac{3}{8}$.
He sells the stock at $62\frac{1}{2} - \frac{1}{8}$ (brokerage), or $62\frac{3}{8}$.
Therefore he loses \$1 a share.
On 100 shares he loses 100 × \$1, or \$100.

$$\begin{array}{r} \$63\frac{3}{8} \\ 62\frac{3}{8} \\ \hline \$1 \\ 100 \\ \hline \$100 \end{array}$$

WRITTEN EXERCISE

1. What is the par value of 50 shares of stock, the par value of a share being \$100? the par value of a share being \$50? the par value of a share being \$25?

2. If a man has \$7500 to invest, and he can buy some shares of stock at par value without paying any brokerage, how many shares can he buy if the par value of each share is \$100? if the par value of each share is \$25?

**3.** How much will 75 shares of stock cost when quoted at $96\frac{1}{8}$ a share? at $102\frac{5}{8}$? at $68\frac{1}{2}$? at $99\frac{7}{8}$?

**4.** A 6% bond is purchased at $149\frac{7}{8}$. What is the rate of income on the money invested?

**5.** Which gives the better income, 5% stock bought when quoted at $139\frac{7}{8}$ or $3\frac{1}{2}$% bonds bought when quoted at $99\frac{7}{8}$?

Compare \$5 ÷ \$140 with \$3.50 ÷ \$100.

**6.** Not considering brokerage, which gives the better rate of income, a 6% bond at 120 or a 5% promissory note?

Compare \$6 ÷ \$120 with \$5 on \$100.

**7.** A man buys, when quoted at $199\frac{7}{8}$, stocks that pay 7% dividends. What is the rate of income on the money invested?

**8.** When United States 4% bonds are quoted at $116\frac{7}{8}$, what rate of income does a purchaser receive on his investment, not considering the question of the date of maturity of the bonds?

*Find the gain or loss on buying 50 shares, par value \$100, of the following stocks as quoted on page 196, and selling at these prices:*

**9.** At., Top., & S.F., $92\frac{1}{8}$.

**10.** Balt. & Ohio, $68\frac{1}{2}$.

**11.** N.Y. Central, $82\frac{3}{8}$.

**12.** N.Y. Central, $89\frac{3}{4}$.

**13.** Union Pac., 116.

**14.** Western Un. Tel., $62\frac{3}{4}$.

**15.** If you had some money to invest, which would you prefer, a stock that regularly pays 8%, quoted at $159\frac{7}{8}$, a 5% bond at $109\frac{7}{8}$, or a $5\frac{1}{2}$% mortgage, the security being equally good?

Compare \$8 return on \$160 invested, \$5 return on \$110 invested, and \$5.50 return on \$100 invested. Use short methods of work.

**16.** If you had some money to invest, which would you prefer, a stock quoted at $149\frac{7}{8}$, that regularly pays 7%, a 4% bond at $89\frac{7}{8}$, or a $4\frac{1}{2}$% note, the security being equally good?

From time to time recall the suggestion of estimating the result in advance.

## LITTLE EXAMINATIONS

I. *At $5\frac{1}{2}$ mills find the taxes on:*

1. \$3000. 2. \$5000. 3. \$7500. 4. \$15,000.

*Find the taxes on \$14,000 at these rates:*

5. 6.2 mills. 6. $7\frac{1}{2}$ mills. 7. 8.1 mills. 8. 8.3 mills.

*Find the duties on goods valued as follows:*

9. \$17,500, 15%. 10. \$25,750, 25%.

II. *Find the premiums on the following fire-insurance policies:*

1. \$8500, \$1.10. 2. \$12,500, \$1.20. 3. \$37,000, \$1.60.

4. Compute a dividend of $4\frac{1}{4}\%$ on \$150,000.

5. If a mining company declares a dividend of $5\frac{1}{2}\%$, and this amounts to \$8250, what is the capital?

6. Which will bring the better income, if any, a 5% note or a 6% stock @ $119\frac{7}{8}$?

7. Which will bring the better income, a 5% note or a 7% stock quoted @ 150? how much better?

8. Which will bring the better income, a 5% note or an 8% stock quoted @ $149\frac{7}{8}$? how much better?

9. Which will bring the better income, a 6% stock quoted @ $139\frac{7}{8}$ or a 5% stock quoted @ $119\frac{7}{8}$? how much better?

10. If a man receives \$24 on a share of stock, and this is 4% on what he paid for it, how much did it cost him?

# REVIEW AND DRILL

## I. DRILL SECTION

### WRITTEN EXERCISE

**1.** Write in Roman numerals the numbers 75, 86, 93, and 39.

**2.** Write in ordinary numerals the numbers two hundred and five thousandths, and two hundred five thousandths.

*Add, and also subtract these numbers:*

| **3.** | **4.** | **5.** | **6.** |
|---|---|---|---|
| 1742.8 | 4183.42 | 3978.41 | 9002.07 |
| 396.9 | 968.75 | 699.56 | 361.56 |

*Multiply, and also divide these numbers:*

**7.** 172.8 by 1.44. **9.** 473.48 by 6.23. **11.** 47.595 by 5.01.

**8.** 409.53 by 5.1. **10.** 517.42 by 63.1. **12.** 161.463 by 32.1.

*Add, and also subtract these numbers:*

| **13.** | **14.** | **15.** | **16.** | **17.** | **18.** |
|---|---|---|---|---|---|
| $3\frac{9}{10}$ | $4\frac{7}{8}$ | $7\frac{5}{8}$ | $9\frac{1}{16}$ | $8\frac{1}{32}$ | $6\frac{4}{5}$ |
| $2\frac{2}{3}$ | $3\frac{1}{5}$ | $2\frac{5}{16}$ | $3\frac{5}{8}$ | $2\frac{3}{4}$ | $3\frac{3}{8}$ |

*Multiply, and also divide these numbers:*

**19.** $\frac{2}{3}$ by $\frac{4}{5}$. **22.** $\frac{7}{32}$ by $\frac{1}{5}$. **25.** $\frac{9}{10}$ by $\frac{2}{3}$. **28.** 37 ft. 8 in. by 2.

**20.** $\frac{5}{8}$ by $\frac{3}{4}$. **23.** $\frac{7}{16}$ by $\frac{3}{4}$. **26.** $\frac{3}{10}$ by $\frac{4}{5}$. **29.** 48 lb. 6 oz. by 2.

**21.** $\frac{5}{8}$ by $\frac{2}{3}$. **24.** $\frac{7}{16}$ by $\frac{2}{3}$. **27.** $\frac{3}{16}$ by $\frac{2}{5}$. **30.** 46 ft. 3 in. by 3.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

**1.** If you know the assessed valuation and the tax rate, how do you find the tax to be paid?

**2.** If you know the assessed valuation of a county and the amount of tax to be raised, how do you find the tax rate?

**3.** If you know the amount of tax to be raised by a village and the tax rate, how do you find the assessed valuation?

**4.** If you know the face of a fire-insurance policy and the premium per \$100, how do you find the premium?

**5.** If you know the face of a life-insurance policy and the premium per \$1000, how do you find the premium?

**6.** There are companies that insure employers against accidents to the workmen. If you know the annual pay roll of an employer, and the insurance company charges a certain per cent of the pay roll, how do you find the total premium?

**7.** If you know the newspaper quotation on some railroad stock, how do you find the cost of a share?

**8.** If you know the newspaper quotation on a certain stock, how do you find the amount you would receive for a share?

**9.** If some mining stock is quoted at a specified amount above par, how do you find the cost of a share?

**10.** What would have to be the rate of dividend on a first-class stock, in comparison with the average rate of income on good investments, to have it at par? to have it below par?

**11.** If you put some money in the postal savings bank, how can you find the amount of the principal and interest at the end of a given time?

## III. INDUSTRIAL PROBLEMS

### THE FAMILY BUDGET

**1.** On New Year's Day Fred's father said: "Let us figure up last year's expenses. I received an income of \$1500. I set aside certain per cents that I would spend for the family. This is what I thought we ought to spend: rent, 17%; heat, 4%; light, $1\frac{1}{2}$%; food, 28%; wages, $8\frac{1}{2}$%; incidentals, 6%; other personal expenses, 15%; books, music, church, and pleasure, 15%. How much money did I allow for these different purposes?" How much did Fred find this to be in each case?

**2.** Fred's father really paid for rent, \$240; for heat, \$52.75; for light, \$17.60; for food, \$406.85; for wages, \$120; for incidentals, \$43.60; for other personal expenses, \$248.65; and for books, music, church, and pleasure, \$258.80. He asked Fred to see how much each expenditure differed from the estimate, and how much he saved during the year.

**3.** He also asked Fred to find what per cent of the amount spent for heat and light was spent for light alone, and what per cent was spent for heat alone. Find these per cents.

**4.** He told Fred that when he began to work he received only \$840 a year, and on this income he was married. He said that the first year he paid for rent, \$168; for heat and light, \$42.50; for food, \$340; for clothing, \$85.50; for insurance, \$32.50; and for other expenses, \$145. He asked Fred to find how much he saved that year. How much was it?

**5.** He asked Fred to find what per cent of his income he paid for rent then. What was it? Was it more or less than he pays now? What was the per cent of his income spent for food then? Was it greater or less than the per cent spent now?

### MONEY VALUE OF AN EDUCATION

1. Kate, who left school at the end of the seventh grade, went into a shop to work at \$17.40 a month. Her sister Alice went on through the high school and got a position paying \$24.70 a month. How much more did Alice earn in a year than Kate?

2. The second year that Alice worked she received an increase of 10% and Kate received an increase of 5%. How much more did Alice earn the second year than Kate?

3. Kate had a friend in the shop who was working for \$16 a month, but who left and studied in a trade school. When the friend finished her course she got work at \$6.25 a week. How much more did she now receive a year than she did in the shop?

4. A girl who is just leaving the public school finds she can enter a city shop at \$4 a week the first year, with 10% more the second year, and a 15% increase the third year over the second. Instead of this she enters a trade school for a year, tuition free. She then receives \$7 a week the first year and 20% more the second year. Counting 50 working weeks a year, how much more does she receive in three years by this plan?

5. A college graduate earns on an average \$1125 a year, while the average wages of a day laborer, who has no more than completed the elementary school, are \$475. Find the difference in the total earnings of two such men after 25 yr. of work.

6. The college man mentioned in Ex. 5 has spent on an average 180 da. a year for 8 yr. on his course beyond the elementary school. He has also averaged \$275 a year for his education. Deducting what he spent from the excess of his earnings over those of the day laborer, we have the net surplus. Divide this by the number of days he spent in the high school and in college, and thus find the approximate money value of each of those days.

# CHAPTER VII

## SQUARE ROOT AND MENSURATION

**134. Squares and Square Roots.** If a square has a side of 4 units, it has an area of 16 square units. Therefore 16 is called the *square* of 4, and 4 is called the *square root* of 16.

**135. Square Roots of Areas.** Therefore, considering the abstract numbers representing the sides and area,

*The side of a square equals the square root of the area.*

**136. Symbols.** The square of 4 is written $4^2$, and the square root of 16 is written $\sqrt{16}$.

**137. Perfect Squares.** Such a number as 16 is called a *perfect square*, but 10 is not a perfect square. We may say, however, that $\sqrt{10}$ equals 3.16 +, because $3.16^2$ nearly equals 10.

**138. Square Roots of Perfect Squares.** Square roots of perfect squares may often be found by simply factoring the numbers.

For example, $\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7}$
$= \sqrt{3 \times 7 \times 3 \times 7}$
$= \sqrt{21 \times 21} = 21.$

$$\begin{array}{r|l} 3 & 441 \\ 3 & 147 \\ 7 & 49 \\ & 7 \end{array}$$

### ORAL EXERCISE

*State the square roots of the following:*

| | | | | | |
|---|---|---|---|---|---|
| 1. 64. | 3. 81. | 5. 121. | 7. 400. | 9. 1.44. | 11. 1600. |
| 2. 9. | 4. 49. | 6. 144. | 8. 900. | 10. 0.64. | 12. 4900. |

### WRITTEN EXERCISE

*By factoring, find the square roots of the following:*

| | | | |
|---|---|---|---|
| **1.** 625. | **4.** 729. | **7.** 1296. | **10.** 12.25. |
| **2.** 324. | **5.** 576. | **8.** 1089. | **11.** 40.96. |
| **3.** 484. | **6.** 2025. | **9.** 65.61. | **12.** 14,641. |

*Find the sides of squares whose areas are:*

| | | |
|---|---|---|
| **13.** 4.84 sq. in. | **16.** 4.41 sq. in. | **19.** 5929 sq. yd. |
| **14.** 1.96 sq. ft. | **17.** 4356 sq. in. | **20.** 10.89 sq. yd. |
| **15.** 2.25 sq. ft. | **18.** 15,625 sq. in. | **21.** 0.1225 sq. ft. |

*Find the perimeters of squares whose areas are:*

| | | |
|---|---|---|
| **22.** 6561 sq. ft. | **25.** 19,600 sq. ft. | **28.** 16,384 sq. in. |
| **23.** 12,100 sq. in. | **26.** 146.41 sq. in. | **29.** 16,900 sq. ft. |
| **24.** 11,025 sq. ft. | **27.** 20,736 sq. in. | **30.** 129,600 sq. in. |

**31.** From a corner of a square piece of land that contains 576 sq. rd. a small square lot containing 64 sq. rd. is cut. Draw a plan of the lots and find the perimeter of each.

**32.** A square lot has an area of 169 sq. rd. How far is it around the lot? How far is it around a lot four times this area? The second perimeter is how many times the first?

**33.** A man has two adjacent building lots fronting on the street, each lot being square. The area of the two together is 89 sq. rd., that of the larger being 64 sq. rd. What is the frontage of each lot?

**34.** A square lot has an area of 289 sq. rd. How far is it around the lot? (Try the prime numbers between 10 and 20.) How far is it around a lot of nine times this area? The second perimeter is how many times the first?

**139. Square of the Sum of Two Numbers.** Since $47 = 40 + 7$, the square of 47 may be obtained as follows:

$$\begin{array}{rl} & 40+7 \\ & 40+7 \\ \hline & (40\times 7)+7^2 \\ 40^2+ & (40\times 7) \\ \hline & 40^2+2\times(40\times 7)+7^2 \\ =1600+2\times 280 & +49 \\ =1600+560 & +49 \\ =2209. & \end{array}$$

| | |
|---|---|
| 280 | 49 |
| 1600 | 280 |
| 40 | 7 |
| $t$ | $+u$ |

This relationship is seen in the annexed figure, where the side of the square is 40 + 7.

Every number consisting of two or more figures may be regarded as composed of tens and units. Therefore,

*The square of a number contains the square of the tens, plus twice the product of the tens and units, plus the square of the units.*

This is the most important principle in square root and should be clearly understood, both from the multiplication and from the illustration.

**140. Separating into Periods.** The first step in extracting the square root of a number is to separate the figures of the number into groups of two figures each, called *periods.*

Since $1 = 1^2$, $100 = 10^2$, $10{,}000 = 100^2$, and so on, it is evident that the square root of any number between 1 and 100 lies between 1 and 10, and the square root of any number between 100 and 10,000 lies between 10 and 100. In other words, the square root of any integral number expressed by *one* figure or *two* figures is a number of *one* figure; the square root of any integral number expressed by *three* or *four* figures is a number of *two* figures; and so on.

Therefore, if an integral number is separated into periods of two figures each, from the right to the left, the number of figures in the square root will be equal to the number of the periods of figures. The last period at the left may have one figure or two figures; for example, 22 09, and 7 89 04 81.

**141. Extracting the Square Root.** The process of extracting the square root of a number will now be considered.

For example, required the square root of 2209.

Separating the figures of the number into periods (§ 140), we see that there will be two integral places in the root.

The first period, 22, contains the square of the tens' number of the root. Since the greatest square in 22 is 16, hence 4, the square root of 16, is the tens' figure of the root.

```
   22 09 ( 47
   16
 80  6 09
 87  6 09
```

Subtracting the square of the tens, the remainder contains twice the tens × the units, plus the square of the units (§ 139). If we divide by twice the tens (that is, by 80, which is 2 × 4 tens), we shall find approximately the units. Dividing 609 by 80 (or 60 by 8), we have 7 as the units' figure.

Since twice the tens × the units, plus the square of the units, is equal to (twice the tens + the units) × the units, that is, since $2 \times 40 \times 7 + 7^2 = (2 \times 40 + 7) \times 7$, we add 7 to 80 and multiply the sum by 7. The product is 609, thus completing the square of 47.

Checking the work, $47^2 = 2209$.

### WRITTEN EXERCISE

*Find the square roots of the following:*

**1.** 3249. **3.** 3721. **5.** 3969. **7.** 5041.

**2.** 1681. **4.** 1225. **6.** 2809. **8.** 3481.

*Find the sides of the squares, given the following areas:*

**9.** 6724 sq. ft. **11.** 9025 sq. ft. **13.** 7921 sq. rd.

**10.** 7569 sq. ft. **12.** 9409 sq. ft. **14.** 6889 sq. yd.

*Find the square roots of the following by taking the square root of each term of the fractions:*

**15.** $\frac{144}{529}$. **17.** $\frac{961}{1024}$. **19.** $\frac{1089}{1156}$. **21.** $\frac{1936}{5041}$.

**16.** $\frac{3025}{3249}$. **18.** $\frac{361}{8281}$. **20.** $\frac{529}{9801}$. **22.** $\frac{289}{9409}$.

**142. Square Root with Decimal Fractions.** Find the square root of 151.29.

The greatest square of the tens in 151.29 is 100, and $\sqrt{100} = 10$.

Then 51.29 contains $2 \times 10 \times$ the units' number of the root, plus the square of the units' number. Why is this?

Dividing by $2 \times 10$, or 20, we find that the next figure of the root is 2.

We have now found 12, the square being $100 + 44 = 144$.

Then 7.29 contains $2 \times 12 \times$ the tenths' number of the root, plus the square of the tenths' number, because we have subtracted 144, which is the square of 12.

Dividing by 24, we find that the tenths' figure of the root is 3.

Hence the square root of 151.29 is 12.3.

If the number is not a perfect square, we may annex pairs of zeros at the right of the decimal point and find the root to as many decimal places as we choose.

| | 1 51.29 (12.3 |
|---|---|
| | 1 |
| 20 | 51 |
| 22 | 44 |
| 240 | 729 |
| 243 | 729 |

**143. Summary of Square Root.** We now see that the following are the steps to be taken in finding square root:

*Separate the number into periods of two figures each, beginning at the decimal point.*

*Find the greatest square in the left-hand period and write its root for the first figure of the required root.*

*Square this root, subtract the result from the left-hand period, and to the remainder annex the next period for a dividend.*

*Divide the new dividend thus obtained by twice the part of the root already found.*

*Annex the figure thus found to this divisor and multiply by the same figure.*

*Subtract this result, bring down the next period, and proceed as before until all the periods have been thus annexed.*

*The result is the square root required.*

### WRITTEN EXERCISE

*Find the square roots of the following:*

| | | |
|---|---|---|
| **1.** 12,321. | **5.** 19.4481. | **9.** 63,001. |
| **2.** 54,756. | **6.** 0.2809. | **10.** 21,224,449. |
| **3.** 110.25. | **7.** 1176.49. | **11.** 49,112,064. |
| **4.** 8046.09. | **8.** 82.2649. | **12.** 96,275,344. |

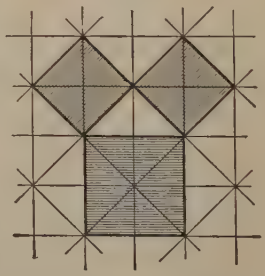
In Exs. 13–18 carry the square roots to two decimal places only.

**13.** 2. **14.** 5. **15.** 7. **16.** 8. **17.** 11.

**18.** Find the number of rods in the side of a square containing 1 acre (160 sq. rd.).

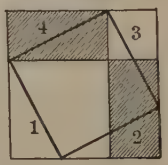
**144. Square on the Hypotenuse.** In a right triangle the side opposite the right angle is called the *hypotenuse* of the triangle.

If a floor is made up of triangular tiles like this, it is easy to mark out a right triangle. In the figure it is seen that the square on the hypotenuse contains eight small triangles, while each square on a side contains four such triangles. Hence



*The square on the hypotenuse equals the sum of the squares on the other two sides.*

This is also true of any right triangle, for we see that if the four triangles, $1 + 2 + 3 + 4$, are taken away from this figure, there remains the square on the hypotenuse. But if we take away the two shaded rectangles, which together have exactly the same area as the four triangles, the squares on the two sides remain. Therefore the square on the hypotenuse must equal the sum of these two squares.



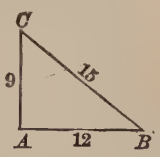
**145. Illustrative Problem.** If $AB = 12$ and $AC = 9$, find $BC$.

Since $\overline{AB}^2 + \overline{AC}^2 = \overline{BC}^2$,

therefore $12^2 + 9^2 = \overline{BC}^2$,

or $144 + 81 = 225 = \overline{BC}^2$,

and $\sqrt{225} = BC$. Therefore $BC = 15$.




**WRITTEN EXERCISE**

**1.** How long is the diagonal of a hall 51 ft. by 68 ft.?

**2.** Find to two decimal places the length of the diagonal of a square that contains 4 sq. ft.

**3.** The two sides of a right triangle are 20 in. and 30 in. Find to two decimal places the length of the hypotenuse.

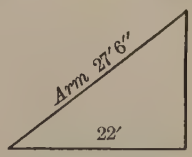
**4.** The two sides of a right triangle are 57 in. and 76 in. Find the length of the hypotenuse.

**5.** What is the length of a wire drawn taut from the cornice of a 100-foot building to a spot 75 ft. from the foot?

**6.** Find the length of the hypotenuse of a right triangle when the sides are 321 in. and 428 in.; 40 in. and 75 in.; 72 ft. and 135 ft.; 36 rd. and $67\frac{1}{2}$ rd.; 20 yd. and 37 yd. 18 in.

**7.** A telegraph pole is set perpendicular to the ground, and a wire is fastened to it 18 ft. from the ground, and then to a stake 13 ft. 6 in. from the foot of the pole, so as to hold it in place. How long is the wire?

**8.** A derrick for hoisting coal has its arm 27 ft. 6 in. long. It swings over an opening 22 ft. from the base of the arm. How far is the top of the derrick above the opening?




Reversing the procedure in § 144, the square on either side equals the difference of what squares?

*Find to two decimal places the hypotenuse of each of the right triangles of which the sides are here given:*

| | | |
|---|---|---|
| **9.** 35 ft., 26 ft. | **12.** $42\frac{1}{2}$ ft., $63\frac{1}{4}$ ft. | **15.** $3\frac{3}{4}$ in., $8\frac{1}{2}$ in. |
| **10.** 81 ft., 35 ft. | **13.** 4.5 in., 7.2 in. | **16.** 23 in., 34 in. |
| **11.** 10 rd., 13 rd. | **14.** 6.25 in., 7.5 in. | **17.** 3.4 in., 2.7 in. |

**18.** A room is 16 ft. long, 12 ft. wide, and 9 ft. high. Find to two decimal places the distance from an upper corner through the room to the opposite lower corner.

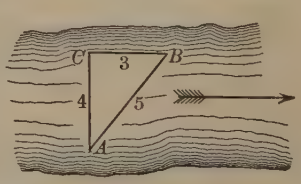
First find the hypotenuse on the floor and then the hypotenuse required. This hypotenuse is called the *diagonal* of the room.

**19.** Find to two decimal places the length of the diagonal of a cube whose volume is 8 cu. in.

First find the edge. Then proceed as in Ex. 18.

**20.** A school flag pole is broken by the wind 16 ft. from the ground. The two pieces hold together, and the top of the pole touches the ground 30 ft. from the foot of the pole. Find the length of the flag pole.

**21.** If I start to row directly across a stream, in the direction $AC$, at the rate of 4 mi. an hour, and if the stream carries me in the direction $CB$, at the rate of 3 mi. an hour, my course will really be $AB$, the result of these two motions. Suppose I row at the rate of 4.5 mi. per hour, and the stream flows 6 mi., what is my rate of progress?




**22.** The captain is walking at the rate of 4 mi. per hour directly across the deck of a steamer, while the boat is moving at the rate of 8 mi. per hour. Find to two decimal places the rate at which the captain is moving. Draw a plan to scale.

It is a good exercise to make up problems like those in Exs. 18–22.

**23.** A square lot has an area of 27,556 sq. ft. How many feet in the length of a side?

**24.** A square piece of oilcloth has an area of 148 sq. ft. 4 sq. in. How many inches in the length of a side?

**25.** The bottom of a square box has an area of 1936 sq. in. How long is the box?

**26.** A square lot has an area of 62,001 sq. ft. How many feet in the length of a side?

**27.** Determine the number of rods in the perimeter of a square field the diagonal of which is 1520 rd.

**28.** The diagonal of a square is 15 ft. Find to two decimal places the length of a side.

**29.** The side of a square is 12.5 ft. Find to two decimal places the length of the diagonal.

**30.** The area of a square is $3\frac{1}{2}$ sq. ft. Find to two decimal places the length of a side.

**31.** The foot of a 45-foot ladder is 27 ft. from the wall of a building against which the top rests. How high does the ladder reach on the wall?

**32.** How far from the wall of a house must the foot of a 45-foot ladder be placed in order that the top of the ladder may touch a window sill 40 ft. from the ground?

**33.** A rope stretched from the top of a 30-foot pole just reaches the ground 18 ft. from the foot of the pole. Assuming the rope to be straight, how long is it?

In practice, tables showing the square and cube roots of numbers are used. The extraction of cube root by the old method of arithmetic is obsolete as a working plan, and is therefore omitted from the essentials which make up this series. A short table of square and cube roots is given in the Supplement on page 277 and, if desired, may be used for many of the above examples.

**146. Circumference, Diameter, and Radius.** The entire distance around a circle is called the *circumference.* A line drawn through the center of a circle and terminated at each end by the circumference is called a *diameter.* Half a diameter is called a *radius.*

A circumference is like the length of a tire of a wheel; a radius is like a spoke of a wheel; a diameter is like two opposite spokes. The word "circumference" is used for the line which bounds the circle as well as for the length of the line.

**147. Ratio of Circumference to Diameter.** By measuring several circles and dividing each circumference by its diameter, the circumference will be found to be about $3\frac{1}{7}$ times the diameter.

It is proved in geometry that this ratio of the circumference to the diameter is more nearly 3.1416. For our work use $3\frac{1}{7}$, or $\frac{22}{7}$, instead of 3.1416, unless the contrary is stated.

**1.** Required the circumference when the radius is 7 in.

Since the radius is 7 in., the diameter is $2 \times 7$ in. Therefore the circumference is

$$3\tfrac{1}{7} \times 2 \times 7 \text{ in.} = 44 \text{ in.}$$

**2.** Required the diameter when the circumference is 28.6 in.

Since the circumference is the product of the diameter and $\frac{22}{7}$, we divide the circumference by $\frac{22}{7}$ to find the diameter.

Therefore

$$\frac{28.6}{\frac{22}{7}} = \frac{7 \times 28.6}{22} = 9.1.$$

(1.3)

Hence the diameter is 9.1 in.

### ORAL EXERCISE

*State the circumferences of circles of the following diameters:*

**1.** 7 in. **2.** 21 in. **3.** 35 in. **4.** 42 in.

*State the circumferences of circles of the following radii:*

**5.** 7 in. **6.** 14 in. **7.** 21 in. **8.** 28 in.

### WRITTEN EXERCISE

*Find the circumferences, given the diameters:*

| | | | |
|---|---|---|---|
| **1.** 68.2 in. | **4.** 5.11 ft. | **7.** 4.69 ft. | **10.** 13 ft. 6 in. |
| **2.** 48.3 ft. | **5.** 53.9 ft. | **8.** 58.1 in. | **11.** 17 ft. 8 in. |
| **3.** 423 in. | **6.** 6.37 in. | **9.** 6.02 ft. | **12.** 64.4 ft. |

*Find the diameters, given the circumferences:*

| | | | |
|---|---|---|---|
| **13.** 132 ft. | **16.** 176 in. | **19.** 68.2 in. | **22.** 198 in. |
| **14.** $58\frac{2}{7}$ ft. | **17.** 770 ft. | **20.** 3.96 ft. | **23.** 15.4 in. |
| **15.** $97\frac{3}{7}$ ft. | **18.** $96\frac{4}{5}$ ft. | **21.** 0.484 ft. | **24.** 0.11 mi. |

*Using 3.1416 instead of $\frac{22}{7}$, find the circumferences, given the diameters:*

| | | | |
|---|---|---|---|
| **25.** 17 in. | **27.** 2.8 in. | **29.** 4.2 in. | **31.** 2.25 ft. |
| **26.** 13 in. | **28.** 4.37 in. | **30.** $3\frac{1}{2}$ ft. | **32.** 6 ft. 2 in. |

*Using 3.1416, find the diameters, given the circumferences:*

| | | |
|---|---|---|
| **33.** 53.4072 in. | **35.** 97.3896 in. | **37.** 182.2228 ft. |
| **34.** 84.8232 in. | **36.** 94.2478 ft. | **38.** 138.2304 ft. |

*Using $\frac{22}{7}$, find the circumferences, given the radii:*

| | | | |
|---|---|---|---|
| **39.** 49 in. | **41.** 91 in. | **43.** 9.8 in. | **45.** 15.4 in. |
| **40.** 77 in. | **42.** 8.4 in. | **44.** 105 in. | **46.** $1\frac{3}{4}$ in. |

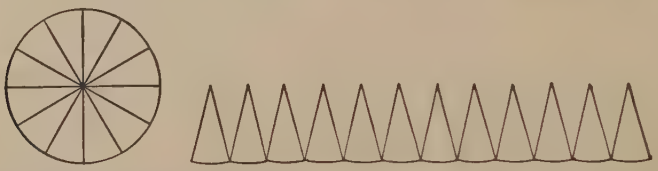
**47.** What is the diameter when the circumference is 74.8 ft.?

**48.** What is the circumference of a rod whose diameter as measured by calipers is 2.1 in.?

**49.** What is the circumference of a wire whose diameter as measured by calipers is 0.63 in.?

**50.** What radius must be used in making a pattern for a wheel that is to be $91\frac{1}{7}$ in. in circumference?

**148. Area of a Circle.** A circle can be separated into figures which are nearly triangles. The height of each triangle is the radius, and the sum of the bases is the circumference. If these



were exact triangles, the area would be $\frac{1}{2} \times$ radius $\times$ circumference. It is proved in geometry that this is the true area.

**149. Given the Radius, to find the Area.** Because the circumference is $\frac{22}{7} \times$ diameter, the area must be

$$\frac{1}{2} \times \text{radius} \times \frac{22}{7} \times \text{diameter},$$

or $$\frac{1}{2} \times \text{radius} \times \frac{22}{7} \times 2 \times \text{radius},$$

or $$\frac{22}{7} \times \text{the square of radius.}$$

*The area of a circle is $\frac{22}{7}$ times the square of the radius.*

Required the area of a circle whose radius is 5 in.

Since $5^2 = 25$,

we have $\text{area} = \frac{22}{7} \times 25 \text{ sq. in.} = 78\frac{4}{7} \text{ sq. in.}$

**ORAL EXERCISE**

*State the areas, given the radii as follows:*

| | | | | |
|---|---|---|---|---|
| **1.** 7 in. | **3.** 14 in. | **5.** 21 in. | **7.** $\frac{1}{22}$ in. | **9.** 1.4 in. |
| **2.** 70 in. | **4.** 28 in. | **6.** 42 in. | **8.** $\frac{7}{22}$ in. | **10.** 2.8 in. |

**11.** If you take a radius half as long as a given radius, the area of the circle will be what part as great?

**12.** If you double the length of the radius, the area of the circle will be how many times as large?

**150. Illustrative Problem.** Required the radius of the circle whose area is $50\frac{2}{7}$ sq. in.

The area is the product of $\frac{22}{7}$ by the square of the radius.

Therefore, dividing by $\frac{22}{7}$ we have the square of the radius.

$$50\tfrac{2}{7} \div \tfrac{22}{7} = \tfrac{7}{22} \times \tfrac{352}{7} = 16.$$

Since $\sqrt{16} = 4$, the radius is 4 in.

### WRITTEN EXERCISE

**1.** What is the area of the cross section of a water pipe that is 2 in. in diameter?

**2.** How many square feet in the base of a water tank that is 42 ft. in diameter?

**3.** A horse tethered by a rope 21 ft. long can graze over how many square feet of ground?

**4.** A horse tethered by a rope can graze over 3850 sq. ft. of ground. How long is the rope?

**5.** How long is the equator on a globe of diameter 12.67 in.? What is the area of the equator circle cut from such a globe?

**6.** A school flag pole has a circumference of 24.2 in. at the base. What is the diameter? What is the radius? What is the area of a cross section?

**7.** What is the area of the cross section of a circular iron beam whose circumference is 31.416 in.? (Use 3.1416.)

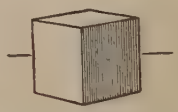
**8.** A boy has a ball tied to a string 1 yd. 6 in. long. As he swings it around, how long is the circumference traveled by the ball? What is the area of the circle inclosed?

**9.** A tinsmith wishes to make a pattern for the bottom of a pail, the area being 154 sq. in., and to allow $\frac{1}{4}$ in. all around for soldering. What radius should he use in drawing the circle?

**151. Prism.** A solid having two parallel faces exactly alike and all the rest of the faces parallelograms is called a *prism.*

The solid shown in the picture is a prism, the parallel faces in this case being triangles. The *base* of a prism is one of the two parallel faces, and the *altitude* of the prism is the distance between these faces. If all the faces of a prism are rectangles, the prism is called a *rectangular prism.* An ordinary crayon box is a rectangular prism.

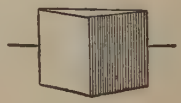
**152. Volume of a Prism.** Contractors, builders, and mechanics frequently have to find the volume of a prism of some kind. For example, a contractor computing the amount of earth to be removed in excavating a canal uses the rule for the volume of a prism. To find this rule first consider the following questions:



If this kind of prism, a cube, is 1 in. on an edge, what is its volume?

If a cube is 2 in. on an edge, what is its volume? If a cube is 3 in. on an edge, what is its volume?

This figure represents half of the above cube. If the cube is 1 in. on an edge, what is the volume of this half?



If the cube is 4 in. on an edge, what is the volume of this half?

If the area of the base of the first figure on this page is 5 sq. in., what is the volume of the lower shaded part which is 1 in. high? What is the total volume?

We have now found the volume of a special kind of prism, a cube; of another special kind, half a cube; and of a triangular prism. The volume of any prism is found in the same way as that of a triangular prism.

We see from the above that

*The volume of a prism equals the area of the base multiplied by the altitude.*

### WRITTEN EXERCISE

*Find the volumes of prisms with bases and altitudes as follows:*

1. 375 sq. in., 29 in.
2. 67.9 sq. in., 4.8 in.
3. 61.3 sq. ft., 2.91 ft.
4. 1.28 sq. ft., 3.2 in.
5. 42.6 sq. in., 7.3 in.
6. 62.8 sq. in., 3.17 in.

*Find the altitudes of prisms with volumes and bases as follows:*

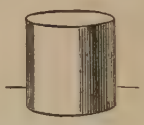
7. 243 cu. in., 29 sq. in.
8. 140 cu. in., $17\frac{1}{2}$ sq. in.
9. 783 cu. in., 87 sq. in.
10. $178\frac{1}{4}$ cu. in., $19\frac{1}{4}$ sq. in.

*Find to the nearest tenth the volumes of rectangular prisms with dimensions as follows:*

11. 42 in. long by 6.8 in. wide by 3.5 in. high.
12. 2.9 in. wide by 3.8 in. long by 6.4 in. high.
13. 3.91 ft. wide by 4.27 ft. long by 6.8 ft. deep.
14. 2.8 ft. deep by 3.25 ft. wide by 4.75 ft. long.
15. 6.5 ft. long by 4.75 ft. wide by 3.25 ft. high.
16. 21.3 in. wide by 29.2 in. long by 3.7 in. deep.
17. The area of one face of a cube is 3721 sq. in. What is the volume of the cube?
18. A prism with a square base of 1156 sq. in. is twice as high as it is wide. Find the volume of the prism.
19. A prism has a square base, a volume of 4851 cu. in., and a height of 11 in. Required the lengths of the edges.
20. A prism has a square base, a volume of $1687\frac{1}{2}$ cu. in., and a height of $7\frac{1}{2}$ in. Find the area of the base.
21. In Ex. 20, find the edge of the base of the prism.

Although pupils at this stage do not need to have an artificial motive for work, as in the earlier grades, teachers sometimes find it helpful to consider such local measures as those relating to a water tank or standpipe before beginning page 218.

**153. Cylinder.** A solid bounded by two equal parallel circular faces and a curved surface is called a *cylinder*.



The solid shown in the picture is a cylinder.

The two parallel circular faces are called the *bases* of the cylinder, and the distance between the bases is called the *altitude* of the cylinder.

The above definition is sufficient for the purposes of arithmetic. There are other cylinders than those having circular bases, but we do not need to study them at this time.

**154. Volume of a Cylinder.** It is often necessary to find the volume of a cylinder, as in finding the volume of a tank or pipe.

In the same way that we found the volume of a prism (§ 152) we may find the volume of a cylinder.

*The volume of a cylinder equals the area of the base multiplied by the altitude.*

Since the area of the base of a cylinder is $\frac{22}{7}$ times the square of the radius, we may say that *the volume of a cylinder equals the product of the altitude multiplied by* $\frac{22}{7}$ *times the square of the radius of the base.*

### WRITTEN EXERCISE

1. Find the volume of a cylindrical water tank 40 ft. high and 40 ft. in diameter.

2. How many cubic feet in a section of water pipe 16 ft. long, the diameter being 1 ft. 11 in.?

3. How many gallons (231 cu. in.) in a cylindrical tank 32 ft. high and 30 ft. in diameter?

4. Find the number of cubic feet in a boiler 15 ft. long and 3 ft. 6 in. in diameter, internal measure.

5. Find the number of cubic inches in a pipe 18 ft. long, 4 in. in external diameter, the metal being $\frac{1}{4}$ in. thick.

**155. Curved Surface of a Cylinder.** If we slit the curved surface of a cylinder and spread it out flat, what will it become? How may we find the area of this surface? Try it.

What is the area of the curved surface of a cylinder 6 in. high and 8 in. in circumference?

The area is the same as that of a rectangle 6 in. by 8 in., or 48 sq. in.

We see that *the area of the curved surface of a cylinder equals the circumference multiplied by the altitude.*

That is, area = circumference × altitude,

or area = $\frac{22}{7}$ × diameter × altitude.

ORAL EXERCISE

*State the areas of the curved surfaces of cylinders with altitudes and circumferences as follows:*

1. 25 in., 80 in.
2. $33\frac{1}{3}$ in., 69 in.
3. 75 in., 40 in.
4. 50 in., 106 in.
5. $66\frac{2}{3}$ in., 48 in.
6. $12\frac{1}{2}$ in., 24 in.
7. $16\frac{2}{3}$ in., 36 in.
8. $33\frac{1}{3}$ in., 33 in.

9. How many square inches in the curved surface of a wire of circumference 1 in. and length 200 ft.?

10. How many square feet of tin are needed for a pipe 8 ft. long and 6 in. in circumference, allowing 1 sq. ft. for overlapping?

11. A tin cup is 7 in. in circumference and 3 in. high, both measures including allowance for soldering. How many square inches of tin are needed for the curved surface?

12. A tin water pipe has a circumference of 9 in. and a length of 10 ft., both measures including allowance for soldering. How many square inches of tin are needed?

### WRITTEN EXERCISE

**1.** How many square inches of surface on a wire 10 ft. long and $\frac{7}{8}$ in. in circumference?

**2.** If the height and diameter of a solid cylinder are both 7 in., what is the total area of surface?

To the curved surface add the areas of the circles forming the top and bottom.

**3.** How many square inches of tin are needed for a cylindrical cup 4 in. high and 4 in. in diameter?

**4.** How many square feet in the curved surface of a water tank that is 40 ft. high and 127.3 ft. in circumference?

**5.** Write a rule for finding the circumference of a circle, given the radius; also for finding the area of the circle.

**6.** How many square feet of surface on the outside of a smokestack 30 ft. high and 2 ft. in exterior diameter?

**7.** The curved surface of a granite cylindrical shaft 3 ft. 6 in. in diameter and 22 ft. 3 in. long is to be polished. How many square feet in the surface to be polished?

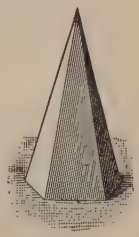
**8.** In a certain factory there is a room heated by 210 ft. of steam pipe, 2 in. in diameter. Required the radiating surface; that is, the area of the curved surface which radiates the heat.

**9.** A large suspension bridge has 4 cables, each 1872 ft. long and 1 ft. 2 in. in diameter. In letting the contract for painting these cables it is necessary to know their surface. Compute it.

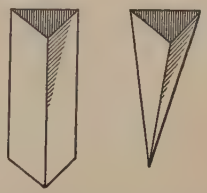
**10.** In making metal bedsteads some iron rods 0.75 in. in diameter are covered with thin rolled brass. Suppose a shop needs 6000 ft. of such rods, how many square feet of rolled brass will be needed, not allowing for waste?

Teachers should not fail to keep in mind the importance of estimating results in advance.

**156. Pyramid.** A solid of this shape, in which the base is any polygon and the other faces are triangles meeting at a point, is called a *pyramid.* The point at which the triangular faces meet is called the *vertex* of the pyramid. The distance from the vertex to the base is called the *altitude* of the pyramid.



**157. Volume of a Pyramid.** A contractor or builder occasionally needs to find the volume of a pyramid. The rule for finding the volume is easily seen by taking a hollow prism and a hollow pyramid of the same base and the same height, as here shown, filling the pyramid with water and pouring the water into the prism. It will be found that the prism can be exactly filled with three times the amount that fills the pyramid. That is,



*The volume of a pyramid equals one third the product of the base and altitude.*

WRITTEN EXERCISE

*Find the volumes of pyramids with bases and altitudes as follows:*

1. 129.3 sq. in., 3.72 in.
2. 202.11 sq. ft., 8.92 ft.
3. 702.39 sq. in., 16.8 in.
4. 187.02 sq. ft., 10.7 ft.

*Find the bases of pyramids with volumes and altitudes as follows:*

5. 89.6 cu. in., 12.8 in.
6. 178.2 cu. in., 19.8 in.
7. 123.2 cu. in., 17.6 in.
8. 140.4 cu. in., 23.4 in.

9. The Great Pyramid in Egypt is $480\frac{3}{4}$ ft. high, and has a square base, 764 ft. on a side. What is the volume? If it weighs 168 lb. per cubic foot, what is the weight?

## LITTLE EXAMINATIONS

I. *Find the square roots of the following:*

1. $\frac{49}{144}$. 2. $\frac{225}{361}$. 3. $\frac{529}{729}$. 4. $\frac{1369}{1681}$. 5. $\frac{2209}{2601}$.

*Find to two decimal places the square roots of the following:*

6. 13. 7. 17. 8. 30. 9. 125. 10. $\frac{5}{9}$.

II. *Find the circumferences, given the diameters:*

1. 34.1 in. 2. 32.2 ft. 3. 3 in. 4. $4\frac{1}{2}$ in. 5. 9 in.

*Find to two decimal places the square roots of the following:*

6. 19. 7. 15. 8. 50. 9. 650. 10. $\frac{7}{12}$.

III. *Find the hypotenuses, given the other sides as follows:*

1. 39 ft., 52 ft. 2. 21 ft., 72 ft. 3. 51 ft., 68 ft.

*Find the volumes of cylinders whose altitudes and areas of base are as follows:*

4. 48 in., 135 sq. in. 5. 9 ft. 6 in., 5 sq. ft. 36 sq. in.

*Find the volumes of cylinders whose altitudes and radii of base are as follows:*

6. 37 in., 5 in. 7. 35 in., 9 in. 8. 3 ft. 7 in., 3 in.

9. Find the volume of a pyramid of base 87 sq. in. and height $14\frac{1}{2}$ in.

10. Find the volume of a pyramid of base 4 sq. ft. 9 sq. in. and height 1 ft. 3 in.

## REVIEW AND DRILL

### I. DRILL SECTION

1. Write in ordinary numerals:

MDCCCCXIX MCMXXI LXXXIX

2. Find the value of $48{,}998 + 76{,}554 + 83{,}456 + 96{,}387$.

3. Find the value of $82{,}021{,}322 - 67{,}137{,}487$.

4. Multiply the sum of \$27.63 and \$99.75 by 346.

5. Divide the difference between \$910.07 and \$756.59 by 1.37, and carry the result to the nearest cent.

*Add the following:*

6. $\frac{1}{2} + \frac{7}{8}$.
7. $\frac{5}{8} + \frac{2}{3}$.
8. $4\frac{1}{2} + 5\frac{3}{4}$.
9. $2\frac{3}{4} + 6\frac{3}{8}$.
10. $6\frac{2}{3} + 2\frac{1}{2}$.
11. $5\frac{1}{2} + 3\frac{3}{5}$.
12. $3\frac{1}{8} + 2\frac{3}{5}$.
13. $4\frac{7}{8} + 3\frac{1}{3}$.

*Subtract the following:*

14. $\frac{5}{8} - \frac{1}{2}$.
15. $2\frac{1}{2} - \frac{7}{8}$.
16. $6\frac{1}{2} - 2\frac{7}{8}$.
17. $9\frac{1}{3} - 2\frac{1}{2}$.

*Multiply the following:*

18. $\frac{7}{8}$ of $\frac{4}{5}$.
19. $\frac{1}{2}$ of $2\frac{7}{8}$.
20. $2\frac{1}{2} \times 3\frac{1}{2}$.
21. $3\frac{1}{4} \times 5\frac{3}{8}$.

*Divide the following:*

22. $\frac{7}{8} \div \frac{1}{2}$.
23. $2\frac{1}{2} \div 1\frac{1}{4}$.
24. $9\frac{3}{4} \div 3\frac{1}{4}$.
25. $18\frac{3}{4} \div 3\frac{3}{4}$.

*Find the values of the following to two decimal places:*

26. 25% of \$17,256.75.
27. $5.293 \div 0.971$.
28. $1 \div 0.7854$.

## II. PROBLEMS WITHOUT NUMBERS

### ORAL EXERCISE

1. How do you find the square of a number?

2. If you know the side of a square, how do you find the area? Illustrate by an example.

3. If you know the area of a square, how do you find the side? Illustrate by an example.

4. How do you find by factoring the square root of a number that is a perfect square? Illustrate.

5. If you know three terms of a proportion, how do you find the fourth term?

6. How do you extract the square root of an integer that is a perfect square?

7. How do you extract the square root of a common fraction? How do you extract the square root of a decimal?

8. How do you find the hypotenuse of a right triangle when you know the other two sides?

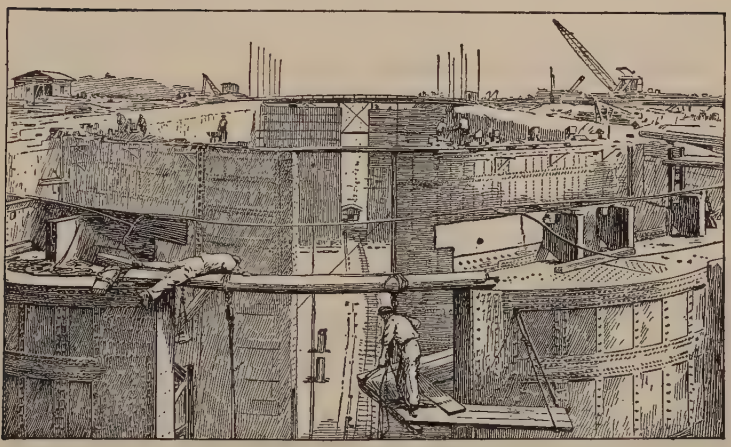
9. Suppose that a fly starts from the upper northeast corner of this room and flies straight to the lower southwest corner. How can you find the distance he flies, knowing the three dimensions of the room?

10. How would you find the number of cubic feet in a cylindrical water tank?

11. If you know the base and altitude of a pyramid, what else can you find, and how do you find it?

12. If you know the altitude of a cylinder, and the diameter of the base, what else can you find, and how do you find them?

## III. INDUSTRIAL PROBLEMS



### WHY WE BUILT THE PANAMA CANAL

**1.** The Panama Canal cost about $375,000,000. Taking our population as 100,000,000, this means that every man, woman, and child must pay how much on an average?

**2.** If the government has borrowed the money at 2%, what are the interest charges every year that all of us must help pay?

**3.** Goods can be shipped just as well by water as by rail, unless there is necessity for haste. It is cheaper to ship by water. The rate on a certain class of freight from Boston to Chicago wholly by rail is 30¢ per 100 lb., but by rail and lake it is $5.54 per ton. What is the per cent of saving by rail and lake?

**4.** The rate on a certain class of goods from New York to San Francisco is $2 per 100 lb. wholly by rail, or $1.20 per 100 lb. by water and the Panama Canal. What is the per cent of saving by the latter route?

**5.** If the rate by the Panama Canal is 40% cheaper than by the rail route, how much would a shipper save on 40 tons of the class of goods mentioned in Ex. 4, over the all-rail rate?

**6.** It is estimated that on a certain class of goods there is a saving of 46% by shipping by the Panama Canal over shipping by rail from the Atlantic coast to the Pacific. If $23,500,000 represents the all-rail freight charges on this class of goods for a given period, and it is just as well to ship by water as by rail, what is the saving by sending the freight through the Canal?

**7.** Before the Panama Canal was opened, the western coast of Central and South America had an annual foreign trade of $600,000,000. It was estimated that this would be increased 55% as a result of the opening of the Canal. What would this make the value of the annual foreign trade?

**8.** It costs us about $15,000,000 for a battleship, to defend our country in case of war. The Panama Canal allows our ships to be sent from the Atlantic to the Pacific and back again, with very little delay. If we need build one less battleship a year on this account, how much will be saved in twenty-five years?

**9.** If the trade of the Atlantic coast of this country with Australasia was $22,000,000 a year before the Panama Canal was opened, and increased 32% as a result of the opening of this route, what did it amount to after that?

**10.** Suppose England's trade with the west coast of South America was $86,000,000 a year before this route was opened, and increased 15% as a result of the opening of the Canal, what did it amount to after that?

**11.** The toll on merchant vessels is $1.20 per net ton of actual earning capacity. If a steamer has a net capacity of 5275 tons, how much toll must be paid in passing through the Canal?

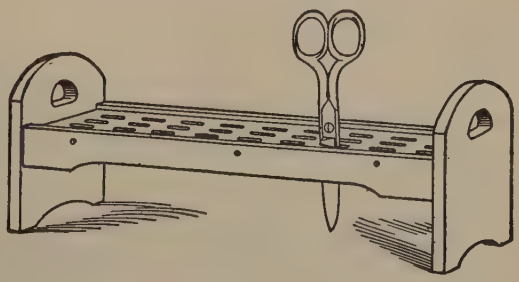
# CHAPTER VIII

## GENERAL REVIEW

**158. General Review.** We have now studied the subjects that are regarded as the essentials of arithmetic. In this chapter we shall consider further illustrations of the applications of arithmetic to the affairs of life, and these will also serve as a general review of the entire subject. The latter part of the chapter comprises a carefully arranged series of examinations.

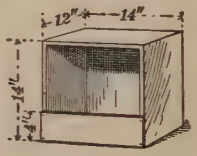
### PROBLEMS IN WOODWORKING

1. The class is making this scissors rack for the sewing room. There are four strips of wood with holes for the scissors, the side piece hiding one of the strips in the picture. If each of the four strips is $\frac{5}{8}''$ wide, and each side piece is $\frac{1}{2}''$ wide, what is the width of the body of the rack?



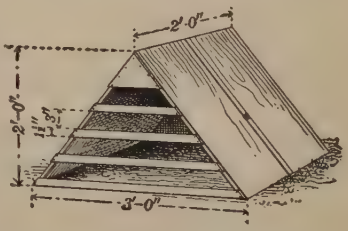
2. There are nine openings in each of the four strips, the distance from the center of each opening to the center of the next being $1\frac{1}{2}''$. The distance from the centers of the first and last openings to the ends of the strips is also $1\frac{1}{2}''$. What is the length of each strip?

**3.** Edward helped his father make some nests and coops. This is a picture of one of the nests for a sitting hen. Edward figured out the number of feet of lumber for each nest. How many feet were required?



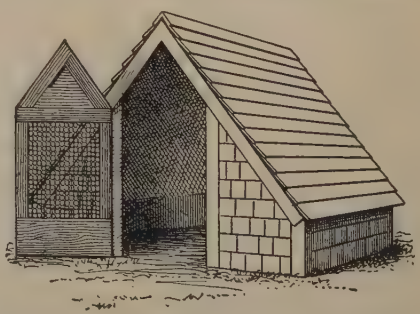

**4.** Edward and his father made eight of these nests, using lumber that cost \$40 per M. How much did the lumber cost?

**5.** For the hens and chickens Edward and his father made eight coops like the one pictured below. Edward drew the figure to the scale of 1 in. to 1 ft., and then measured the size of each part of the coop. Do this, and find the number of feet of lumber required and its value at \$40 per M.




Some approximation is allowed here, but this is exactly what happens in practical work, and is very desirable. The pupils may compare their answers and thus check one another's errors.

**6.** Edward's father built a sufficient number of poultry houses like this for his hens. The length of each was 9 ft., the width 7 ft., the height in the center 6 ft., and the height at the eaves 18 in. Allowing 20 cu. ft. for each hen, how many hens will this house accommodate?



If Edward gets an answer like 11.8 hens, he will say that the house will accommodate a dozen. In other words, he will use his common sense with such an answer, just as you always should.

### BUSINESS STRATEGY

1. A druggist is considering whether to move his store to a more central location. He asks his son George to do some figuring with him, to see whether it will pay to move. The new store would cost him \$330 more per year. How much is this a month?

2. If 20 more customers will enter the store each day, and each will buy 45¢ worth of goods on which the profit is 12% above all expenses, these figures all being averages and the drug store being open 30 days each month, does George find that the change of location is wise or not? Give his reason.

3. After moving to the new store, George's father decides to do some advertising. He has a cough medicine which he manufactures at a cost of 15¢ a bottle, and his annual sales average 200 bottles at 50¢ a bottle. If \$250 spent in advertising enables him to sell 1250 more bottles during the year, is this advertising a good investment? Find his profit on this medicine before he advertised and also after deducting the amount spent for advertising. What was his per cent of profit in each case?

4. George thinks that the \$250 spent in advertising was not as good an investment as the money spent in manufacturing. What was the per cent of returns on each? Was George right?

5. It occurs to George that instead of idling away his time after school and on Saturdays, he would do better to help his father in the store. His father offers him 5% on such money as he may take in. If he takes in \$2.50 on each of the five school days and \$9.50 on Saturday, how much does he make for himself in a week?

6. If George needs \$225 to carry him through the trade school, beginning 5 yr. hence, how much of his money must he save each year, not considering interest, and how much may he spend?

**159. Problem Data.** The following price list may be used in solving the problems on page 231, and similar problems.

The data may also be secured by the pupils by inquiry at home or at some grocery. This may be made the basis of practical problems in simple domestic bookkeeping.

**Coffee,** Maracaibo, 20¢ per pound; 5 lb. for 85¢.
Java and Mocha, 31¢ per pound; 5 lb. for $1.45.
Old Government Java, green, 27¢ per pound; 5 lb. for $1.30.

**Tea,** Black India, 50¢ per pound.
English Breakfast, 40¢ per pound.
Ceylon, finest, 80¢ per pound.

**Cocoa,** half-pound cans, 33¢; $3.75 per dozen cans.

**Sugar sirup,** half-gallon cans, 50¢; 5-gallon cans, $4.

**Maple sirup,** pints, 25¢; gallon cans, $1.45; $16.50 per dozen cans.

**Honey,** 8-ounce bottles, 40¢; $4.50 per dozen bottles.

**Flour,** buckwheat, 5¢ per pound; a bag of 24½ lb., $1.10.
Self-raising, 3 lb. for 19¢; 6 lb. for 35¢.
Wheat, 4¢ per pound; $5 per barrel of 196 lb.; 70¢ per sack of 24½ lb.

**Breakfast cereal,** 14¢ per package; $1.60 per dozen packages.

**Macaroni,** 12¢ per package; 25 packages for $2.75.

**Crackers,** Alberts, 23¢ per tin; $2.65 per dozen tins.
Soda, 5¢ per carton; 55¢ per dozen cartons.

**Soups,** half-pint cans, 10¢; pint cans, 16¢; quart cans, 28¢; $3.25 per dozen quart cans.

**Bacon,** American, 25¢ per pound.
Sliced, in jars, 26¢ per pound; $3.10 per dozen jars.

**Herrings,** 15¢ per can; $1.75 per dozen cans.

**Asparagus,** 50¢ per can; $5.75 per dozen cans.

**Domino sugar,** 5 lb. for 50¢.

**Loaf sugar,** 7½¢ per pound.

**Granulated sugar,** 5¢ per pound.

**Olives,** 32¢ per bottle; $3.75 per dozen bottles.

**Olive oil,** 65¢ per bottle; 38¢ per half bottle; $7.75 per dozen bottles.

**Allspice,** 15¢ per can.
**Cinnamon,** 40¢ per pound.
**Pepper,** 22¢ per pound.
**Cloves,** 40¢ per pound.
**Nutmegs,** 55¢ per pound.
**Ginger,** 20¢ per pound.

### INDUSTRY OF THE KITCHEN

**1.** If a family needs a dozen cans of cocoa, how much is saved by purchasing at the dozen rate?

**2.** If a family needs 5 gal. of sugar sirup, how much is saved by purchasing a 5-gallon can instead of buying ten half-gallon cans? What per cent is saved?

**3.** How much would a hotel save by purchasing 120 gal. of maple sirup by the dozen gallon cans instead of buying single cans? What per cent would the hotel save?

**4.** In which is the per cent of saving greater, in buying honey by the dozen bottles instead of by the bottle, or in buying maple sirup by the dozen cans instead of by the can?

**5.** How much is buckwheat flour a pound? At this rate, what will $24\frac{1}{2}$ lb. cost? How much is saved in buying a bag?

**6.** What is the per cent of saving in buying self-raising flour by the 6-pound package instead of by the 3-pound package?

**7.** By inquiry at home, make out a grocery list for a week, from page 230. Make two pages of a home account, the left-hand page showing the amount received, and the right-hand page showing the amounts spent for groceries.

**8.** Make out a bill for six items of groceries, making the proper extensions and footing. Receipt the bill.

**9.** Make out a bill for ten items of groceries, as in Ex. 8.

**10.** Make out a bill for twelve items of groceries, as in Ex. 8.

**11.** If gas costs \$1 per 1000 cu. ft., and your neighbor uses 2320 cu. ft. in May, how much was his gas bill for that month?

**12.** At the beginning of the month the gas meter registers 14,260, and at the end of the month 17,140. How much is the gas bill for the month, at \$1 per 1000 cu. ft.?

### THE DAIRY INDUSTRY

1. A farmer sells 26,250 lb. of milk to a creamery in a certain month. The milk averages 4.2% butter fat. With how many pounds of butter fat does the creamery credit him in that month?

2. If 6 lb. of butter fat are needed in making 7 lb. of butter, what is the value of the butter produced from 1236 lb. of butter fat, the butter being worth 24¢ a pound?

3. A certain dairy during a certain month sells to a creamery milk averaging 3.75% of butter fat. The butter fat weighs 630 lb. How many pounds of milk does the dairy sell?

4. A farmer has two cows, one supplying 986 lb. of milk testing 3.1% butter fat in a certain month, and the other 812 lb. testing 4.2%. If the creamery allows him 27¢ a pound for butter fat, which cow pays him the more for that month, and how much more, the feed costing the same?

5. A creamery uses 7500 lb. of milk in a week. The skim milk amounts to 80% of the whole milk and contains 0.6% butter fat. How many pounds of butter fat are lost in the skim milk?

### PROBLEMS IN IRRIGATION

1. An acre-inch of water is the amount of water 1 in. deep that it will take to cover a perfectly flat acre. What is an acre-foot of water, how many acre-inches does it contain, and how many cubic feet of water does it contain?

These terms are used in irrigation, particularly in the mountain states. The exercise may be omitted in those parts of the country in which irrigation is not common. The pupils should bring local problems to class. The technical terms on this page need not be learned unless needed in the locality, and they are sufficiently explained in the problems to make formal definitions unnecessary.

2. A second-foot of water is a cubic foot passing a particular point in 1 sec. If 7 cu. ft. of water pass in 1 sec., the irrigation ditch is delivering 7 second-feet of water. By timing a floating stick in a flume 4 ft. wide, the water being 6 in. deep, it was found that the water was flowing 10 ft. per second. How many second-feet of water was the flume carrying?

Allowance has to be made for friction of flowing water, but this need not be considered in an elementary treatment of the subject.

3. If a flume is delivering 20 second-feet of water, how long will it take to supply the water to irrigate 2 A. of land, the soil requiring 8 acre-inches per acre?

4. A farmer has 80 A. of beets which he wishes to irrigate from a reservoir. How many cubic feet should he store in the reservoir if each acre requires 2 acre-feet for the season?

There will have to be an allowance for evaporation and other waste, but this problem is supposed to require the net amount after such allowance is made.

5. A company is organized to furnish 25 sections (or 25 sq. mi.) of land with $1\frac{1}{2}$ acre-feet of water per acre annually. If 25% is allowed for loss by seepage and evaporation, so that only 75% of the water is available, and if the reservoir has an average depth of 40 ft., and must store all the water necessary for a year at one time, how many acres will the reservoir cover?

### PROBLEMS OF THE WESTERN FARM

**1.** A farmer has 200 A. on which he rotates his crops, with 3 yr. given entirely to alfalfa and 2 yr. entirely to sugar beets. The beets require 2 acre-feet of water per acre each season, and the alfalfa requires four fifths as much. How many acre-feet of water will the farmer need during the five-year period?

**2.** In a recent year the reclamation service of the government expended $86,500,000 in reclaiming about 2,500,000 acres of land. What was the average expense per acre?

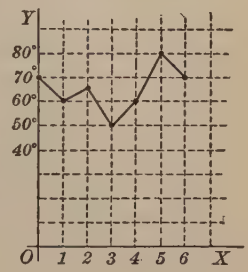
**3.** An experiment was made in raising alfalfa. A farmer rented 50 A. of irrigated land for 5 yr. at $5 per acre per year. The seed cost 40¢ per acre per year, the preparation of the land and the seeding cost $1 per acre per year, and the fertilization 80¢ per acre per year. It cost $3 per acre every time the alfalfa was cut, and it was cut twelve times in the 5 yr. The yield was 2.8 T. per acre per year. Find the total cost for 5 yr., the amount of alfalfa grown, and the cost per ton.

The crops as well as the industries vary in different parts of the country. The teacher may omit pages which relate to industries with which the pupils are not familiar. Thus, the raising of alfalfa may mean little in certain regions.

**4.** Some experiments in Utah show that alfalfa grows best with 1.6 acre-feet of water per season, wheat and oats with 1.2 acre-feet, barley with 1.7 acre-feet, and sugar beets with 1.9 acre-feet. A Utah farmer had 100 A. of alfalfa, 50 A. of wheat, 75 A. of oats, 40 A. of barley, and 50 A. of sugar beets. He used uniformly 2 acre-feet of water per acre on all this land. Disregarding any damage to the crops, how much water did he waste?

**5.** Some farmers use this rule for finding the number of bushels of corn in a bin: Take eight times the product of the three dimensions of the bin, and divide by ten. Using this rule, find the number of bushels in a bin 10 ft. by 6 ft. by 4 ft.

**160. Graph.** In this figure the successive days of the week are represented on the line $OX$, and the temperature at noon on the different days is represented on the lines parallel to $OY$. The broken line shows that the temperature one day at noon was $+70°$, the next day, $60°$; then $65°$, $50°$, $60°$, $80°$, $70°$. This broken line is called a *graph*. In this particular case it does not show the temperature at any time except noon.




**WRITTEN EXERCISE**

1. On ten successive January days in Duluth the thermometer at noon registered $40°$, $60°$, $55°$, $50°$, $20°$, $17°$, $12°$, $8°$, $10°$, $30°$. Trace the graph.

2. The number of ounces of silver produced in Colorado, stated in millions, was 0.5 in 1870; 13 in 1880; 19 in 1890; 20 in 1900; and 8 in 1910. Trace the graph.

3. The production of oats in the United States, in hundred million bushels, for eight consecutive years was 8, 9, 10, 10, 8, 8, 10, 11. Using any convenient scale, trace the graph.

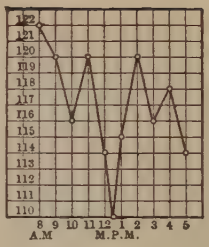
4. The production of cotton in the United States, in million bales, for seven consecutive years was 14, 11, 14, 12, 14, 11, 12. Using any convenient scale, trace the graph.

5. The number of tons of copper produced in this country in 1850 was 650; in 1880 it was 27,000; in 1900 it was 271,000; in 1914 it was 555,000. Trace the graph.

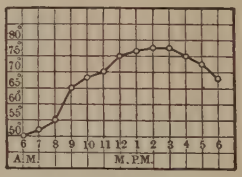
6. The value of the silver produced in this country in 1850 was \$51,000; in 1880 it was \$35,000,000; in 1900 it was \$36,000,000; in 1914 it was \$41,000,000. Trace the graph.

The graph is often found to be very helpful in the study of geography.

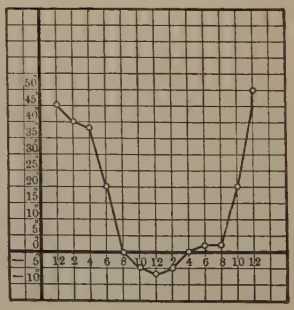
**7.** In a certain mill the steam gauge on a boiler showed a pressure of 122 lb. to the square inch at 8 A.M. The pressure varied through the day as shown by this graph. Write the pressures for every hour from 8 A.M. to 5 P.M., and also for 12.30 P.M. Why did the pressure go so low between 12 M. and 1 P.M.?




**8.** The temperature between 6 A.M. and 6 P.M. on the first of last May, at our school, was as shown by this graph. Write the approximate temperature for each hour and half hour of the day.

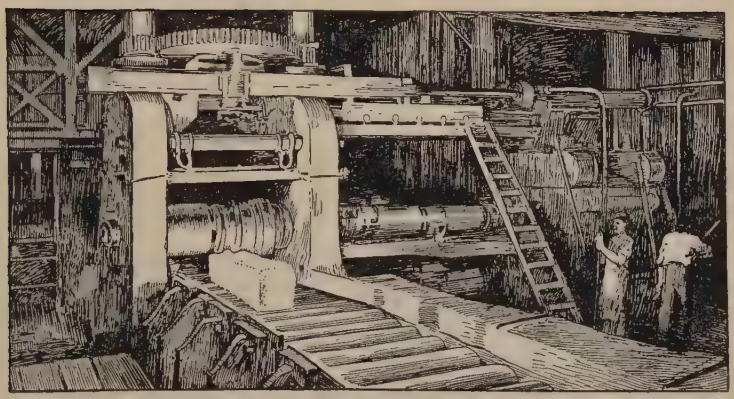



**9.** When the temperature is below zero this fact is indicated by a minus sign. That is, $-5°$ means $5°$ below zero. This graph shows the changes in temperature from noon on a day in January to noon on the following day. Write the approximate temperatures for every hour of this period, and a statement telling when the temperature fell the most rapidly and when it rose the most rapidly.




**10.** Draw a graph showing approximately, according to your estimate, the changes in temperature from 9 A.M. to 4 P.M. yesterday.

**11.** The average farm price of hay per ton in this country was \$8.52 in 1905, \$10.37 in 1906, \$11.68 in 1907, \$8.98 in 1908, \$10.50 in 1909, \$12.14 in 1910, \$14.29 in 1911, \$11.79 in 1912, and \$12.43 in 1913. Trace the graph on the scale of 1 in. to a dollar.



**THE IRON INDUSTRY**

1. What is the weight of a steel girder rolled out at this mill, the girder being 26′ 6″ long, weighing $42\frac{1}{2}$ lb. to the foot?

2. The mill also rolls out iron rods. What is the cost of 12′ 8″ of iron rod, $2\frac{1}{8}$ lb. to the foot, at $1\frac{3}{4}¢$ a pound?

3. The wooden pattern from which an iron casting is made weighs $6\frac{1}{4}\%$ as much as the iron. The pattern weighs $45\frac{5}{8}$ lb. How much does the casting weigh?

4. If steel rails weighing 120 lb. to the yard are used between New York and Chicago, a distance of 980 mi., how many tons will be required for a double-track road between these cities?

5. An iron tire expands $1\frac{9}{16}\%$ on being heated for shrinking on a wheel. A wooden wheel needs a tire 4′ 8″ in diameter. How much longer will the tire be when thus heated?

6. In a certain blast furnace the casting machine turns out 20 pigs of iron per minute, averaging in weight 110 lb. each. If this machine runs at this rate for 308 days, 16 hours a day, how many long tons of pig iron will it turn out?

### THE SHIPPING INDUSTRY

**1.** One of the fastest day's runs recorded for a steamer is 676 knots. How many statute miles is this?

**2.** The tonnage of the *Bismarck* is 60,000 tons, which is 10% more than that of the *Vaterland*. What is the tonnage of the *Vaterland*?

**3.** If a steamer's hull is 65 ft. deep, 955 ft. long, and 100 ft. broad, the depth is what per cent of the length? The breadth is what per cent of the length?

**4.** One of the steamship lines landed in New York on an average 1206 passengers per ship in a certain year, which was 90% of the average the year before. What was the average then?

**5.** One of the largest steamers will carry 80 barge loads of grain, at 8000 bu. to a barge. A freight car 40 ft. long will carry 1280 bu. What is the total length of a train carrying enough grain to load this steamer?

**6.** One of our American steamers pays \$4760 in monthly wages to the 136 stokers who keep up the fires for its great engines. A German steamer pays \$2864 to its 179 men who do this work. Each American stoker receives what per cent more monthly wages than the German?

### TRANSPORTATION

**1.** If a freight car is 36 ft. long, 8 ft. 3 in. wide, and 7 ft. 6 in. high, inside measure, how many cubic feet does it contain?

**2.** If the width of the car as given in Ex. 1 were increased 1 in., what would be the per cent of increase in capacity?

**3.** If a wooden coal car can carry 80,000 lb., and a steel car 100,000 lb., the steel car can carry what per cent more than the wooden car? The wooden car can carry what per cent less than the steel car?

**4.** A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Reckoning a bushel as containing $1\frac{1}{4}$ cu. ft., and a bushel of wheat as weighing 60 lb., how many pounds does the wheat weigh?

**5.** If the car of Ex. 4 has a carrying capacity of 80,000 lb., to what depth must it be filled with wheat to contain exactly this amount?

**6.** If a locomotive weighing $115\frac{1}{2}$ tons can exert a pull of $22\frac{1}{4}\%$ of its weight in starting a train, how many pounds of pull does it exert?

**7.** If a locomotive and coal tender together weigh 95 tons, and there are seven cars to the train, averaging 21 tons each, and a horizontal pull equal to 0.5% of the total weight of the train is required to maintain the necessary speed on a level track, what is this pull in pounds?

**8.** A 20,000-ton ship carries 12 times as much freight as the old-style ocean steamers, makes 25% better time, and the expenses per year are only four times as much. In a year such a modern boat will do how many times the work of the old kind? At the same cost, it will carry how many times as much freight?

### PROBLEMS OF THE MACHINIST

**1.** If the diameter of a wheel is 7 in., find the circumference.

**2.** What is the circumference of a fly wheel of which the diameter is 7 ft. 6 in.?

**3.** The diameter of the piston of an engine is 8 in. Required the circumference of the piston.

**4.** If the pressure of steam in a boiler is 140 lb. to the square inch, find the pressure on a valve $2\frac{7}{8}$ in. in diameter.

**5.** A boiler contains 480 flues, each $2\frac{3}{8}$ in. in internal diameter. Find the total area for draft through them.

**6.** If a pressure of 130 lb. to the square inch is applied to the circular plunger of a cylinder 2 ft. in diameter, what is the total pressure?

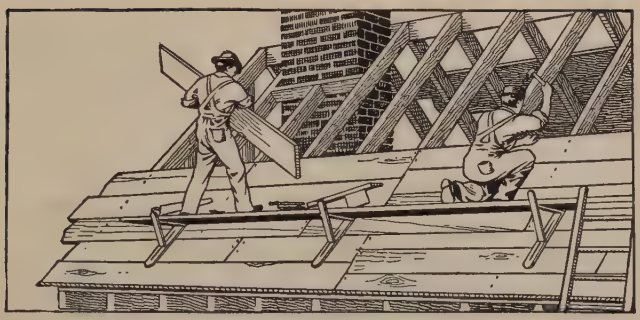
**7.** If an emery wheel of which the diameter is 10 in. makes 2000 revolutions per minute, its grinding speed per minute is 2000 times the circumference. What is the grinding speed?

**8.** A fly wheel of which the diameter is 6 ft. makes 100 revolutions per minute. At what rate per minute does a point on the rim travel?

**9.** How many revolutions per minute must be made by a circular saw of which the diameter is 28 in., in order that the saw may have a cutting speed of 8800 ft. per minute?

**10.** Two cogwheels are so geared that for every revolution of the larger wheel the smaller revolves 12 times. The larger wheel has 204 cogs. How many cogs has the smaller?

**11.** The fly wheel of an engine 8 ft. in diameter makes 250 revolutions per minute. It is directly connected by a belt with a wheel 2 ft. in diameter. How many revolutions per minute does the smaller wheel make?



### PROBLEMS OF THE BUILDER

1. From the eaves to the ridge of this roof is 13 ft., and the span is 24 ft. Find the rise of the roof; that is, the height of the peak above the level of the eaves.

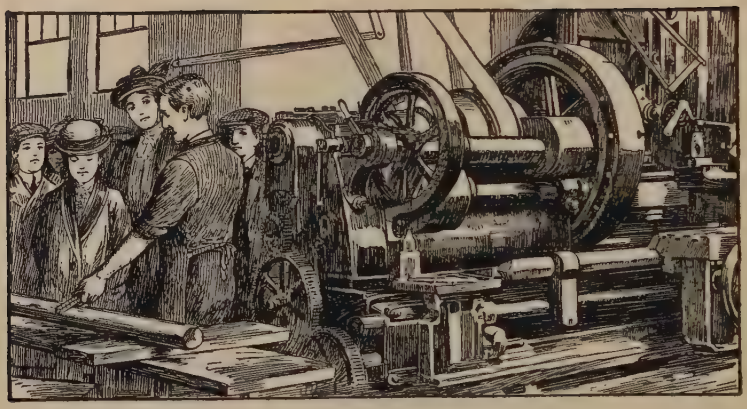
2. The span of a common A-shaped roof is 26 ft., and the rise of the roof is 6 ft. What is the distance, to two decimal places, from the eaves to the ridge?

3. A semicircular arch over a door has a diameter of 4′ 1″. Required the length of this semicircumference.

4. A builder is putting in a steam pipe 165 ft. long and 2 in. in diameter. What is the radiating surface, that is, the curved surface that gives out heat?

5. A contractor places on the top of a city apartment house a cylindric water tank 20 ft. high and 10 ft. in diameter. How many cubic feet of water will the tank hold?

6. The excavation needed for the cellar of a city apartment house is to be 92 ft. long, 60 ft. wide, and 7 ft. deep. In addition to this the excavation is to be extended 8 in. beyond each of the four sides, so as to allow space for working. How many loads (cubic yards) of earth must be removed?



### VISIT TO THE MACHINE SHOP

**1.** Miss Adams took her class to visit a shop, and the foreman gave them some problems. He showed them a casting that was 3 ft. 7 in. long and explained that iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat, and he asked them to compute the length of this casting when red hot. How long would it be?

**2.** Miss Adams asked if all metals expanded at the same rate, and the foreman told her that they did not. He said that in casting brass hinges they had to allow for a shrinkage of $\frac{1}{64}$ in length when the brass cooled. The boys were told to figure out the length of the mold for a hinge that was to be $5\frac{7}{8}$ in. long when cooled. How long was it?

**3.** The foreman showed them a piece of steel shafting that the men were just setting up. He told them that it was 3 in. in diameter, 20 ft. long, and weighed 490 lb. to the cubic foot. He said they had to know the weight of the shafting in order to be sure to give it ample support. The girls were told to figure it out. How much should they find that it weighed?

### PROBLEMS IN MANUFACTURING

1. How much will a manufacturer receive for 8 rockers at \$7.25, 9 dressers at \$14.50, and 16 bedsteads at \$18.75, discounted at 8% ?

2. A manufacturer sells the following bill of goods: 8 doz. plates @ \$1.42, 9 doz. cups @ \$2.35, $\frac{3}{4}$ doz. covered dishes @ \$9.75, 16 doz. plates @ \$1.65, $\frac{5}{6}$ doz. pitchers @ \$6.75. The bill is discounted at 3%, and he gains 20% on the goods. Required the cost of manufacturing the lot.

3. A manufacturing company has a capital of \$350,000, its annual business is $3\frac{1}{4}$ times its capital, and its profits are $7\frac{1}{2}\%$ of its business. How much are its annual profits?

4. A manufacturer sells brass lamps to a dealer at such a price that the dealer can sell them at \$6.60 and make a profit of 32%. The manufacturer himself makes a profit of 25%. What is the actual cost of manufacture per lamp?

5. On which will a manufacturer make the greater per cent of profit: an article which costs \$2.90 to make and sells for \$3.33$\frac{1}{3}$ less 3%, or one which costs \$8.70 to make and sells for \$12 less $\frac{1}{6}$? How much greater is this per cent of profit?

6. A manufacturer has a water tank 8 ft. 3 in. wide, 12 ft. 6 in. long, and 9 ft. 8 in. deep. If a cubic foot of water weighs $62\frac{1}{2}$ lb., what is the weight of the water required to fill the tank?

7. A factory pays \$3782.40 for materials, labor, and overhead charges, and sells the product at $12\frac{1}{2}\%$ profit. Find the receipts.

8. A manufacturer pays a salesman a salary of \$1250 a year and a commission of $1\frac{1}{2}\%$ on all sales, and allows him \$1000 a year for expenses. If the salesman sells goods to the value of \$172,500, how much does the manufacturer pay him, including salary, commission, and expenses?

### THE POTTERY INDUSTRY

**1.** In a shipment of a carload of earthenware to Chicago there were 15 lavatories weighing 150 lb. each, 10 bathtubs weighing 900 lb. each, and 50 laundry tubs weighing 250 lb. each. What was the freight on the shipment at 50¢ per hundred?

**2.** A manufacturer ships a carload of bathtubs. As packed, each tub is 5 ft. 6 in. long, 30 in. wide, and 21 in. deep. They are stood on end, one deep, in a car that is 36 ft. long and 8 ft. wide. How many tubs can be put in the car? Draw a plan showing how they should be placed.

The problem requires practical judgment. In the first place, one dimension is not used. In the second place, more tubs can be put in the car if placed one way than can be put in if they are placed another way.

**3.** Each of the tubs mentioned in Ex. 2 weighs 1121 lb. Required the total weight of the shipment.

**4.** The cost of material used in making each of the tubs mentioned in Exs. 2 and 3 is 2¢ per pound. Find the value of the raw material in the shipment.

**5.** The cost of a 1000-pound charge of glaze needed for 150 lavatories is 7¢ per pound. Find the cost of the glaze for 75 lavatories.

**6.** It requires $18\frac{1}{2}$ T. of anthracite egg coal to fire a kiln. How many kilns will a carload of coal weighing 106,720 lb. fire, and how much will it be worth at $4 per ton?

**7.** A press of clay weighing 3200 lb. costs $21 per ton. How many lavatories, each weighing 200 lb., will this press of clay turn out, and what is the cost of the material in each?

Whenever possible the class should visit some such industries and obtain problems for themselves. Avoid technical expressions as far as possible. Such problems should be discussed as informational exercises.

### THE JEWELRY INDUSTRY

**1.** In a recent year there were in New Jersey 420 manufacturing jewelers employing on an average 12 men each. This was an increase of 5% over the total number of men in the preceding year. How many men were employed in each year?

**2.** In the year in which 420 of these establishments employed on an average 12 men each, the total value of the product was $13,272,000. Find the average value of the product per factory, and the average value of the product per workman.

**3.** The girls from one of the schools in a city visited one of the shops where jewelry is manufactured, and found that pure gold was marked 24 k., which means 24 carats. They found that rings were commonly marked 14 k., meaning that they were $\frac{14}{24}$ gold. They brought to class the problem to determine what per cent of such a ring is pure gold. What is the answer?

**4.** The girls also found that the cost of 24 k. gold is $20.70 per ounce, and they were shown 4 ingots of such gold weighing 400 oz. each. What was the cost of this gold?

**5.** They also found that the ounce used by jewelers was not quite the same as that used by grocers, and that it was divided into 20 pennyweights. They were given a problem, to find the value of a lot of 14 k. gold chains weighing 464 dwt. (pennyweights) at 85¢ per pennyweight. What is the answer?

**6.** Find the value of a shipment of 64 wedding rings weighing 5 dwt. per ring at 83¢ per pennyweight.

**7.** What is the difference in cost between a 10 k. ring weighing 5 dwt. and a 14 k. ring of the same weight? Consider only the cost of the gold as given in Ex. 4.

**8.** How much more valuable is an 18 k. gold chain weighing 4 oz. than a 14 k. chain of the same weight?

### THE PEARL BUTTON INDUSTRY

1. A manufacturer bought 980 lb. of white pearl shell at 78¢ per pound less 10%. Find the cost of this raw material.

2. A Chicago manufacturer bought 1225 lb. of white pearl shell at 82¢ per pound less 8% and 2%. How much did this raw material cost him?

3. A manufacturer bought 40 cases of black pearl shell at 45¢ per pound less 8%. The average weight of each case being 65 lb., how much did this shipment of raw material cost him?

4. A manufacturer put up pearl buttons in boxes of 6 gross each. He sold 64 boxes to Roberts & Jameson and 256 boxes to Peabody & Jones. How many gross of buttons did he sell to each firm? how many buttons?

5. The manufacturer received $1.90 per gross for the buttons mentioned in Ex. 4, but allowed a discount of 12%. How much was due him from Roberts & Jameson? How much was due him from Peabody & Jones?

6. A button company received from London a consignment of shells billed as follows: 440 lb. fancy white @ 43 d. (that is, 43 English pence), 1560 lb. dull white @ 31 d., 440 lb. black @ 21 d., 740 lb. medium gray @ 18 d., and 880 lb. sea snail @ $10\frac{1}{2}$ d. Taking 2¢ to the penny, find the value of this consignment in our money.

The English bill would give 3 shillings and 7 pence (3 s. 7 d.) instead of 43 d., there being 12 pence in a shilling. This is the plan followed in Ex. 7.

7. A London exporter bills to a certain manufacturer a consignment of shells as follows: 760 lb. fancy white @ 3 s. 5 d., 380 lb. medium gray @ 1 s. 8 d., and 1275 lb. dull white @ 2 s. 8 d. Find the value of this consignment in our money.

### PROBLEMS ON SPRAYING

**1.** To lessen potato scab some seed potatoes are soaked in a solution of $\frac{1}{2}$ pt. of formalin to 15 gal. of water. What is the per cent of formalin in the mixture?

In Exs. 1 and 2 answer to the nearest tenth per cent.

**2.** In Ex. 1, what would be the per cent of formalin in the mixture if $\frac{5}{8}$ pt. had been used to 15 gal. of water?

**3.** Bordeaux mixture, for spraying, contains $1\frac{1}{4}\%$ by weight of copper sulphate, $1\frac{1}{4}\%$ of unslaked lime, and the rest water. How many pounds of each in a barrel containing 250 lb. of the mixture?

**4.** A cubic foot of Bordeaux mixture weighs 63 lb. From the statements made in Ex. 3, find the number of pounds of copper sulphate and unslaked lime in a tank that is 6 ft. long, 4 ft. wide, and 1 ft. 6 in. deep.

**5.** Half of a peach tree was sprayed with Bordeaux mixture and the other half was not. From the sprayed half 284.8 lb. of fruit were taken, and from the other half 95% less. How many pounds were taken from the tree?

**6.** When copper sulphate costs 10¢ a pound, lime 1¢ a pound, and arsenate of lead 20¢ a pound, and we need for a spray 3 lb. of arsenate of lead and 4 lb. each of lime and copper sulphate to 50 gal. of water, and using $2\frac{1}{2}$ gal. of the spray to a tree, what will it cost for material to spray 125 trees?

**7.** For spraying potato plants a man prepares a solution of Paris green, using $\frac{1}{2}$ lb. of Paris green costing 30¢ a pound to 100 gal. of water. He uses 300 gal. to an acre, and sprays a field of $3\frac{1}{2}$ acres. What is the cost of the material? If the labor costs \$7.25, what is the total cost? If he sells his crop for \$28 more than if he had not sprayed, how much is gained by spraying?

### PROBLEMS OF THE CONTRACTOR

**1.** If a contractor makes $\frac{1}{4}$ of the amount he receives for a \$4500 job, how much does he make?

**2.** If a contractor makes $\frac{1}{6}$ of the amount he receives for a \$15,000 job, how much does he make?

**3.** If a contractor makes \$675 on a \$3375 job, how much does he make on each dollar of the amount he receives?

**4.** If a contractor makes $\frac{1}{10}$ of the amount he receives for one \$4500 job, and loses $\frac{1}{10}$ of the amount he receives on another contract of \$4500, has he gained or lost on the two together?

**5.** A contractor builds a brick wall 68′ 8″ long, 5′ 8″ high, and 2 bricks thick. Allowing 14 bricks per square foot of outside surface for a 2-brick wall, how much will the bricks cost at \$8.50 per M?

**6.** A man contracted to paper a house. The rooms are $14' \times 16'$, $8' \times 12'$, $15' \times 18'$, $10' \times 12'$, $13' \times 15'$, $16' \times 16'$, $6' \times 10'$, and $12' \times 12'$. He deducts 50′ from the combined perimeters of the rooms and allows one roll of paper for every $3\frac{1}{2}$ ft. of perimeter. How many rolls does he need?

**7.** A man has a contract to put up the four walls of a building which is to be 80′ long, 35′ wide, and 78′ high. The walls are to be 18″ thick, and 750 cu. ft. is to be allowed for openings. Allowing 22 bricks to a cubic foot, how much will the bricks cost at \$9.25 per M?

**8.** A contractor wishes to build a concrete wall $8' \times 66'$, and 18″ thick. Concrete is $\frac{1}{9}$ Portland cement, which costs \$1.90 a barrel of 400 lb., and the rest is gravel costing 88¢ a load (cubic yard), delivered. A cubic foot of dry cement weighs 150 lb. How much will the materials cost?

**161. Review Examinations.** Pages 249–264 are devoted to oral and written review examinations, ten examples to a set.

### WRITTEN EXAMINATION

1. Find the interest on \$428.50 for 3 mo. 5 da. at 6%. Solve in two ways, stating the operations in steps.

2. From a lot 12 rd. by 16 rd. a path 4 ft. wide is cut on all four sides. How many square yards are left?

3. A man sold a lot for \$500, gaining 20%. He sold another lot for \$500, losing 20%. Did he gain or lose on the two transactions together, and how much?

4. Which is the better investment of \$1500 for a year: a note bearing $4\frac{1}{2}\%$ interest, or a savings-bank account at 4% per year, the interest being payable semiannually and no interest being withdrawn until the end of the year?

5. If $x : 17\frac{1}{2} = 9\frac{1}{2} : 35$, what is the value of $x$?

6. A woman living in a city earned \$819 last year by shopping for her friends. On half of her purchases she received 4% commission, and on half she received 5%. How much money did she spend in shopping?

7. Multiply 7 ft. 6 in. by 4, and divide the product by 5.

8. A wholesale jeweler advertised his stock to be sold at 50% and 25% off. I bought a watch for \$15. What was the list price?

9. A man owns a house valued at \$12,800. He rents it for \$768, paying taxes at the rate of 2 mills, \$24.20 for insurance, and \$144 for other expenses. Find his net rate of income.

10. How many yards of carpet, $\frac{1}{2}$ yd. pattern and $\frac{3}{4}$ yd. wide, will it take to cover a room 16 ft. 8 in. long and 12 ft. wide, cutting so as to match patterns and running the strips lengthwise?

### WRITTEN EXAMINATION

**1.** Find to the nearest hundredth of a foot the side of a square whose area is 50 sq. ft.

**2.** The time from one new moon to the next (a lunar month) is 29 da. 12 hr. 44 min. 3 sec. Express 6 lunar months in days and the decimal of a day.

**3.** Find the difference in the number of acres in two fields, one of which contains 120 sq. rd., and the other of which is 120 rd. square.

**4.** If a certain gun metal is composed of 84% copper and the remainder tin, how many pounds of tin will be needed with 638.4 lb. of copper to make the gun metal?

**5.** A company that has a capital of $1,250,000 earned net $103,750.75 last year. It paid a dividend of 6% and carried the balance to surplus. How much did the company pay in dividends, and how much did it carry to surplus?

**6.** A farmer has a bin 18 ft. long, 5 ft. 3 in. wide, and 5 ft. deep which holds 378 bu. How many bushels in a bin that is 4 ft. longer, 1 ft. 9 in. narrower, and has the same depth?

**7.** Water is flowing into a tank whose base is 9 ft. 4 in. long and 6 ft. 9 in. wide, at the rate of 3 cu. ft. in 2 min. How long will it take the water to fill the tank to a depth of 4 ft. 6 in.?

**8.** One face of a cube has an area of 3 sq. in. What is the volume of the cube?

**9.** A woman pays taxes at the rate of $1.09 per $100 on property assessed at $7500. Find the amount of her taxes.

**10.** How high is a tree that casts a shadow 85 ft. long at the same time that an upright post 3 ft. high casts a shadow 4 ft. 3 in. long?

### ORAL EXAMINATION

1. How long must a man have \$250 invested at 6% for the amount of principal and interest to be \$265? to be \$280? to be \$295? to be \$310? to be \$325?

2. A man borrowed \$8000 in the East at 5% and invested it in the West at 8%. How much did he make per year by the transaction? Why does not every one who has money to invest follow this plan?

3. What sum must be invested at 6% for the interest at the end of 2 yr. to be \$180?

4. At what rate must \$1000 be invested for the amount of principal and interest to be \$1120 at the end of 3 yr.? for the amount to be \$1180 at the end of 3 yr.?

5. A dealer sells \$1000 worth of goods discounted at 20% and 10%. What is the net price?

6. A man having \$10,000 invested at $4\frac{1}{2}\%$ reinvests half of it at 5% and leaves the rest as before. What is the increase in income?

7. During the year a merchant borrows at various times \$100, \$200, \$1200, \$1500, each time for 60 da. at 6%. How much interest does he pay?

8. A bank makes a loan of \$800 at 6% and receives \$12 interest (bank discount). Required the time.

9. A bank lends \$800 for 60 da. and receives \$8 interest. Required the rate.

10. A bank lends some money for 60 da. at 6% and receives \$10 interest. How much is the amount of the loan? How much is the loan on which a bank receives \$20 interest in 60 da. at 6%?

### WRITTEN EXAMINATION

1. The rates of the express and local trains are 45.6 mi. an hour and 28.4 mi. an hour respectively. How much time is saved by taking the express train for a distance of 161.88 mi.?

2. From a tank full of water $\frac{3}{5}$ of the water was drawn off. The tank then lacked 70 gal. of being half full. What is the capacity of the tank?

3. A tank of capacity $166\frac{1}{2}$ gal. is $\frac{2}{5}$ full of water. How long will it take a pipe supplying water at the rate of 3.7 gal. a minute to fill the rest of the tank?

4. A man who owned some shares of stock in a mill sold $\frac{1}{3}$ of his shares to A, $\frac{1}{5}$ to B, and $\frac{1}{6}$ to C. He then had 90 shares left. How many shares had he at first?

5. A boy lost $\frac{1}{6}$ of his kite string in a tree and 30 ft. of it in some wires. If he had $\frac{3}{4}$ of it left, how long was it at first?

6. How many bricks will be required for the walls of a house 42 ft. long, 28 ft. wide, and 18 ft. high, deducting 3 doors 7 ft. 6 in. by 4 ft., and 9 windows 5 ft. by 3 ft., the walls to be 1 ft. thick, allowing 22 bricks to 1 cu. ft.?

7. The sugar beet contains by weight $14\frac{1}{2}\%$ of sugar. In a recent year the United States produced 2,884,750 tons of sugar beets. How much sugar did this represent?

8. A man has a salary of \$1750 a year. He pays 15% of his salary for rent, 25% for groceries, 12% for meats, 13% for clothing, and \$112.50 for other expenses. What per cent of his salary does he save?

9. Find the square root of 7 to three figures.

10. How many tons of coal can be placed in a bin 16 ft. 6 in. long, 7 ft. wide, and 6 ft. deep, allowing 33 cu. ft. to the ton?

### WRITTEN EXAMINATION

**1.** Find the difference in acres between two fields, one containing 100 sq. rd., and the other being 100 rd. square.

**2.** Find the cost of a stair carpet at \$1.50 a yard for a straight flight of 16 steps 12 in. wide and 8 in. rise, allowing 20 in. extra at the bottom of the stairs and the same amount, besides the 12 in. for the top step, at the top.

**3.** Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% alumina. Find the number of pounds of each of these substances in 650 lb. of flint glass.

**4.** A bell weighing 875 lb. is composed by weight of 77% copper, 2% zinc, and the rest tin. How many pounds of each metal does the bell contain?

**5.** Gun metal is composed by weight of $85\frac{1}{2}$% copper and the rest tin. How many pounds of copper must be put with $159\frac{1}{2}$ lb. of tin to make gun metal?

**6.** Bronze is composed by weight of 18% tin and the rest copper. How many pounds of tin must be put with 1066 lb. of copper to make this quality of bronze?

**7.** The driving wheels of a locomotive have a diameter of 6 ft. 5 in. How many revolutions a minute will each wheel make when the locomotive is traveling at the rate of 50 mi. an hour?

**8.** The front wheels of a wagon are 3 ft. 6 in. in diameter and the rear wheels are 4 ft. 1 in. How many more revolutions will a front wheel make than a rear wheel in going a mile?

**9.** Multiply 17 ft. 9 in. by 6.

**10.** It is required to make a rectangular tin box that shall contain exactly 1 gal. Required to know the three dimensions such that they shall each be in exact inches.

### WRITTEN EXAMINATION

1. A boy computing the cost of 56 articles made the mistake of multiplying by 65 instead of by 56, and obtained $487.50. What was the cost of the lot?

2. Two masts of a ship are 52 ft. apart, one being 80 ft. high and the other 70 ft. high. Supposing them perpendicular to the deck, how far is it from the top of one to the top of the other?

3. A merchant asked 25% less than the cost of a certain article, but at an auction he received $33\frac{1}{3}\%$ more than the asked price. Did he gain or lose on the cost, and what per cent?

4. Water is flowing at the rate of 1 cu. ft. every 90 sec. into a tank whose base measure is $3'\ 6''$ by $4'\ 2''$. How long will it take to fill the tank to a depth of $1'\ 6''$?

5. A 12-inch gun can fire a shell weighing 850 lb. every 30 sec. At this rate how many pounds of shell could a battleship fire from four 12-inch guns in 2 min. 30 sec.?

6. A train leaves a city at 10.45 A.M. and reaches another city $127\frac{1}{2}$ mi. distant at 5 min. past 2 P.M. Allowing 20 min. for stops, what is the rate per hour?

7. A farmer finds that a bin 8 ft. long, 3 ft. 6 in. wide, and 5 ft. deep holds 112 bu. How many bushels in a bin that is half as long, 6 in. wider, and 1 ft. less in depth?

8. If 12 men can lay the asphalt blocks in a certain length of a city street in 16 da., how many men must be added to the force to complete the work 4 da. sooner?

9. Divide 124 ft. 3 in. by 7, and check the result.

10. A village has a water tank 42 ft. in diameter. In 3 hr. 45 min., when no water is being pumped in, the water is lowered 3 ft. How many gallons per hour are drawn out on an average?

### ORAL EXAMINATION

**1.** If you have a field in the shape of a triangle, and can make all the measurements you wish, and know the value of the land per acre, how do you find the value of the field?

**2.** If you know the assessed valuation of a man's property and the tax rate, how do you find the amount of his taxes?

**3.** If you know the face of a draft and the rate of premium, how do you find the cost of the draft?

**4.** If you have the first, third, and fourth terms of a proportion given, how do you find the second term? Given any three terms, how do you find the other term?

**5.** If you know the number of times a boy is at bat in a series of baseball games and the number of base hits, how do you find the per cent of base hits?

**6.** If you send to a city for a book, and wish to send the money with the order, how do you send the money? Why do you take this method? How much does it cost?

**7.** If you know the height and the diameter of a cylindrical silo, how do you find the number of cubic feet of ensilage that the silo will contain?

**8.** If you know the length of a strip of matting in feet, how do you find the length in yards? If you know the length in yards, how do you find the length in feet?

**9.** If you know the number of rods in one side of a square field, how do you find the number of square rods in the field? the number of acres?

**10.** In dividing one decimal fraction by another, where do you place the decimal point in the quotient? Give the reason for your answer.

### WRITTEN EXAMINATION

1. The amount of water that flows over Niagara Falls averages 264,000 cu. ft. a second. Find the average number of gallons carried over the falls in a minute.

2. If $\frac{7}{8}$ in. on a map represents an actual distance of $35\frac{1}{2}$ mi., what distance on the map represents 28.4 mi.?

3. How many loads of gravel averaging 1 cu. yd. will be required to grade $2\frac{3}{4}$ mi. of road, the gravel to be laid 14 ft. wide and 6 in. deep?

4. A dealer pays \$9737.50 for some wool, including the purchasing agent's commission of $2\frac{1}{2}\%$. Find the commission.

5. The sugar contained in the sugar beet is $14\frac{1}{2}\%$ of the weight of the beet. How many pounds of beets will be required to produce 284.2 lb. of sugar?

6. How many boards each 12 ft. long will be required to build a straight fence 4 boards high about a rectangular field 40 rd. wide containing 20 acres?

7. Find the circumference of a circle whose diameter is 22.6 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

8. Find the diameter of a circle whose circumference is 471.24 in., using 3.1416 as the ratio of circumference to diameter; also using $3\frac{1}{7}$; also using the ratio 355 : 113.

9. If 24 burners, each consuming $7\frac{1}{2}$ cu. ft. of gas per hour, are used on an average $3\frac{1}{2}$ hr. per day for 365 da., what will be the gas bill at 85¢ per thousand?

10. A bell weighing 1110 lb. is composed of 76.5% copper, 21.5% tin, and the rest zinc. How many pounds of each metal does the bell contain?

### ORAL EXAMINATION

1. Last year a gardener had 3 A. of potatoes and raised 200 bu. to the acre. How many bushels did he raise in all? If he sold these potatoes at 60¢ a bushel, how much did he get for them all? Potatoes weigh 60 lb. to the bushel. What was the weight of his potato crop?

2. For spraying his potato plants a gardener uses 1 lb. of Paris green to 100 gal. of water. Since 1 gal. of water weighs 8.3 lb., the weight of the Paris green is what part of the weight of water?

3. A gardener soaked his seed potatoes in a solution of 1 pt. of formalin to 25 gal. of water, to keep off the potato scab. What part of the solution is formalin?

4. If $\frac{3}{4}$ of the weight of potatoes is water, and if a bushel of potatoes weighs 60 lb., what is the weight of water in a bushel of potatoes?

5. A farmer had 2 A. of celery, and the yield was 1550 doz. heads to the acre. How many dozen did he have in all?

6. A farmer bought seed for 1000 celery plants at 24¢ an ounce. Allowing an ounce of seed to 2000 plants, how much did the seed cost?

7. When wheat costs $\$1.12\frac{1}{2}$ a bushel, how much will 100 bu. cost? 1000 bu.? 2000 bu.?

8. If wheat is worth \$1 a bushel, how much will the yield of 100 acres be worth if the average yield is 25 bu. per acre?

9. If wheat is worth \$1.10 a bushel, how much will the yield of 50 acres be worth if the average yield is 20 bu. per acre?

10. If wheat is worth $\$1.12\frac{1}{2}$ a bushel, how much will the yield of 80 acres be worth if the average yield is 25 bu. per acre?

### WRITTEN EXAMINATION

**1.** The following numbers are the given per cents more than what numbers and the same per cents less than what other numbers? That is, 390 is $62\frac{1}{2}\%$ more than what number and $62\frac{1}{2}\%$ less than what other number?

| | | | |
|---|---|---|---|
| 390, $62\frac{1}{2}\%$ | 175, $66\frac{2}{3}\%$ | 2.4, 20% | 9.1, 30% |
| 150, $87\frac{1}{2}\%$ | 350, $16\frac{2}{3}\%$ | 42.3, 50% | 17.6, 60% |
| 640, $33\frac{1}{3}\%$ | 143, $8\frac{1}{3}\%$ | 210, 40% | 56.1, 70% |

**2.** If a man spends 5% of his salary for clothes and then has \$855 left, what is his salary? How much does he spend for clothes?

**3.** If you have read all but 40% of a book and have read 234 pages, how many pages has the book?

**4.** If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

**5.** If you lose $\frac{1}{10}$ of a kite string and have 585 ft. left, how long was the string? How many feet did you lose? What per cent of the string was left?

**6.** A man sold some damaged goods for \$1547, at a loss of 9%. How much did the goods cost him?

**7.** A village has a population of 3384, having lost 6% in the last ten years. What was the population ten years ago? What was the number lost?

**8.** A man sold a car for \$1000, at a loss of 20%. How much did the car cost him?

**9.** A man sold some property and gained \$360, which was 12% on the money he invested. How much did he invest?

**10.** A man sold some property for \$3360, at a profit of 24%. How much did the property cost him?

### ORAL EXAMINATION

**1.** A ruler is marked in thirty-seconds of an inch. A piece of veneer is measured and found to be $\frac{6}{32}$ in. thick. Express this thickness in lower terms.

**2.** A surveyor uses a chain 66 ft. long, made of 100 links. He measures a certain distance and finds it to be 1 chain and 50 links. Express this as a chain and a fraction; as links; as yards; as feet.

**3.** How much picture molding is needed for a frame $5\frac{3}{8}$ in. long and $3\frac{1}{8}$ in. wide?

**4.** Four packages weigh $12\frac{1}{2}$ lb., $13\frac{1}{4}$ lb., $10\frac{3}{4}$ lb., and $10\frac{3}{4}$ lb. respectively. What is the total weight?

**5.** The page of a book is $4\frac{3}{8}$ in. wide and $7\frac{1}{16}$ in. long. What is the sum of the two sides? What is the perimeter? What is the difference between the length and width?

**6.** A picture is $3\frac{1}{4}$ in. wide and the mat is $1\frac{1}{8}$ in. wide on each side of the picture. How wide are the picture and mat together?

**7.** A plank $1\frac{7}{8}$ in. thick is planed down $\frac{1}{16}$ in. on one side. How thick is the plank then? How thick would it be if it were planed down $\frac{1}{16}$ in. on both sides?

**8.** From a piece of picture molding 7 ft. $2\frac{1}{8}$ in. long there is cut a piece 2 ft. $1\frac{1}{4}$ in. long. How long is the remainder?

**9.** A piece of marble $8\frac{3}{32}$ in. thick is polished down $\frac{1}{16}$ in. What is then the thickness? How much more will it have to be polished down to be exactly 8 in. thick? to be $7\frac{15}{16}$ in. thick? to be $7\frac{7}{8}$ in. thick?

**10.** A piece of cloth $7\frac{1}{8}$ in. wide is folded so as to be $\frac{3}{4}$ in. less in width. How wide is it then? How much more will have to be folded in order that it may be $6\frac{1}{4}$ in. wide?

### WRITTEN EXAMINATION

**1.** A family with an income of \$1200 spent 0.25 of it on food. How much did it spend on food?

**2.** A family with an income of \$1200 spent \$300 on food. Express the amount spent on food as a common fraction of the income; as a decimal fraction. Tell why the answers in Exs. 1 and 2 check each other.

**3.** A family with an income of \$2000 spent 0.23 of it on food. How much did it spend on food? A family with an income of \$2000 spent \$460 on food. What fractional part of the income was spent on food?

**4.** A family with an income of \$800 spent 0.45 of it on food. How much did it spend on food? Make a problem that shall check Ex. 4, as Ex. 2 checked Ex. 1, and as the second part of Ex. 3 checked the first part. Solve the problem.

**5.** With an income of \$1100 a family spent \$275 for food. This was what decimal fraction of the income?

**6.** With an income of \$900 a family spent \$252 for food. This was what decimal fraction of the income?

**7.** A man whose income is \$2800 a year spends 0.26 of it for rent. How much does he spend for rent?

**8.** A man whose income is \$2800 a year spends \$728 for rent. This is what decimal fraction of the income?

**9.** A man spends 0.23 of his income for food, 0.26 of it for rent, and 0.1 of it for clothing. If his income is \$2400 a year, how much does he spend for each? for all three?

**10.** A man earns \$200 a month and spends 0.22 of it for rent. How much does he pay for rent each month? How much does he pay for rent in a year?

### WRITTEN EXAMINATION

1. Find the side of a square whose area is 529 sq. in.

2. What will it cost for rails for 5 mi. of double track if the rails cost \$28 a ton and weigh 150 lb. per yard?

3. A and B together have \$153. If $\frac{3}{4}$ of A's money is equal to $\frac{2}{3}$ of B's, how much has each?

4. A horse tied to a stake can reach the grass 30 ft. in any direction from the stake. Over how many square feet of land can the horse graze?

5. Allowing 1000 shingles to 120 sq. ft. of surface, find how many thousand shingles will cover a pitched roof each side of which is 64 ft. long and 28 ft. 6 in. wide.

6. A corner lot has 72 ft. 6 in. front and is 144 ft. 6 in. deep. Find the cost of laying a 5-foot stone walk on the front and side, at 21¢ per square foot.

7. Find in inches the depth of a cylindrical tank 5 ft. in diameter that has the same capacity as a rectangular cistern 8 ft. square and 6 ft. deep.

8. A boat travels 15 mi. downstream in $2\frac{1}{2}$ hr. The boat's rate of travel in still water is $4\frac{1}{2}$ mi. an hour. In what time can the boat return?

9. A man has $\frac{5}{8}$ of his property invested in a farm and $\frac{1}{6}$ in railway stock; the remainder, \$1500, is deposited in a bank. Find the value of the farm and of the railway stock. Find the value of the entire property.

10. Two men start from points 33 mi. apart and walk toward each other, the first at the rate of $4\frac{1}{2}$ mi. per hour, and the second at the rate of $3\frac{3}{4}$ mi. per hour. How far from the point where the second man started will they meet?

### WRITTEN EXAMINATION

1. How many steps of 2 ft. 9 in. each would you have to take a minute in order to walk at the rate of 3 mi. an hour?

2. A machine does some work in 8 hr. If its capacity is increased $14\frac{2}{7}\%$, how long will it take to do the work?

3. Of three partners, the first is entitled to $37\frac{1}{2}\%$ and the second to $43\frac{3}{4}\%$ of the profits. If they make \$4800, what is the share of each of the three?

4. A certain alloy of metal is composed of $17\frac{1}{2}\%$ tin and the rest copper. How many pounds of tin are needed with $742\frac{1}{2}$ lb. of copper to make the alloy?

5. Two cans have a capacity of 5 pt. and 8 pt. respectively. How could you measure out 14 pt. into a third can by using these alone?

6. A dealer marked a set of furniture 25% above cost. He then sold it at 10% less than the marked price, and made a profit of \$5. How much did the furniture cost him?

7. A piece of carpet 27 in. wide weighs 4 lb. to the yard. What is the weight of this carpet necessary to cover a room 18 ft. wide and 24 ft. long?

8. If a gallon of water weighs $8\frac{1}{4}$ lb. and a cubic foot of water weighs 1000 oz., how many gallons are there in a cubic foot of water?

9. A cylindrical water tank has an internal diameter of 14 ft. and a height of 16 ft. How many cubic feet of water will it contain? How many gallons, taking a gallon as 231 cu. in.?

10. It has been determined that the equatorial radius of the earth is 20,926,202 ft. and the polar radius 20,854,895 ft. Express each in miles and find the difference.

### WRITTEN EXAMINATION

1. How much will 275 cu. in. of steel weigh if 1 cu. ft. weighs 490 lb. ?

2. What is the weight of a circular piece of steel shafting 3 in. in diameter and 20 ft. long, 1 cu. ft. of steel weighing 490 lb. ?

3. If the pressure of steam in a boiler is 120 lb. to the square inch, find the pressure on a valve $2\frac{1}{2}$ in. in diameter.

4. A boiler contains 420 flues, each $2\frac{1}{2}$ in. in internal diameter. Find the total area for draft through them.

5. If a pressure of 120 lb. to the square inch is applied to the circular plunger of a cylinder 2 ft. in diameter, what is the total pressure ?

6. The rim of a flywheel contains 1.56 cu. ft. of cast iron when it is 8 in. wide. If the rim were made a foot wide, how much would it weigh, 1 cu. ft. of cast iron weighing 450 lb. ?

7. From a rectangular sheet of steel which is $14'\ 3''$ long, $3'\ 3\frac{1}{2}''$ wide, and $\frac{1}{2}''$ thick a circular plate $13''$ in diameter is cut. If 1 cu. ft. of steel weighs 490 lb., what is the weight of the part of the sheet that is left ?

8. If the drivewheel of an engine makes 140 revolutions a minute, and the cylinder is 9 in. in diameter and 14 in. long and is filled with steam twice during each revolution, how many cubic feet of steam are used in a minute ?

9. A hollow steel shaft has an external diameter of $9''$ and an internal diameter of $5''$. If 1 cu. ft. of steel weighs 490 lb., find the weight of 10 ft. of this shaft.

10. How high is a smokestack that casts a shadow 80 ft. long at the same time that the shadow of a boy 4 ft. 6 in. tall is 5 ft. long ?

### WRITTEN EXAMINATION

**1.** Multiply $3\frac{7}{16}$ by $2\frac{1}{4}$, and check the answer by division.

**2.** Divide $3\frac{3}{8}$ by $2\frac{1}{2}$ and divide $2\frac{1}{2}$ by $3\frac{3}{8}$. What is the product of the two quotients? Why is this?

**3.** How many boards each 5 ft. 4 in. long can be cut from a board 16 ft. long, cutting crosswise?

**4.** If it takes 1 bbl. of lime and $\frac{5}{8}$ cu. yd. of sand to make mortar for 1000 bricks, and if $22\frac{1}{2}$ bricks are allowed to 1 cu. ft., how much of each is needed for a wall $6' \times 28' \times 1'\ 6''$?

**5.** If a horse is tethered by a 40-foot rope, the length of the largest circumference he can walk around is how many times what it would be if he were tethered by a 20-foot rope?

**6.** In Ex. 5 the horse can graze over how many times as much ground with the first rope as with the second?

**7.** If a man can do a piece of work in 8 da., what part of it can he do in 1 da.? in 4 da.? in $4\frac{1}{2}$ da.?

**8.** If X can build a wall in 18 da. and Y in 12 da., how many days will it take both together to build the wall?

**9.** A tank can be filled by three pipes, running separately, in 4 hr., 5 hr., and 6 hr. respectively. In how many hours will the empty tank be filled by all three running together?

Exs. 8 and 9 are introduced because they are interesting old questions, and not because they are of practical value.

**10.** If the eye is 8 in. above the surface of a lake, it can see about 1 mi.; if it is $2^2 \times 8$ in. above the surface, 2 mi.; if it is $3^2 \times 8$ in. above the surface, 3 mi.; and so on, the height being approximately 8 in. multiplied by the square of the distance in miles. Express to two decimal places the number of miles a man can see in one direction, if his eye is 36 ft. above the sea.

## SUPPLEMENT

**162. Nature of this Supplement.** As frequently stated in this series, there is a tendency at present to limit arithmetic to the absolute essentials. This has been the policy of the authors in preparing the series. There are certain topics which are essential in some places and which should be where the teacher can use them if necessary. Such are the topics treated in this supplement.

**163. Land Measure.** In surveying land the following measures are used:

LENGTH

**7.92 inches (in.) = 1 link (li.)**
**100 links, or 4 rods (rd.) = 1 chain (ch.)**
**80 chains, or 5280 feet (ft.) = 1 mile (mi.)**

City property is usually measured by feet and decimals of a foot.

AREA

**16 square rods (sq. rd.) = 1 square chain (sq. ch.)**
**10 square chains = 1 acre (A.)**
**640 acres = 1 square mile (sq. mi.)**
**36 square miles = 1 township (T.)**

The square rod is sometimes called a *perch*.

How many acres in a field 32 ch. long and 6 ch. 30 li. wide?

6 ch. 30 li. = 6.30 ch.

$32 \times 6.30$ sq. ch. $= 32 \times 6.30 \times 0.1$ A. $= 20.16$ A.

Actual measurements of this kind are valuable in connection with the school ground and other pieces of land. The pupils should have a clear conception of an acre, by being shown a field containing one acre or two acres or a half acre.

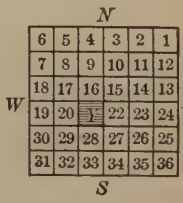
**164. Laying out Public Lands.** In the more recently settled parts of the country, land is laid out as here described.

**165. Principal Meridian and Base Line.** Through a given tract a meridian is chosen as the *principal meridian*. An east and west line is chosen as the *base line*. The principal meridian and base line are here shown.

**166. Township.** Lines are run parallel to the principal meridian and base line at intervals of 6 mi. This divides the land into *townships*.

**167. Range.** The north and south rows of townships are called *ranges*. On the first map $X$ is numbered T. 2 N., R. 3 W.; that is, the second township north of the base line, in the third range west of the meridian.

**168. Section.** A township is divided into *sections*, each 1 mi. square. This map shows the method of numbering these sections.




Each section is then divided as shown in the third map.

If this map represents the shaded part $Y$ of the second map, and the second map represents the shaded part $X$ of the first map, the shaded part here shown would be thus described: S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 21, T. 2 N., R. 3 W. This means the southwest quarter of the northwest quarter of section 21, second township north, third range west.

In sections of the country where land is not laid out in this way little attention should be given to this subject. In rural schools, however, some attention should always be given to the actual measuring of land, particularly by dividing fields of irregular shape into triangles.

### WRITTEN EXERCISE

*Write the description, plot, and find the area:*

1. S.E. $\frac{1}{4}$, Sec. 5, T. 3 S., R. 3 W.

2. N.E. $\frac{1}{4}$, Sec. 8, T. 2 N., R. 2 W.

3. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 2, T. 2 N., R. 3 E.

4. S. $\frac{1}{2}$ of S.E. $\frac{1}{4}$, Sec. 20, T. 2 S., R. 3 W.

5. N. $\frac{1}{2}$ of S.W. $\frac{1}{4}$, Sec. 10, T. 3 S., R. 2 E.

6. E. $\frac{1}{2}$ of N.W. $\frac{1}{4}$, Sec. 30, T. 2 S., R. 3 E.

7. N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$, Sec. 5, T. 1 N., R. 1 W.

8. S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 32, T. 1 S., R. 3 E.

9. How much is this farm worth at \$65 an acre? W. $\frac{1}{2}$ of S. $\frac{1}{2}$, Sec. 3, T. 2 N., R. 2 W.

10. How much is this farm worth at \$75 an acre? N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$, Sec. 5, T. 2 S., R. 2 E.

11. Find the area of this farm: S.W. $\frac{1}{4}$, Sec. 10, T. 2 S., R. 2 E. Draw a plan of the township and locate the farm.

12. Find the area of this farm: N. $\frac{1}{2}$, Sec. 6, T. 1 N., R. 1 E. Draw a plan of the township and locate the farm.

13. Mr. Simmons owns the S. $\frac{1}{2}$, N.E. $\frac{1}{4}$, Sec. 3, T. 2 N., R. 3 E. How many rods of fence are needed to inclose his land?

14. How far is it from Mr. Taylor's farm, N.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 16, T. 1 N., R. 3 W., to Mr. Hunt's farm, S.W. $\frac{1}{4}$ of N.W. $\frac{1}{4}$, Sec. 28, T. 1 N., R. 3 W.? Draw the map.

15. A road runs straight through a farm and is $\frac{1}{2}$ mi. long and 3 rd. wide. How many acres in the road? If hay can be cut from the sides, averaging $\frac{1}{4}$ of the area of the road, and the amount of hay averages 2 tons to the acre and is worth \$9.50 a ton, what is gained by attending to this crop?

**169. Table of English Money.** The English table of money is as follows:

**12 pence (d.) = 1 shilling (s.)**
**20 shillings = 1 pound (£)**

The pound is worth \$4.8665, or about \$5; the shilling is worth \$0.243, or about 25¢; and the penny is worth about 2¢.

**170. Table of French Money.** The French table of money, used in several other European countries also, is as follows:

**100 centimes (c.) = 1 franc (fr.)**

The franc is worth \$0.193, or about 20¢.

**171. Table of German Money.** The German table of money is as follows:

**100 pfennigs (pf.) = 1 mark (M.)**

The mark is worth \$0.238, or about 25¢; and 4 pf., about 1¢.

WRITTEN EXERCISE

*Taking £1 as equal to \$4.87, express in our money:*

1. £75. 2. £96. 3. £68. 4. £16 10 s.

*Taking £1 as equal to \$4.87, express in English money:*

5. \$38.96. 6. \$48.70. 7. \$24.35. 8. \$43.83.

*Taking 1 M. as equal to 23.8¢, express in our money:*

9. 75 M. 10. 96 M. 11. 125 M. 12. 3750 M.

*Taking 1 M. as equal to 23.8¢, express in German money:*

13. \$9.52. 14. \$11.90. 15. \$14.28. 16. \$71.40.

*Taking 1 fr. as equal to 19.3¢, express in our money:*

17. 85 fr. 18. 250 fr. 19. 230 fr. 20. 750 fr.

**172. Longitude and Time.** In geography you learned about the longitude of places, and you know that longitude is reckoned in degrees, minutes, and seconds from the prime meridian, which passes through Greenwich, England.

**173. Correspondence of Longitude to Time.** Since the earth turns about on its axis once every 24 hours, the place in which we live will pass through 360° between now and this time to-morrow. That is, to the time 24 hours will correspond the longitude 360°. Therefore

360° of longitude corresponds to 24 hr.

1° of longitude corresponds to $\frac{1}{360}$ of 24 hr., or $\frac{1}{15}$ hr., or 4 min.

1′ of longitude corresponds to $\frac{1}{60}$ of 4 min., or $\frac{1}{15}$ min., or 4 sec.

1″ of longitude corresponds to $\frac{1}{60}$ of 4 sec., or $\frac{1}{15}$ sec.

**174. Earlier West, Later East.** Because the earth revolves 360° in 24 hours, the sun appears to pass through 15° an hour. Therefore, when it is noon here it is an hour later, or 1 P.M., 15° east of here; an hour earlier, or 11 A.M., 15° west of here; and 6 hours earlier, or 6 A.M., 90° west of here.

Since this work belongs partly to geography, no illustrations are here introduced. The teacher should use the globe as needed.

### ORAL EXERCISE

**1.** When it is 9 A.M. here, what time is it 15° east of here? 30° west of here? directly south of here?

**2.** When it is noon here, what time is it 30° east of here? 90° west of here? 150° east of here? directly north of here?

**3.** What difference of time corresponds to a difference of longitude of $7\frac{1}{2}$°? 45°? 60°? 105°? 120°? 165°? 180°?

**4.** Since 1′ of longitude corresponds to 4 sec. of time, to what does 7′ correspond? 11′? 15′? 30′? 45′? 60′? 120′?

**175. Illustrative Problem.** Two ships at sea are 65° 7′ 30″ of longitude apart. What is their difference in time?

Since 1° corresponds to $\frac{1}{15}$ hr., 65° corresponds to $65 \times \frac{1}{15}$ hr., or $4\frac{1}{3}$ hr., or 4 hr. 20 min.

Since 1′ corresponds to $\frac{1}{15}$ min., 7′ corresponds to $7 \times \frac{1}{15}$ min., or $\frac{7}{15}$ min., or 28 sec.

Since 1″ corresponds to $\frac{1}{15}$ sec., 30″ corresponds to $30 \times \frac{1}{15}$ sec., or 2 sec.

Therefore the difference in time between the ships is 4 hr. 20 min. 30 sec.

It is apparent that the mere figures of the answer could be obtained by this division, although the reason would not be so clear.

$$15\overline{)65\ \ 7\ 30}$$
$$4\ 20\ 30$$

WRITTEN EXERCISE

1. A difference of 27° 4′ 15″ in longitude corresponds to what difference in time?

2. Two ships are 75° 30′ 30″ of longitude apart. What is their difference in time?

3. A ship 62° 3′ 40″ W. receives at 11 A.M. a wireless telegram from one 60° 1′ 10″ W. When was it sent?

4. For time purposes the longitude of Berlin is 15° E., and that of Chicago is 90° W. What is the difference in longitude? Illustrate by a rough map. When it is 2 P.M. in Chicago, what time is it in Berlin?

5. For time purposes the longitude of San Francisco is taken as 120° W. When it is noon in London (on the prime meridian), what time is it in San Francisco?

6. A steamer 68° 10′ 30″ W. sends a wireless telegram to the Nantucket lightship, 70° W., at 8 hr. 30 min. A.M. At what time is it received? If transmitted to New York, 75° W., without loss of time, when is it received there?

**176. Correspondence of Time to Longitude.** Since 24 hours of time corresponds to 360° of longitude, we have the following:

24 hr. corresponds to 360° of longitude.
1 hr. corresponds to $\frac{1}{24}$ of 360°, or 15°, of longitude.
1 min. corresponds to $\frac{1}{60}$ of 15°, or $\frac{1}{4}$°, or 15′, of longitude.
1 sec. corresponds to $\frac{1}{60}$ of 15′, or $\frac{1}{4}$′, or 15″, of longitude.

The difference in time between two ships is 3 hr. 7 min. 3 sec. What is the difference in longitude?

Since 1 hr. corresponds to 15°, 3 hr. corresponds to 3 × 15°, or 45°.

Since 1 min. corresponds to 15′, 7 min. corresponds to 7 × 15′, or 105′, or 1° 45′.

Since 1 sec. corresponds to 15″, 3 sec. corresponds to 3 × 15″, or 45″.

Therefore the difference in longitude is 46° 45′ 45″.

| | | |
|---|---|---|
| 3° | 7′ | 3″ |
| | | 15 |
| 46° | 45′ | 45″ |

Evidently the mere figures of the answer could be obtained by calling 3 hr. 7 min. 3 sec. 3° 7′ 3″ and multiplying by 15, although the reason for the work would not be so clear.

### WRITTEN EXERCISE

**1.** The difference of time between two ships is 2 hr. 3 min. 10 sec. What is the difference in longitude?

**2.** When it is noon at Denver, it is 7 P.M. at Greenwich (0°). What longitude does Denver use for its time purposes?

**3.** When it is noon at Greenwich, it is 6 A.M. in Iowa. What longitude does Iowa use for its time purposes?

**4.** The difference in time between the Harvard and Columbia university observatories is 8 min. 22.7 sec. What is the difference in longitude?

**5.** At 10 hr. 4 min. A.M. a steamer in longitude 26° 30′ W. sends a wireless message to another steamer. It is received at 10 hr. 19 min. A.M. What is the longitude of the second steamer?

**177. Standard Time.** For practical purposes most of the civilized world is now divided into sections, and all places in the same section use the *standard time* of a certain meridian.

The details of this plan, like the study of railway time-tables, are matters of geography and have no value in arithmetic.

The following is the standard-time map of the United States:

When it is noon by Eastern time (75° W.), it is 11 A.M. by Central time (90° W.), 10 A.M. by Mountain time (105° W.), 9 A.M. by Pacific time (120° W.), and 5 P.M. by Greenwich time (0°).

### ORAL EXERCISE

1. When it is 10 A.M. in Iowa, what time is it in New York?
2. What time is it now in your school? in Boston?
3. When it is 10 A.M. in San Francisco, what time is it in Denver? in St. Louis? in Cleveland? in Albany?
4. When it is 9 A.M. in Denver, what time is it in Los Angeles?
5. When it is noon in California, what time is it in Illinois?
6. When it is 3 P.M. in Milwaukee, what time is it in Oregon?

**178. Six Per Cent Method of Finding Interest.** This method, known as the Six Per Cent Method, will often be convenient.

Required the interest on \$420 for 5 mo. 10 da. at 6%.

Since 2 mo. $= \frac{1}{6}$ yr., the rate for 2 mo. is $\frac{1}{6}$ of 6%, or 1%.

| | |
|---|---|
| The interest for 2 mo. is 1% of \$420 | = \$4.20 |
| The interest for 2 mo. more | = 4.20 |
| The interest for 1 mo. more is $\frac{1}{2}$ of \$4.20 | = 2.10 |
| The interest for 10 da. more is $\frac{1}{3}$ of \$2.10 | = .70 |
| The interest for 5 mo. 10 da. | = \$11.20 |

Hence *the interest at 6% for 60 days is 0.01 of the principal, and for 6 days 0.001 of the principal.* For other periods the interest can be found from the interest for these periods.

The method is sometimes stated as follows:

*To find the interest for 600 da. at 6%, move the decimal point one place to the left; for 60 da., two places to the left; for 6 da., three places to the left.*

After finding the interest at 6%, the interest for other rates is easily found. Thus the interest at 5% is $\frac{5}{6}$ of that at 6%.

ORAL EXERCISE

*Find the interest at 6% on the following:*

1. \$750, for 60 da.
2. \$300, for 30 da.
3. \$200, for 90 da.
4. \$275, for 60 da.
5. \$250, for 30 da.
6. \$400, for 90 da.
7. \$500, for 60 da.
8. \$500, for 30 da.
9. \$500, for 600 da.
10. \$1200, for 720 da.

*Find the interest at 5% on the following:*

11. \$600, for 60 da.
12. \$1200, for 30 da.
13. \$120, for 30 da.
14. \$720, for 90 da.

### WRITTEN EXERCISE

1. Find the interest on \$1750 for 60 da., first using the Six Per Cent Method, then using cancellation, and finally using the ordinary method of finding the interest for 1 yr. and then for the fractional part of a year. Write a statement telling the advantage of the Six Per Cent Method over the other methods.

2. Using the three methods mentioned in Ex. 1, find the interest on \$675 for 90 da. at 6%; on \$1425 for 30 da. at 6%.

3. A note for \$750 is discounted for 60 da. at 6%. Find the discount and the proceeds.

4. A note for \$650 is discounted for 30 da. at 6%. Find the discount and the proceeds.

5. A note for \$2400 is discounted for 90 da. at 6%. Find the discount and the proceeds.

Since bank notes usually run for 30 da., 60 da., or 90 da., since 6% is the most common rate, and since we can tell the discount for 60 da. by simply glancing at the face of the note, we can often find the discount on bank notes for the usual periods without using pencil and paper.

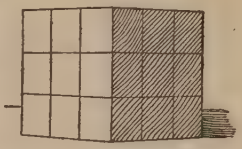
*Find the discounts at 6% on notes for the following amounts:*

6. \$1200, for 90 da.
7. \$1270, for 90 da.
8. \$1525, for 30 da.
9. \$2575, for 90 da.
10. \$3625, for 90 da.
11. \$4250, for 30 da.
12. \$450, for 3 mo. 10 da.
13. \$725, for 2 mo. 15 da.
14. \$850, for 1 mo. 20 da.
15. \$550, for 2 mo. 20 da.
16. \$1275, for 3 mo. 15 da.
17. \$6250, for 1 mo. 18 da.

*Find the interest at 5% on notes for the following amounts:*

18. \$4200, for 60 da.
19. \$3750, for 90 da.
20. \$500, for 1 mo. 10 da.
21. \$750, for 2 mo. 15 da.

**179. Cubes and Cube Roots.** If a cube has an edge of 3 units, it has a volume of 27 cubic units. Therefore 27 is called the *cube* of 3, and 3 is called the *cube root* of 27.



The cube of 3 is written $3^3$; the cube root of 27 is written $\sqrt[3]{27}$.

**180. Cube Roots of Volumes.** Therefore, considering the *abstract* numbers representing the measures,

*The edge of a cube is the cube root of its volume.*

**181. Powers.** Squares and cubes are called *powers*. We also have higher powers, like the fourth, fifth, and so on. Raising to powers is sometimes called *involution*. Extracting roots is sometimes called *evolution*.

**182. Cube Roots of Perfect Cubes.** Cube roots of perfect cubes may often be found by factoring.

For example, $\sqrt[3]{216} = \sqrt[3]{2\times2\times2\times3\times3\times3}$

$= \sqrt[3]{(2\times3)\times(2\times3)\times(2\times3)}$

$= \sqrt[3]{6\times6\times6} = 6.$

$$\begin{array}{r|l} 2 & 216 \\ 2 & 108 \\ 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ & 3 \end{array}$$

The subject of cube root is now so commonly postponed until algebra is studied that extended explanation is not felt to be necessary. A brief treatment will therefore suffice. Teachers are advised to omit the subject unless the local course of study calls for it.

### WRITTEN EXERCISE

*Find the following cube roots by factoring:*

1. $\sqrt[3]{729}$.
2. $\sqrt[3]{1331}$.
3. $\sqrt[3]{512}$.
4. $\sqrt[3]{1728}$.
5. $\sqrt[3]{2744}$.
6. $\sqrt[3]{15{,}625}$.
7. $\sqrt[3]{4096}$.
8. $\sqrt[3]{5832}$.
9. What is the edge of a cube of volume 13.824 cu. in.?
10. What is the edge of a cube of volume 10,648 cu. in.?

**183. Finding Square and Cube Roots.** We have seen (§ 141) that we may need to find the square roots of numbers. We shall also see that we may need to find the cube roots of numbers, particularly in measuring solid figures.

There are methods of finding the square root and the cube root of any number. These methods are seldom used in ordinary computations. When we wish to find squares, cubes, square roots, or cube roots, we generally use tables. For example, on page 277 is a convenient table giving the squares, cubes, square roots, and cube roots of all numbers from 1 to 100.

**1.** Find the square root of 39.

From the table, $\sqrt{39} = 6.245$. This means that $6.245^2$ is approximately 39. By squaring 6.245 we see that the result is 39.000025. That is, $\sqrt{39}$ is 6.245, to three places of decimals.

**2.** Find the cube root of 88.

From the table, $\sqrt[3]{88} = 4.448$. That is, 88 is not an exact cube, and its cube root is between 4.447 and 4.448.

**3.** Find the square root of 841.

The column of numbers (under No.) goes only to 100. We therefore look to see if 841 is in the column of squares. We find that it is, being the square of 29. Therefore $\sqrt{841} = 29$.

**4.** Find the cube root of 884,736.

The column of cubes shows that $884{,}736 = 96^3$. Hence, $\sqrt[3]{884{,}736} = 96$.

**5.** Find the square root of 2000.

Since $2000 = 20 \times 100$, we may find the square roots of 20 and 100 and take their product. We see that $\sqrt{20} = 4.472$, and $\sqrt{100} = 10$. We therefore have $\sqrt{2000} = 10 \times 4.472 = 44.72$.

**6.** Find the square root of 1765.

The table shows that $\sqrt{1764} = 42$, so that $\sqrt{1765} = 42 +$ some fraction; that is, this table gives us only the two integral figures.

**184. Table of Powers and Roots.** This table will be helpful:

| No. | Squares | Cubes | Square Roots | Cube Roots | No. | Squares | Cubes | Square Roots | Cube Roots |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1.000 | 1.000 | 51 | 2,601 | 132,651 | 7.141 | 3.708 |
| 2 | 4 | 8 | 1.414 | 1.260 | 52 | 2,704 | 140,608 | 7.211 | 3.733 |
| 3 | 9 | 27 | 1.732 | 1.442 | 53 | 2,809 | 148,877 | 7.280 | 3.756 |
| 4 | 16 | 64 | 2.000 | 1.587 | 54 | 2,916 | 157,464 | 7.348 | 3.780 |
| 5 | 25 | 125 | 2.236 | 1.710 | 55 | 3,025 | 166,375 | 7.416 | 3.803 |
| 6 | 36 | 216 | 2.449 | 1.817 | 56 | 3,136 | 175,616 | 7.483 | 3.826 |
| 7 | 49 | 343 | 2.646 | 1.913 | 57 | 3,249 | 185,193 | 7.550 | 3.849 |
| 8 | 64 | 512 | 2.828 | 2.000 | 58 | 3,364 | 195,112 | 7.616 | 3.871 |
| 9 | 81 | 729 | 3.000 | 2.080 | 59 | 3,481 | 205,379 | 7.681 | 3.893 |
| 10 | 100 | 1,000 | 3.162 | 2.154 | 60 | 3,600 | 216,000 | 7.746 | 3.915 |
| 11 | 121 | 1,331 | 3.317 | 2.224 | 61 | 3,721 | 226,981 | 7.810 | 3.936 |
| 12 | 144 | 1,728 | 3.464 | 2.289 | 62 | 3,844 | 238,328 | 7.874 | 3.958 |
| 13 | 169 | 2,197 | 3.606 | 2.351 | 63 | 3,969 | 250,047 | 7.937 | 3.979 |
| 14 | 196 | 2,744 | 3.742 | 2.410 | 64 | 4,096 | 262,144 | 8.000 | 4.000 |
| 15 | 225 | 3,375 | 3.873 | 2.466 | 65 | 4,225 | 274,625 | 8.062 | 4.021 |
| 16 | 256 | 4,096 | 4.000 | 2.520 | 66 | 4,356 | 287,496 | 8.124 | 4.041 |
| 17 | 289 | 4,913 | 4.123 | 2.571 | 67 | 4,489 | 300,763 | 8.185 | 4.062 |
| 18 | 324 | 5,832 | 4.243 | 2.621 | 68 | 4,624 | 314,432 | 8.246 | 4.082 |
| 19 | 361 | 6,859 | 4.359 | 2.668 | 69 | 4,761 | 328,509 | 8.307 | 4.102 |
| 20 | 400 | 8,000 | 4.472 | 2.714 | 70 | 4,900 | 343,000 | 8.367 | 4.121 |
| 21 | 441 | 9,261 | 4.583 | 2.759 | 71 | 5,041 | 357,911 | 8.426 | 4.141 |
| 22 | 484 | 10,648 | 4.690 | 2.802 | 72 | 5,184 | 373,248 | 8.485 | 4.160 |
| 23 | 529 | 12,167 | 4.796 | 2.844 | 73 | 5,329 | 389,017 | 8.544 | 4.179 |
| 24 | 576 | 13,824 | 4.899 | 2.884 | 74 | 5,476 | 405,224 | 8.602 | 4.198 |
| 25 | 625 | 15,625 | 5.000 | 2.924 | 75 | 5,625 | 421,875 | 8.660 | 4.217 |
| 26 | 676 | 17,576 | 5.099 | 2.962 | 76 | 5,776 | 438,976 | 8.718 | 4.236 |
| 27 | 729 | 19,683 | 5.196 | 3.000 | 77 | 5,929 | 456,533 | 8.775 | 4.254 |
| 28 | 784 | 21,952 | 5.292 | 3.037 | 78 | 6,084 | 474,552 | 8.832 | 4.273 |
| 29 | 841 | 24,389 | 5.385 | 3.072 | 79 | 6,241 | 493,039 | 8.888 | 4.291 |
| 30 | 900 | 27,000 | 5.477 | 3.107 | 80 | 6,400 | 512,000 | 8.944 | 4.309 |
| 31 | 961 | 29,791 | 5.568 | 3.141 | 81 | 6,561 | 531,441 | 9.000 | 4.327 |
| 32 | 1,024 | 32,768 | 5.657 | 3.175 | 82 | 6,724 | 551,368 | 9.055 | 4.344 |
| 33 | 1,089 | 35,937 | 5.745 | 3.208 | 83 | 6,889 | 571,787 | 9.110 | 4.362 |
| 34 | 1,156 | 39,304 | 5.831 | 3.240 | 84 | 7,056 | 592,704 | 9.165 | 4.380 |
| 35 | 1,225 | 42,875 | 5.916 | 3.271 | 85 | 7,225 | 614,125 | 9.220 | 4.397 |
| 36 | 1,296 | 46,656 | 6.000 | 3.302 | 86 | 7,396 | 636,056 | 9.274 | 4.414 |
| 37 | 1,369 | 50,653 | 6.083 | 3.332 | 87 | 7,569 | 658,503 | 9.327 | 4.431 |
| 38 | 1,444 | 54,872 | 6.164 | 3.362 | 88 | 7,744 | 681,472 | 9.381 | 4.448 |
| 39 | 1,521 | 59,319 | 6.245 | 3.391 | 89 | 7,921 | 704,969 | 9.434 | 4.465 |
| 40 | 1,600 | 64,000 | 6.325 | 3.420 | 90 | 8,100 | 729,000 | 9.487 | 4.481 |
| 41 | 1,681 | 68,921 | 6.403 | 3.448 | 91 | 8,281 | 753,571 | 9.539 | 4.498 |
| 42 | 1,764 | 74,088 | 6.481 | 3.476 | 92 | 8,464 | 778,688 | 9.592 | 4.514 |
| 43 | 1,849 | 79,507 | 6.557 | 3.503 | 93 | 8,649 | 804,357 | 9.644 | 4.531 |
| 44 | 1,936 | 85,184 | 6.633 | 3.530 | 94 | 8,836 | 830,584 | 9.695 | 4.547 |
| 45 | 2,025 | 91,125 | 6.708 | 3.557 | 95 | 9,025 | 857,375 | 9.747 | 4.563 |
| 46 | 2,116 | 97,336 | 6.782 | 3.583 | 96 | 9,216 | 884,736 | 9.798 | 4.579 |
| 47 | 2,209 | 103,823 | 6.856 | 3.609 | 97 | 9,409 | 912,673 | 9.849 | 4.595 |
| 48 | 2,304 | 110,592 | 6.928 | 3.634 | 98 | 9,604 | 941,192 | 9.899 | 4.610 |
| 49 | 2,401 | 117,649 | 7.000 | 3.659 | 99 | 9,801 | 970,299 | 9.950 | 4.626 |
| 50 | 2,500 | 125,000 | 7.071 | 3.684 | 100 | 10,000 | 1,000,000 | 10.000 | 4.641 |

### WRITTEN EXERCISE

*Using the table on page 277, find the square roots of the following:*

| | | | | |
|---|---|---|---|---|
| 1. 25. | 3. 64. | 5. 7. | 7. 29. | 9. 67. |
| 2. 49. | 4. 81. | 6. 13. | 8. 47. | 10. 91. |

*Find the cube roots of the following:*

| | | | | |
|---|---|---|---|---|
| 11. 8. | 13. 64. | 15. 17. | 17. 28. | 19. 51. |
| 12. 27. | 14. 13. | 16. 19. | 18. 46. | 20. 99. |

*Find the square roots of the following:*

| | | | | |
|---|---|---|---|---|
| 21. 225. | 23. 961. | 25. 5184. | 27. 7744. | 29. 9025. |
| 22. 529. | 24. 2116. | 26. 6889. | 28. 8649. | 30. 9604. |

*Find the cube roots of the following:*

| | | | |
|---|---|---|---|
| 31. 5832. | 33. 19,683. | 35. 85,184. | 37. 531,441. |
| 32. 9261. | 34. 42,875. | 36. 97,336. | 38. 804,357. |

*Find the integral parts of the square roots of the following:*

| | | | | |
|---|---|---|---|---|
| 39. 230. | 41. 509. | 43. 1475. | 45. 3850. | 47. 6730. |
| 40. 375. | 42. 850. | 44. 1695. | 46. 4775. | 48. 8675. |

*Find the integral parts of the cube roots of the following:*

| | | | |
|---|---|---|---|
| 49. 2775. | 50. 22,325. | 51. 75,750. | 52. 675,750. |

53. The volume of a cube is 2197 cu. in. Find the edge.

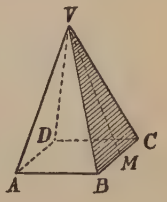
54. The volume of a cube is 39 cu. ft. Find the edge.

55. The area of one face of a cube is 78 sq. in. Find the volume of the cube.

56. The area of all six faces of a cube is 36 sq. in. Find the volume of the cube.

**185. Lateral Area of a Pyramid.** We have already learned how to find the volume of a pyramid, a problem more often found in practical life than the one of finding the surface.

The only kind of pyramid considered in arithmetic is one in which the vertex is exactly over the center of the base, the so-called *regular pyramid* here illustrated.




The altitude of any triangular face of a pyramid is called the *slant height* of the pyramid.

For example, $VM$ is the slant height of the pyramid here shown.

The sum of the areas of the triangular faces of a pyramid is called the *lateral area* of the pyramid.

Since the area of each triangular face of a pyramid equals half the product of the base and altitude,

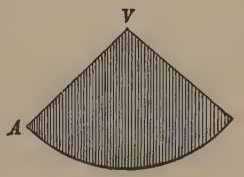
*The lateral area of a pyramid equals the perimeter of the base multiplied by half the slant height.*

**186. Lateral Area of a Cone.** If we should slit the surface of a cone and flatten it out as shown below, we would have part of a circle.

The only kind of cone considered in arithmetic is the so-called *right circular cone* here illustrated. The cone is so familiar as to require no formal definition. The terms "lateral area" and "slant height" will be understood from the study of the pyramid.

In the way that we found the area of a circle we find that

*The lateral area of a cone equals the circumference of the base multiplied by half the slant height.*




In the exercise on page 280 it is understood that regular pyramids and right circular cones are given, these being the ones usually encountered.

**187. Volume of a Cone.** In the way that we found the volume of a pyramid we may find the volume of a cone. Then

*The volume of a cone equals one third the product of the base and height.*

What is the volume of a cone 5 in. high, the radius of the base being 2 in.?

Area of base $= \frac{22}{7} \times 4$ sq. in.

Volume $= \frac{1}{3} \times 5 \times \frac{22}{7} \times 4$ cu. in. $= 20.95$ cu. in.

That is, the volume is a little greater than 20.95 cu. in. Common sense should be used as to rejecting small fractions. The result may be given as $20.95\frac{5}{21}$ cu. in., but we would never measure so closely as this.

### WRITTEN EXERCISE

*Find the lateral areas of pyramids with the following perimeters of bases and slant heights:*

1. 24 in., 16 in. 2. 4 ft. 3 in., 9 in. 3. 4 ft. 9 in., 12 in.

*Find the lateral areas of cones with the following circumferences of bases and slant heights:*

4. $15\frac{3}{4}$ in., $4\frac{3}{4}$ in. 5. 7 ft. 6 in., 4 ft. 6. 3 ft. 6 in., 15 in.

*Find the volumes of cones with the following radii of bases and heights:*

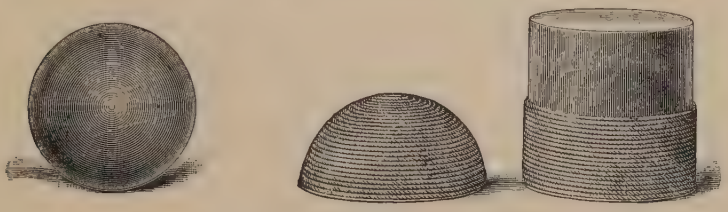
7. 15.4 in., 6 in. 8. 6.3 in., 15 in. 9. 42 in., 18 in.

10. The Great Pyramid of Cheops has a slant height of 500 ft. and a square base 764 ft. on a side. Find the lateral area.

11. How many square feet of slating on a steeple in the form of a pyramid of slant height 40 ft. and perimeter of base 48 ft.?

12. A dome is surmounted by a gilded cone whose slant height is 16 ft. 8 in., the radius of the base being 3 ft. 6 in. How many square feet must be allowed for gilding?

**188. Surface of a Sphere.** If we wind half of the surface of a sphere with a cord as here shown, and then wind with exactly



the same length of the cord the surface of a cylinder whose radius equals the radius of the sphere and height equals the diameter, we find that the cord covers half the curved surface of the cylinder.

Therefore, *the surface of a sphere equals the curved surface of a cylinder of the same radius and height.*

Since the curved surface of the cylinder is $\frac{22}{7} \times$ diameter $\times$ height, and the height is $2 \times$ radius,

$$\text{area of sphere} = \tfrac{22}{7} \times 2 \times \text{radius} \times 2 \times \text{radius}$$
$$= 4 \times \tfrac{22}{7} \times \text{square of radius.}$$

**189. Volume of a Sphere.** It is sometimes desirable to be able to find the volume of a sphere, although this is not one of the essentials of arithmetic. It is shown in geometry that

*The volume of a sphere is equal to $\frac{4}{3}$ times $\frac{22}{7}$ times the cube of the radius.*

**1.** If the earth is a sphere of 4000 mi. radius, find the surface.

$4 \times \frac{22}{7} \times 4000^2 = 4 \times \frac{22}{7} \times 16{,}000{,}000 = 201{,}142{,}857\frac{1}{7}$.

Therefore the surface is about 201,143,000 sq. mi.

**2.** If a ball is 4 ft. in diameter, find the volume.

The radius is $\frac{1}{2}$ of 4 ft., or 2 ft.

The volume is $\frac{4}{3} \times \frac{22}{7} \times 2 \times 2 \times 2$ cu. ft., or 33.52 cu. ft.

### WRITTEN EXERCISE

1. If a ball has a radius of $2\frac{1}{4}$ in., what is the surface?

2. The sun's diameter is 866,500 mi. Find the surface.

3. If a tennis ball is $2\frac{3}{8}$ in. in diameter, what is the surface?

4. A gilded ball is to be put on top of a tower. The diameter of the ball is $2'\ 6''$. How many square inches are to be gilded?

5. If a cubic foot of granite weighs 165 lb., find the weight of a sphere of granite 1 ft. in diameter.

6. A bowl is in the form of a hemisphere 4.2 in. in diameter. How many cubic inches does it contain?

7. What is the weight of a sphere of marble 3 ft. in circumference, marble being 2.7 times as heavy as water and 1 cu. ft. of water weighing 1000 oz.?

8. Taking the radius of the earth as 4000 mi. and the earth as an exact sphere, find the volume to the nearest 1000 cu. mi.

9. A pyramid has a lateral area of 200 sq. in. The slant height is 8 in. Find the perimeter of the base.

10. A conical spire has a slant height of 68 ft. The perimeter of the base is 60 ft. What is the lateral surface?

11. What is the entire surface of a cone whose slant height is 7 ft. and the diameter of whose base is 7 ft.?

12. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindrical marble column that is 10 ft. high and 16 in. in diameter?

13. How many cubic yards of earth must be removed in digging a canal 2 mi. 1740 ft. long, 200 ft. wide, and 16 ft. deep?

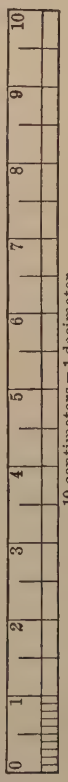
14. A marble 1 in. in diameter is dropped into a cylindrical jar 4 in. high and 3 in. in diameter, half full of water. How much does the marble cause the water to rise?

**190. Metric System.** There is a system of measures used in many countries which is known as the *Metric System.*

The unit of length is the *meter.* The meter is 39.37 in. nearly.

The unit of capacity is the *liter* (lētĕr), a cube 0.1 of a meter on an edge. The liter is 1 qt. nearly.

The unit of weight is the *gram.* It is practically the weight of a cube of water 0.01 of a meter on an edge. The gram is 15.4 grains nearly.




**191. Prefixes.** Learn the following prefixes:

Just as 1 mill = 0.001 of a dollar,
so 1 millimeter = 0.001 of a meter.
Just as 1 cent = 0.01 of a dollar,
so 1 centimeter = 0.01 of a meter.
Just as decimal means tenths,
so 1 decimeter = 0.1 of a meter.

| PREFIX | MEANS | AS IN | WHICH MEANS | |
|---|---|---|---|---|
| myria- | 10,000 | myriameter | 10,000 | meters. |
| kilo- | 1000 | kilogram | 1000 | grams. |
| hekto- | 100 | hektoliter | 100 | liters. |
| deka- | 10 | dekameter | 10 | meters. |
| deci- | 0.1 | decimeter | 0.1 | of a meter. |
| centi- | 0.01 | centigram | 0.01 | of a gram. |
| milli- | 0.001 | millimeter | 0.001 | of a meter. |

This system is as much easier than our common one as the system of United States money is easier than the English system. This is the reason why it is so extensively used on the continent of Europe and in Central America and South America.

Unless the pupil is going to use the metric system at once in some of his measurements in science, the study of this subject may be omitted. It is easily learned when the necessity for its use arises. The system is used almost everywhere in scientific work.

**192. Table of Length.** This table is as follows:

| | | |
|---|---|---|
| A myriameter | = | 10,000 meters |
| A **kilometer** (km.) | = | 1000 meters |
| A hektometer | = | 100 meters |
| A dekameter | = | 10 meters |
| **Meter** (m.) | | |
| A decimeter (dm.) | = | 0.1 of a meter |
| A **centimeter** (cm.) | = | 0.01 of a meter |
| A **millimeter** (mm.) | = | 0.001 of a meter |

The meter is about 39.37 in., $3\frac{1}{4}$ ft., or a little over a yard; the kilometer is about 0.6 of a mile.

The abbreviations in this book are recommended by various scientific associations. Some, however, use Km., Dm., dm., for kilometer, dekameter, and decimeter.

### WRITTEN EXERCISE

*Express as inches, taking 39.37 in. as 1 m.:*

| | | | |
|---|---|---|---|
| **1.** 47 m. | **3.** 52 m. | **5.** 324 cm. | **7.** 4680 mm. |
| **2.** 34.5 m. | **4.** 67.5 m. | **6.** 2.83 cm. | **8.** 3000 mm. |

**9.** Express the diameter of a 7-centimeter gun in inches.

**10.** A certain hill is 203 m. high. Express this in feet.

**11.** A tower is 37.5 m. high. Express this in feet.

**12.** A wheel is 1.3 m. in diameter. Express this in inches.

**13.** The distance from Dieppe (dē-ep′) to Paris is 209 km. Express this in miles.

**14.** The distance between two places in Germany is 178 km. Express this in miles.

**15.** The distance from Paris to Cologne is 306 mi. What is the railway fare at 12 centimes per kilometer?

**16.** The distance from Paris to Brussels is 326 km., and it takes 7 hours to make the trip by railway. What is the average rate in miles per hour?

**193. Table of Square Measure.** This table is as follows:

| | | |
|---|---|---|
| A square myriameter | = | 100,000,000 square meters |
| A **square kilometer** (km.$^2$) | = | 1,000,000 square meters |
| A square hektometer | = | 10,000 square meters |
| A square dekameter | = | 100 square meters |
| **Square meter** (m.$^2$) | | |
| A **square decimeter** (dm.$^2$) | = | 0.01 of a square meter |
| A **square centimeter** (cm.$^2$) | = | 0.0001 of a square meter |
| A square millimeter (mm.$^2$) | = | 0.000001 of a square meter |

The abbreviation sq. m. is often used for m.$^2$, and similarly for sq. cm., etc.

The square dekameter is also called an *are* (är); and since there are 100 square dekameters in 1 hm.$^2$, a square hektometer is called a *hektare* (ha.). The hektare is 2.47 acres, or nearly $2\frac{1}{2}$ acres.

**194. Table of Cubic Measure.** This table is as follows:

| | | |
|---|---|---|
| A cubic myriameter | = | 1,000,000,000,000 cubic meters |
| A cubic kilometer | = | 1,000,000,000 cubic meters |
| A cubic hektometer | = | 1,000,000 cubic meters |
| A cubic dekameter | = | 1000 cubic meters |
| **Cubic meter** (m.$^3$) | | |
| A **cubic decimeter** (dm.$^3$) | = | 0.001 of a cubic meter |
| A **cubic centimeter** (cm.$^3$) | = | 0.000001 of a cubic meter |
| A cubic millimeter (mm.$^3$) | = | 0.000000001 of a cubic meter |

The abbreviation cu. m. is often used for m.$^3$, and similarly for cu. cm., etc.

In measuring wood a cubic meter is called a *stere* (st.).

**WRITTEN EXERCISE**

*Express the following as square centimeters:*

**1.** 750 km.$^2$ **2.** 37 m.$^2$ **3.** 4296 mm.$^2$ **4.** 6.25 m.$^2$

*Express the following as cubic meters:*

**5.** 19.75 cubic dekameters. **7.** 7 st. **9.** 937 cm.$^3$

**6.** 427,653.84 mm.$^3$ **8.** 3.5 st. **10.** 486 dm.$^3$

**195. Table of Weight.** The table of weight is as follows:

| | | |
|---|---|---|
| A **metric ton** (t.) | = | 1,000,000 grams |
| A quintal (q.) | = | 100,000 grams |
| A myriagram | = | 10,000 grams |
| A **kilogram** (kg.) | = | 1000 grams |
| A hektogram | = | 100 grams |
| A dekagram | = | 10 grams |
| **Gram** (g.) | | |
| A decigram | = | 0.1 of a gram |
| A centigram (cg.) | = | 0.01 of a gram |
| A milligram (mg.) | = | 0.001 of a gram |

A kilogram, commonly called a *kilo*, is about $2\frac{1}{5}$ lb. A 5-cent piece weighs 5 g. A metric ton is nearly 2204.6 lb.

**196. Table of Capacity.** The table of capacity is as follows:

| | | |
|---|---|---|
| A **hektoliter** (hl.) | = | 100 liters |
| A dekaliter | = | 10 liters |
| **Liter** (l.) | | |
| A deciliter | = | 0.1 of a liter |
| A **centiliter** (cl.) | = | 0.01 of a liter |
| A milliliter (ml.) | = | 0.001 of a liter |

A liter is the same as 1 $dm.^3$ A liter is about one quart.

### WRITTEN EXERCISE

*Express as kilos:*

| | | | |
|---|---|---|---|
| **1.** 374 lb. | **4.** 352 oz. | **7.** 3275 g. | **10.** 7386 g. |
| **2.** 428 lb. | **5.** 48.4 lb. | **8.** 7275 g. | **11.** 4984 g. |
| **3.** 204 lb. | **6.** 300 T. | **9.** 173.8 lb. | **12.** 6249 g. |

*Express as liters:*

| | | | |
|---|---|---|---|
| **13.** 3 hl. | **16.** 17 hl. | **19.** 5000 ml. | **22.** 15 hl. |
| **14.** 7.5 hl. | **17.** 2.25 hl. | **20.** 2500 ml. | **23.** 724 hl. |
| **15.** 8 pt. | **18.** $12\frac{1}{2}$ gal. | **21.** 500 gal. | **24.** 6.83 hl. |

**197. Letters used in Solving Problems.** Teachers who may wish to use a little algebra in arithmetic will find the material in this section.

For example, I am thinking of a number. When it is multiplied by 2, and 7 is added to the result, the sum is 33. What is the number?

*Solution using a letter*

If I am thinking of $n$, then, letting $2n$ mean $2 \times n$,

$2n$ is twice the number,

$2n + 7$ is 7 added to $2n$,

and $2n + 7 = 33$, as stated in the problem.

Then $2n = 26$, by subtracting 7 from equals.

Therefore $n = 13$, by dividing these equals by 2.

*Check or Proof.* $2 \times 13 + 7 = 33$.

*Solution without using letters*

Because twice the number added to 7 equals 33, therefore if 7 is taken away from 33 there will remain twice the number. Therefore 26 is twice the number. Therefore once the number is half of 26, or 13.

*The solutions compared*

| | |
|---|---|
| $2n + 7 = 33.$ | 33 |
| Subtracting 7, | 7 |
| $2n = 26.$ | $2\overline{)26}$ |
| Therefore $n = 13.$ | 13 |

We therefore see that the two solutions are the same, but that the letters make the reasoning clearer. Therefore in solving a problem by algebra we proceed as follows:

*Write a letter for the number sought.*
*Use this letter in the statement of the problem.*
*This will give an equation, as shown above.*
*Solve this equation.*

**198. Applications of the Equation.** The following are illustrations of some of the applications of the equation:

1. Find what per cent 25 is of 225.

Let $x\%$ = the required rate per cent.

Then $x\% \times 225 = 25.$

Dividing by 225, $x\% = \frac{25}{225}$

$= \frac{1}{9}$, or $11\frac{1}{9}\%$.

2. Find the number of which \$8.40 is 12%.

Let $x$ = the required number.

Then $0.12\,x = \$8.40.$

Dividing by 0.12, $x = \$8.40 \div 0.12$

$= \$70.$

### WRITTEN EXERCISE

*Find what per cent the first number is of the second:*

1. 25, 750.
2. 5 ft., $37\frac{1}{2}$ ft.
3. 528 ft., 2 mi.
4. $1\frac{1}{2}$, $7\frac{1}{2}$.
5. 7 ft., 7 yd.
6. \$4.90, \$29.40.

7. A man's income is \$1650 a year, and he spends \$693. What per cent of his income does he save?

8. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? What per cent are girls?

9. Of what length is 126 ft. 6 in. exactly 50%?

10. A baseball team won 65% of the games played, and won 13 games. How many games did it play?

11. A baseball team lost $33\frac{1}{3}\%$ of the games played, and lost 16 games. How many games did it play?

The use of the algebraic equation is not essential to the study of arithmetic. It simplifies the study of formulas, but only a few of the practical problems of daily life are rendered more simple by its use.

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# ANSWERS

**Page 3.** **1.** 14. **2.** 59. **3.** 80. **4.** 1700. **5.** Two thousand, nine. **6.** Forty thousand, forty. **7.** One hundred twenty-five thousand, one hundred twenty-five. **8.** Seven million, seven thousand, seven. **9.** 152,080½. **10.** 206,400.3. **11.** 440,106. **12.** 999,580. **13.** 1,301,101. **14.** 22,155,002.

**Page 5.** **1.** $19.53. **2.** $12.95. **3.** $34.10. **4.** $24.90. **5.** $29.05. **6.** $21.73. **7.** $17,368.39. **8.** 1692.753. **9.** 294,530. **10.** 1,880,906. **11.** 340,652.2. **12.** 835. **13.** 879. **14.** 767. **15.** 873. **16.** 872. **17.** 33.37. **18.** 53.88. **19.** 17.61. **20.** 530.69. **21.** 805.8. **22.** 12,835. **23.** 16,080. **24.** 167.85. **25.** 494.537. **26.** 68.133.

**Page 7.** **1.** 212. **2.** 309. **3.** 408. **4.** 538. **5.** 608. **6.** 717. **7.** 829. **8.** 892. **9.** 292. **10.** 183. **11.** 262. **12.** 342. **13.** 672. **14.** 782. **15.** 289. **16.** 578. **17.** 685. **18.** 436. **19.** 287. **20.** 377. **21.** 478. **22.** 670. **23.** 557. **24.** 244. **25.** 310. **26.** 401. **27.** 51.2. **28.** 3. **29.** 3.18. **30.** 11.7. **31.** 21.7. **32.** 3.09. **33.** 13.5. **34.** 25.9. **35.** 2.05. **36.** 40.4. **37.** 1.32. **38.** 1.04. **39.** 115. **40.** 20.9. **41.** 2.08. **42.** 8.9. **43.** 1.79. **44.** 2.87. **45.** 17.8. **46.** 18.9. **47.** 17.6. **48.** 1.66. **49.** 172. **50.** 161. **51.** 268. **52.** 213. **53.** 2662. **54.** 3209. **55.** 5108. **56.** 8267. **57.** 2093. **58.** 4492. **59.** 6172. **60.** 4091. **61.** 7093. **62.** 2487. **63.** 5078. **64.** 3057. **65.** 4087. **66.** 6082. **67.** 6185. **68.** 2087. **69.** 1287. **70.** 4895. **71.** 1869. **72.** 14,859. **73.** 9339. **74.** 21,833. **75.** 13,389.

**Page 9.** **1.** 1,469,264. **2.** 1,389,789. **3.** 3,071,880. **4.** 4,015,920. **5.** 5,485,172. **6.** 3,403,944. **7.** 6,195,441. **8.** 39,930,966. **9.** $13,654.75. **10.** $10,174.50. **11.** $13,639.92. **12.** $32,181.32. **13.** 13.75. **14.** 117.36. **15.** 515.43. **16.** 1010.24. **17.** 170.8. **18.** 1708. **19.** 17,080. **20.** 1.708. **21.** 547.4. **22.** 5474. **23.** 54,740. **24.** 5.474. **25.** 2070.6. **26.** 178.7. **27.** 134. **28.** 1.34. **29.** 0.134. **30.** $176. **31.** $43. **32.** $97. **33.** 74.2. **34.** 742. **35.** 74,200. **36.** 742. **37.** 248.2. **38.** 2482. **39.** 24.82. **40.** 0.2482. **41.** $162.80. **42.** $82.80. **43.** $1454.88. **44.** $1286.01. **45.** $985.80. **46.** $1313.64. **47.** $2879.28. **48.** $897.84. **49.** $702. **50.** $800. **51.** 1.284. **52.** 13.3774. **53.** 42.9359. **54.** 67.47. **55.** 345.8. **56.** 3.458. **57.** 345.8. **58.** 347.984. **59.** 345,800. **60.** 1452.36. **61.** 14,523.6. **62.** 3090.36. **63.** 206.64. **64.** 17.36. **65.** 17.36. **66.** 1.736. **67.** 156. **68.** 15.6. **69.** 1.56. **70.** 0.036. **71.** 2.4615. **72.** 21.7761. **73.** 81.68325. **74.** 0.007416. **75.** 0.406525.

**Page 11.** **1.** 56,560; 565,600; 28,280; 141,400; 707,000. 84,480; 844,800; 42,240; 211,200; 1,056,000. 171,840; 1,718,400; 85,920; 429,600; 2,148,000. 150,480; 1,504,800; 75,240; 376,200; 1,881,000. 141,200; 1,412,000; 70,600; 353,000; 1,765,000. **2.** 73,680; 736,800; 36,840; 184,200; 921,000. 87.76; 877.6; 43.88; 219.4; 1097. 118,408; 1,184,080; 59,204; 296,020; 1,480,100. 170.08; 1700.8; 85.04; 425.2; 2126. 507.44; 5074.4; 253.72; 1268.6; 6343. **3.** 49,760; 497,600; 24,880; 124,400; 622,000. 164.24; 1642.4; 82.12; 410.6; 2053. 482,456; 4,824,560; 241,228; 1,206,140; 6,030,700. 150.32; 1503.2; 75.16; 375.8; 1879. 308.32; 3083.2; 154.16; 770.8; 3854. **4.** 565,800; 152,766; 186,714. 2,135,000; 576,450; 704,550. 407,000; 109,890; 134,310. 486,700; 131,409; 160,611. 360,100; 97,227; 118,833. **5.** 4717; 1273.59; 1556.61. 4557; 1230.39; 1503.81. 53,340; 14,401.8; 17,602.2. 5135; 1386.45; 1694.55. 3327.3; 898.371; 1098.009. **6.** 6012; 1623.24; 1983.96. 5342; 1442.34; 1762.86. 53,140; 14,347.8; 17,536.2. 572.1; 154.467; 188,793. 3320.1; 896.427; 1095.633. **7.** 5690; 2845. 8408; 4204. 41,932; 20,966. 198.28; 99.14. 38.884; 19.442. **8.** 23,294; 11,647. 21,874; 10,937. 48,572; 24,286. 283.84; 141.92. 23.688; 11.844.

**Page 12.** **1.** $2.50. **2.** $82,510. **3.** $62.50. **4.** $55. **5.** $37. **6.** $287.50. **7.** $250. **8.** $6375. **9.** $3,211,200. **10.** $37,610. **11.** $43,077.50. **12.** $3600. **13.** $16,000. **14.** $8125. **15.** $16,875. **16.** $637.50. **17.** $1881.25. **18.** $80,000. **19.** $61,000. **20.** $80,000. **21.** $800,000. **22.** $350,000. **23.** 1200. **24.** 1600. **25.** 4800. **26.** $10,410. **27.** 210,500. **28.** $10,605. **29.** 238,324. **30.** $1,053,250. **31.** $10,560,000. **32.** $105,600. **33.** 80,000,000. **34.** $30. **35.** $2900. **36.** $2700. **37.** $62.50. **38.** $310. **39.** $4.50. **40.** $12. **41.** $1200. **42.** $71,320. **43.** $900; $1200. **44.** 300 lb.; 600 lb.

**Page 14.** **1.** 84. **2.** 21. **3.** 31. **4.** 121. **5.** 270. **6.** 446. **7.** 248. **8.** 246. **9.** 302 rem. 198. **10.** 256 rem. 12. **11.** 363. **12.** 475. **13.** 4380. **14.** 1006. **15.** 2120. **16.** 313. **17.** 525. **18.** 730. **19.** 934. **20.** 2025. **21.** 4268. **22.** $2.03. **23.** $1.35. **24.** $3.21. **25.** 1095 rem. 2. **26.** 760 rem. 53. **27.** 1154 rem. 11. **28.** 1007 rem. 29. **29.** 882 rem. 73. **30.** 166 rem. 124. **31.** 58 rem. 725. **32.** 186 rem. 258. **33.** 79 rem. 245. **34.** 994 rem. 95. **35.** 561 rem. 301. **36.** 938 rem. 153. **37.** 1236 rem. 241. **38.** 1526 rem. 154. **39.** 842 rem. 78. **40.** 1846 rem. 50. **41.** 1543 rem. 68. **42.** 709 rem. 217.

**Page 16.** **1.** 3.1. **2.** 31. **3.** 3.1. **4.** 1.7. **5.** 0.17. **6.** 17. **7.** 1.7. **8.** 2.3. **9.** 23. **10.** 2.3. **11.** 2.1. **12.** 21. **13.** 2.1. **14.** 1000. **15.** 10,000. **16.** 10,000. **17.** 2090. **18.** 20.9. **19.** 2090. **20.** 50. **21.** 0.5. **22.** 7.6. **23.** 76. **24.** 0.76. **25.** 340. **26.** 0.34. **27.** 1.7. **28.** 17. **29.** 17. **30.** 8.2. **31.** 8.2. **32.** 9100. **33.** 51. **34.** 6. **35.** 980. **36.** 78. **37.** 7.1. **38.** 81. **39.** 65. **40.** 91. **41.** 0.7. **42.** 21. **43.** 1.2. **44.** 31. **45.** 0.31. **46.** 9.1. **47.** 0.39. **48.** 5. **49.** 182. **50.** 329 rem. 0.12. **51.** 3.07. **52.** 3.1. **53.** 513. **54.** 513. **55.** 0.513.

**56.** 0.51 rem. 2. **57.** 5.1 rem. 0.002. **58.** 1.1. **59.** 11. **60.** 1.85. **61.** 1076. **62.** 10.76. **63.** 2000. **64.** 2.03. **65.** 2.03. **66.** 24,500. **67.** 800. **68.** 174. **69.** 2205. **70.** 2.16. **71.** 216. **72.** 7.059. **73.** 4652. **74.** 14.118. **75.** 35.295.

**Page 17.** **1.** 1589. **2.** 155. **3.** 634. **4.** 290.28. **5.** 27. **6.** 142.29. **7.** 224.34. **8.** 250.41. **9.** 637.52. **10.** 477. **11.** 238.68. **12.** 107.52. **13.** 750. **14.** 146.19. **15.** 540. **16.** 25; $75; $13,459; $115,053. **17.** 26; $7; 3950; $19,280. **18.** $2001; $3003; $2212.20; $3621. **19.** 65; 169; 255; 70. **20.** 51; 168; 276; 375. **21.** 3,618,769; 18,093,845.

**Page 18.** **1.** $49.99. **2.** $41.52. **3.** $48.68. **4.** $45.26. **5.** $438.76. **6.** $4.84. **7.** $1.75. **8.** $1.49. **9.** $0.53. **10.** $32.88. **11.** 16.52. **12.** 73.72. **13.** 2.88. **14.** 82.56. **15.** 14.574. **16.** 28.968. **17.** 0.44448. **18.** 15.48084. **19.** 755.6748. **20.** 16.777. **21.** 1.169. **22.** 0.133. **23.** 3.802. **24.** 12.278. **25.** 2419.599.

**Page 19.** **1.** 1.1 mi. **2.** 1.6 T. **3.** 1500 lb. **4.** 3300; 1100. **5.** 2.1. **6.** 14,256. **7.** 1.1. **8.** 2288. **9.** $280\frac{1}{2}$; $93\frac{1}{2}$. **10.** 51; $3\frac{1}{11}$. **11.** 80; 10. **12.** 71; 568. **13.** 9 mi. 268 ft. **14.** 128 ft. 2 in.

**Page 20.** **1.** 1320. **2.** 440. **3.** 20. **4.** $\frac{1}{4}$. **5.** $\frac{3}{4}$. **6.** 1089 ft. **7.** 15.

**Page 21.** **1.** $\frac{7}{8}$. **2.** $\frac{7}{16}$. **3.** $\frac{9}{16}$. **4.** $1\frac{3}{16}$. **5.** $1\frac{9}{16}$. **6.** $1\frac{5}{16}$. **7.** $1\frac{3}{16}$. **8.** $1\frac{5}{16}$. **9.** $1\frac{9}{16}$. **10.** $1\frac{1}{16}$. **11.** $\frac{23}{32}$. **12.** $\frac{31}{32}$. **13.** $1\frac{1}{32}$. **14.** $1\frac{23}{32}$. **15.** $1\frac{17}{32}$. **16.** $1\frac{1}{24}$. **17.** $1\frac{11}{24}$. **18.** $\frac{5}{8}$. **19.** $1\frac{5}{8}$. **20.** $\frac{21}{64}$. **21.** $2\frac{19}{20}$. **22.** $3\frac{1}{2}$. **23.** $14\frac{1}{2}$. **24.** $15\frac{5}{12}$. **25.** $9\frac{1}{24}$. **26.** $11\frac{17}{24}$. **27.** $18\frac{7}{24}$. **28.** $16\frac{13}{24}$. **29.** $9\frac{1}{6}$. **30.** $10\frac{11}{16}$. **31.** $92\frac{13}{30}$. **32.** $138\frac{13}{16}$. **33.** $168\frac{1}{10}$. **34.** $142\frac{2}{15}$. **35.** $128\frac{2}{3}$. **36.** $122\frac{49}{60}$. **37.** $113\frac{1}{30}$. **38.** $69\frac{49}{60}$. **39.** $16\frac{3}{5}$. **40.** $16\frac{5}{16}$. **41.** $12\frac{29}{30}$. **42.** $14\frac{27}{32}$. **43.** $12\frac{13}{24}$. **44.** 14. **45.** $15\frac{21}{64}$. **46.** $38\frac{5}{12}$. **47.** $\frac{5}{8}$. **48.** $\frac{1}{8}$. **49.** $\frac{7}{16}$. **50.** $\frac{9}{16}$. **51.** $\frac{11}{32}$. **52.** $\frac{23}{32}$. **53.** $\frac{27}{64}$. **54.** $27\frac{1}{8}$. **55.** $57\frac{1}{4}$. **56.** $63\frac{1}{10}$. **57.** $44\frac{25}{32}$. **58.** $61\frac{13}{16}$. **59.** $64\frac{11}{24}$. **60.** $70\frac{43}{48}$. **61.** $2\frac{1}{16}$. **62.** $5\frac{3}{16}$. **63.** $2\frac{1}{16}$. **64.** $4\frac{1}{16}$. **65.** $3\frac{7}{32}$. **66.** $3\frac{17}{32}$. **67.** $14\frac{15}{16}$.

**Page 22.** **1.** $3\frac{11}{16}$ in. **2.** $95.25. **3.** 68. **4.** $\frac{7}{12}$. **5.** $\frac{3}{32}$. **6.** $\frac{1}{2}$. **7.** $\frac{2}{15}$. **8.** $\frac{8}{15}$. **9.** $1\frac{7}{9}$. **10.** $\frac{9}{16}$. **11.** $\frac{8}{35}$. **12.** $4\frac{3}{8}$. **13.** $2\frac{1}{3}$. **14.** $172\frac{31}{32}$. **15.** $7\frac{1}{10}$.

**Page 23.** **1.** $\frac{3}{4}$. **2.** $1\frac{1}{4}$. **3.** $\frac{5}{8}$. **4.** $\frac{7}{8}$. **5.** $1\frac{1}{8}$. **6.** $1\frac{3}{8}$. **7.** $\frac{3}{8}$. **8.** $1\frac{1}{8}$. **9.** $1\frac{1}{8}$. **10.** $1\frac{3}{8}$. **11.** $\frac{1}{4}$. **12.** $\frac{1}{4}$. **13.** $\frac{3}{8}$. **14.** $\frac{1}{8}$. **15.** $\frac{3}{8}$. **16.** $\frac{1}{8}$. **17.** $\frac{1}{8}$. **18.** $\frac{5}{8}$. **19.** $\frac{3}{8}$. **20.** $\frac{1}{8}$. **21.** $\frac{1}{8}$. **22.** $\frac{3}{8}$. **23.** $\frac{1}{16}$. **24.** $\frac{3}{16}$. **25.** $\frac{5}{16}$. **26.** $\frac{7}{16}$. **27.** $\frac{1}{32}$. **28.** $\frac{21}{32}$. **29.** $\frac{9}{32}$. **30.** $\frac{15}{32}$. **31.** 2. **32.** $\frac{1}{2}$. **33.** $1\frac{1}{8}$. **34.** $\frac{5}{8}$. **35.** $\frac{6}{7}$. **36.** $1\frac{1}{6}$. **37.** $\frac{8}{9}$. **38.** $1\frac{1}{8}$. **39.** $\frac{16}{21}$. **40.** $\frac{9}{10}$. **41.** 137. **42.** 262. **43.** 279. **44.** 243. **45.** 252. **46.** 240. **47.** 14. **48.** 16. **49.** 29. **50.** 31. **51.** 60. **52.** 40. **53.** 154. **54.** 304. **55.** $83\frac{1}{3}$. **56.** 2196. **57.** $8\frac{1}{3}$. **58.** $20\frac{1}{4}$. **59.** 7. **60.** 5. **61.** 8. **62.** $\frac{23}{35}$.

**Page 24.** **1.** $2052.76. **2.** Cows; $5. **3.** 5278 lb. **4.** $585. **5.** 17,231 mi. **6.** $101\frac{1}{2}$. **7.** $141.12. **8.** 43,200,000 mi. **9.** 33. **10.** 72. **11.** 2.03. **12.** 1650 bu. **13.** 51,000. **14.** 0.9375. **15.** 11. **16.** 527,796 ft.; 146.61 ft. **17.** 5. **18.** $4\frac{5}{12}$. **20.** $212\frac{1}{2}$. **21.** $195. **22.** 84.7 bu.; 3388 bu.; $154. **23.** 1,440,000. **24.** $28.80. **25.** $70; $35. **26.** $39.60. **27.** $385. **28.** $37\frac{5}{7}$ in. **29.** 907.5.

**Page 29.** 1. 90%. 2. \$912. 3. \$505.44. 4. \$28.30. 5. $3\frac{2}{6}\frac{3}{3}$ in. 6. 115 lb. 7. 914 lb. 8. 576 sq. in. 9. $23\frac{1}{5}$. 10. \$201.60. 11. $56\frac{8}{9}$ in. 12. \$20.93. 13. 3825. 14. \$105.12. 15. 468 lb. 16. 484. 17. 962.5; 262.5; 25. 18. 108; 21; 18; 3. 19. 120. 20. \$145.80. 21. $25\frac{5}{7}$. 22. 6%; 5%; 7%. 23. $\frac{28}{183}$. 24. $66\frac{5}{7}°$. 25. $29.79\frac{5}{7}$ in. 26. 10 lb.; 50 lb. 27. $4\frac{1}{2}$ hr.

**Page 32.** 1. \$518.42. 2. 16; 170; 1860. 3. \$7.77; \$4.41; \$5.18; \$9.03; \$18.06. 4. \$0.56; \$0.91; \$1.16; \$1.59. 5. 12,300; 11,300; 77,000. 6. 7840 ft. 7. 1948 lb.; 3409 lb.; 7792 lb.; 12,175 lb. 8. 24 ft. 9. 30 ft. 10. 54 ft.

**Page 33.** 1. \$640; \$1920. 2. \$26.25. 3. \$4875. 4. 350. 5. \$4200. 6. \$42,000; \$10,500. 7. \$7.

**Page 34.** 1. \$362,700. 2. $2\frac{13}{16}$. 3. $3\frac{2}{11}$. 4. 40,040. 5. \$13.20. 6. 124.8 lb. 7. \$1704. 8. \$6.30. 9. 213.6 mi. 10. 122. 11. 284. 12. \$960. 13. 22,815, carbon; 696, ash. 14. 91.4 lb.; $91\frac{2}{5}$%. 15. 275.

**Page 37.** 1. \$159.51. 2. \$217.29. 3. \$203.59. 4. \$136.69. 5. \$410. 11. \$183.15. 12. \$351. 13. $126\frac{2}{6}\frac{1}{4}$.

**Page 39.** 3. 171 sq. in. 4. 342 sq. in. 5. $46\frac{1}{2}$ in.

**Page 40.** 1. 2437. 2. 255,000,000. 3. 1750 cu. ft. 4. 8. 5. 1.44 meters. 6. 3 ft. $11\frac{1}{12}$ in. 7. \$761,400.

**Page 43.** 1. \$122.50. 2. 27. 3. 24. 4. 27. 5. 200 lb., 25 lb. 6. 60 cu. ft.; 103,680 cu. in.; 65 cu. ft.; 112,320 cu. in. 7. $54\frac{1}{8}$. 8. 800. 9. $19\frac{1}{3}$. 10. 13,858. 11. 4056.25. 12. 1082.5. 13. 6084.375. 14. 2706.25. 15. 9731.25. 16. 4330. 17. 4461.875. 18. 2706.25. 19. 1623.75. 20. 2165.

**Page 45.** 1. 1024. 2. 512. 3. 576. 4. 1680. 5. 640. 6. \$14.28.

**Page 46.** 1. 737 bd. ft. 2. 2632 bd. ft. 3. 3602 bd. ft. 5. \$481.26. 6. \$279.26. 7. \$1291.70. 8. \$71.68.

**Page 48.** 1. 96. 2. 125. 3. 234. 4. $220\frac{1}{2}$. 5. 400. 6. 405. 7. 1183. 8. $318\frac{1}{2}$. 9. 352.

**Page 49.** 1. $93\frac{5}{9}$. 2. $852\frac{8}{9}$. 3. \$44.20. 4. \$667.80.

**Page 50.** 1. $32\frac{5}{6}$; 6 in. 2. $55\frac{1}{12}$. 3. $51\frac{1}{2}$. 4. 26. 5. \$51.20. 6. \$51.20. 7. \$33.07. 8. \$58.53. 9. \$39.67. 10. \$19.07. 11. \$72.20. 12. \$115.20.

**Page 51.** 1. \$7.20. 2. 21; \$7.35. 3. \$4.50.

**Page 52.** 1. 3200; 4800. 2. $\frac{500}{1089}$ A.; $\frac{80}{1089}$ A.; $\frac{40}{363}$ A.; $\frac{100}{363}$ A. 3. 935 sq. ft. 4. 4675 cu. ft. 5. $5921\frac{2}{3}$ cu. ft. 6. 780; $47\frac{3}{11}$; 260. 7. 144; 54 in each; 108 in each. 8. 8865.

**Page 53.** 2. $365\frac{5}{8}$ sq. in. 3. $251\frac{7}{8}$ sq. in. 9. $\frac{1}{8}$. 10. 4 ft. 1 in. 11. $2\frac{1}{2}$; $\frac{5}{21}$.

**Page 56.** 1. 9 ft. 11 in. 2. 2 ft. $7\frac{1}{2}$ in. 3. $5\frac{5}{8}$. 4. 3 ft. $3\frac{1}{8}$ in. 5. $\frac{43}{48}$; 0.896. 6. \$0.68.

**Page 57.** **1.** $187\frac{1}{2}$. **2.** 216. **3.** $191\frac{1}{4}$. **4.** $162\frac{1}{2}$. **5.** 1080.

**Page 58.** **1.** 14,148; \$594.22; \$112.20. **2.** \$53.40. **3.** \$68.75. **4.** \$231.25. **5.** $22\frac{6}{7}$; \$67.50.

**Page 61.** **1.** \$80, \$20; \$70, \$30; \$10, \$90. **2.** 33 : 40. **3.** 18 clear days, 12 cloudy days. **4.** \$9; \$6; \$7; \$10.50.

**Page 63.** **1.** 36. **2.** 18. **3.** 82. **4.** 175. **5.** 4. **6.** 56. **7.** 160. **8.** 455. **9.** 114. **10.** 13. **11.** $22.68\frac{6}{7}$. **12.** 4.135. **13.** $15.78\frac{2}{11}$. **14.** 0.24. **15.** $2.003\frac{9}{11}$. **16.** $1.281\frac{7}{9}$. **17.** $2.37\frac{1}{107}$. **18.** $77\frac{3}{8}$. **19.** 1.5. **20.** 3120. **21.** 5. **22.** 46. **23.** 107.1 in. **24.** 5. **25.** 4 hr. **26.** 4. **27.** $21\frac{3}{5}$ sec. **28.** 12 min. 40 sec. **29.** 75 mi. **30.** 3. **31.** 5. **32.** 10 min. 18.75 sec. **33.** 29 min. $59\frac{1}{9}$ sec. **34.** \$26.67; \$33.33. **35.** 409.6; 3686.4. **36.** \$14.70. **37.** \$4.

**Page 66.** **1.** 29 ft. **2.** 43 ft. 2 in. **3.** 55 ft.; 51 ft. **4.** 66 ft. **5.** 40 ft. **6.** $101\frac{1}{2}$ ft. **7.** 3. **8.** $12\frac{1}{2}$ ft. **10.** \$25.06; \$54.94. **11.** 2500. **12.** $1\frac{7}{10}$. **13.** 50 ft. **14.** 160 ft. **15.** 27 ft. **16.** 37 ft. **17.** 27 ft. **18.** 38 ft. 6 in. **19.** $33\frac{1}{3}$ ft. **20.** \$5.50. **21.** 4 ft. **22.** $8\frac{1}{3}$ lb. **23.** \$224; \$288; \$336. \$266; \$342; \$399.

**Page 71.** **1.** \$51.37. **2.** 54 ft. 1 in. **3.** 53 lb. 5 oz. **4.** 52 yd. 1 in. **5.** \$13.92. **6.** 13 ft. 4 in. **7.** 13 lb. 8 oz. **8.** 13 yd. 28 in. **9.** \$565.88. **10.** 579 ft. 8 in. **11.** 575 lb. 12 oz. **12.** 569 yd. 8 in. **13.** \$2.90. **14.** 2 lb. 15 oz. **15.** 2 ft. 11.44 in. **16.** 2 pk. 7.89 qt. **17.** 2 yd. 32.78 in. **18.** 2 mi. 285.22 rd. **19.** 2 sq. yd. 1152.78 sq. in. **20.** 2 cu. ft. 1536.78 cu. in.

**Page 73.** **1.** Cement, 10; sand, 20; stone, 40. **2.** 20.88 bbl. cement, 5.868 cu. yd. sand, 11.736 cu. yd. stone or gravel. **3.** 54.23 bbl. cement, 15.2405 cu. yd. sand, 30.481 cu. yd. stone or gravel. **4.** 45 bags cement, $4\frac{1}{2}$ cu. yd. sand, 9 cu. yd. stone or gravel; \$32.40. **5.** \$102. **6.** \$156.

**Page 74.** **1.** 1080. **2.** 1800. **3.** 10,560. **4.** $\frac{3}{22}$.

**Page 76.** **1.** No. **2.** 3; 3; 3.2. **3.** 160; 160; 222. **4.** 864; 864; 2640. **5.** 6330; 6330; 1254. **6.** 1080; 1080; 1245. **7.** $\frac{1}{8}$; 8. **8.** $\frac{3}{8}$; 270. **9.** 70; 1400. **10.** 22; 1224.

**Page 77.** **1.** 0.00125; $\frac{1}{800}$. **2.** 0.00875; $\frac{7}{800}$. **3.** $3.33\frac{1}{3}$; $\frac{10}{3}$. **4.** \$45; \$45. **5.** \$812.50; \$812.50; \$812.50. **6.** \$1150; \$1150. **7.** 0.50. **8.** 0.75. **9.** 0.125. **10.** $0.66\frac{2}{3}$. **11.** 0.005. **12.** 0.007. **13.** 2. **14.** 1.50. **15.** 1.25. **16.** 10. **17.** 0.00005. **18.** 0.00065.

**Page 78.** **1.** $12\frac{1}{2}\%$; $3\frac{1}{8}\%$. **2.** $12\frac{1}{2}\%$; $3\frac{1}{8}\%$. **3.** 80%. **4.** 520%. **5.** 37%. **6.** 62%. **7.** $22\frac{1}{2}\%$. **8.** 45%. **9.** $333\frac{1}{3}\%$. **10.** $\frac{1}{8}\%$.

**Page 79.** **1.** $8\frac{1}{3}\%$; $2\frac{7}{8}\%$. **2.** 50%; $12\frac{1}{2}\%$. **3.** 75%; 50%; $62\frac{1}{2}\%$. **4.** $6\frac{1}{4}\%$; $12\frac{1}{2}\%$; 20%. **5.** $43\frac{3}{4}\%$; $46\frac{7}{8}\%$; $68\frac{3}{4}\%$. **6.** $6\frac{2}{3}\%$; $18\frac{2}{11}\%$; 100%. **7.** $83\frac{1}{3}\%$. **8.** $11\frac{1}{9}\%$.

**Page 80.** 1. $137. 2. $93. 3. $64. 4. $211. 5. $6.66. 6. $7.45. 7. $30.17. 8. $25.72. 9. $53.38. 10. $59.25. 11. 81. 12. 132. 13. 108. 14. 140.

**Page 81.** 1. 16.5. 2. 30.1. 3. 8.64. 4. 31.98. 5. 57.66. 6. 0.884. 7. 2.881. 8. 2.067. 9. 0.196. 10. 0.2205. 11. 0.231. 12. 0.0285. 13. 0.1292. 14. 1743.7. 15. 4.4784. 16. 4836 yd. 17. $1202. 18. $1520.50. 19. 3017 yd. 20. 4744. 21. 30,940. 22. $2301. 23. $9390. 24. $18,625. 25. 1185 cu. ft. 26. 972 cu. yd. 27. 1360 sq. ft. 28. $10.50. 29. $350. 30. $2,980,800. 31. $348. 32. 14.625 lb. 33. 12.92 lb.; $3.36. 34. Second; 0.46¢. 35. $155.25. 36. $17.93.

**Page 86.** 1. 5%; $3\frac{1}{3}$%. 2. 3%; 20%. 3. 4%; 5%. 4. $6\frac{1}{4}$%; $16\frac{2}{3}$%. 5. $33\frac{1}{3}$%; $6\frac{2}{3}\frac{2}{3}$%. 6. 25%; 1%. 7. $266\frac{2}{3}$%; 150%. 8. $33\frac{1}{3}$%; 20%; 10%. 9. 50%; $33\frac{1}{3}$%; 25%. 10. 50%; 25%; $16\frac{2}{3}$%. 11. 30%. 12. $7\frac{1}{7}$%. 13. $20\frac{5}{6}$%. 14. 26%. 15. 75%. 16. 58%. 17. 48%; 52%. 18. 12%. 19. 75%. 20. $66\frac{2}{3}$%; 50%. 21. $6\frac{1}{4}$%; $12\frac{1}{2}$%. 22. 30%; 70%; 100%.

**Page 88.** 1. 91,375. 2. 17,200. 3. 87. 4. 288. 5. $16.64. 6. 140. 7. Yes. 8. 800. 9. 800. 10. 800. 11. 1700. 12. 1400. 13. 1300. 14. 1600. 15. 3700. 16. 5600. 17. 776. 18. 2670. 19. 186. 20. 416. 21. 200. 22. 168. 23. 63.09. 24. 74.88. 25. 170. 26. 82. 27. 62. 28. 11,320. 29. 5740. 30. $21; $15.75. 31. $39; $62.40. 32. 56 ft. 8 in.; 249 ft. 4 in. 33. 181 lb. 4 oz.; 3171 lb. 14 oz. 34. $190. 35. 220. 36. 120%; $150. 37. $2110. 38. 145.

**Page 91.** 1. $12\frac{1}{2}$%. 2. 25%; 20%; 25%. 3. 8%; 92%. 4. 132. 5. 371. 6. 19.18 mi. 7. $0.8\frac{3}{12}\frac{2}{1}$%. 8. $1512. 9. $270. 10. 25%. 11. $28\frac{4}{7}$%. 12. $6\frac{1}{4}$%. 13. 25%. 14. $78\frac{2}{3}\frac{6}{3}$%. 15. $60\frac{5}{1\,7}$%. 16. $23\frac{1}{5}\frac{6}{9}\frac{9}{7}$%; $26\frac{3}{5}\frac{8}{4}\frac{2}{3}$%. 17. 3%. 18. $15\frac{1}{5}$%. 19. $19\frac{3}{8}\frac{5}{0}\frac{4}{9}$%; $11\frac{1}{1}\frac{1}{1}\frac{3}{7}$%. 20. 25%. 21. $0.11\frac{1}{2}\frac{0}{1}$%. 22. 48¢. 23. $210. 24. $27. 25. $9; $8. 26. $20.40; $17. 27. 500. 28. 500. 29. 120; 180. 30. 120; 200. 31. 70; 130. 32. 88; $113\frac{1}{7}$. 33. 300; 420. 34. 400; 880. 35. 5.46; 10.92. 36. 10.56; 31.68. 37. 240; 1040. 38. 26.4; 396. 39. 20.5; 184.5. 40. 23.4; 32.76. 41. 61,000. 42. $175. 43. $165. 44. 1700. 45. 2400. 46. 55 lb. 47. 7134.

**Page 96.** 1. $380. 2. $16.50. 3. $15.75. 4. $13.20. 5. $22.20. 6. $48.30. 7. $64.95. 8. $36.45. 9. $48.20. 10. $25.80. 11. $36.25. 12. $72. 13. $64.40. 14. $13.85. 15. $31.50. 16. $27.80. 17. $52.20.

**Page 97.** 1. $112. 2. $432.25. 3. $318.75. 4. $431.20. 5. $526.50. 6. $252. 7. $229.50. 8. $222.75. 9. $140. 10. $450. 11. $50. 12. No difference.

**Page 98.** 1. $1.34. 2. $5.67. 3. $5.43; $1.17. 4. $5.87; $8.80. 5. $1.03; $0.20. 6. $6.43. 7. $3.74; $11.22.

**Page 99.** 1. $84.67. 2. $1606.50. 3 $249.98. 4. $97.43. 5. $1444.27. 6. $1426.96.

EA

**Page 102.** 1. \$191.50. 2. \$254. 3. \$253.64. 4. \$94.40. 5. \$325.20. 6. \$233.21. 7. \$547.26. 8. \$1311.28.

**Page 104.** 1. \$17.50. 2. \$28.13. 3. \$45. 4. \$51.75. 5. \$59.38. 6. \$97.50. 7. \$165.75. 8. \$157.50. 9. \$840. 10. \$405. 11. \$1035. 12. \$3327.50. 13. \$5468.75. 14. No difference.

**Page 105.** 1. \$27.31. 2. \$24.75; \$20.63. 3. \$9.79; \$10.77. 4. \$100.94; \$84.11; \$67.29. 5. \$88.40; \$99.45; \$121.55. 6. \$6. 7. \$22.05. 8. \$57.89. 9. \$44.16. 10. \$28.37. 11. \$9.22. 12. \$37.21. 13. \$18.90. 14. \$45.14. 15. \$34.08. 16. \$106.40. 17. \$78.31. 18. \$36.43. 19. \$623.19. 20. \$19.47. 21. \$48.67. 22. \$73.78. 23. \$234.21. 24. \$103.42. 25. \$127.50. 26. \$68.75. 27. \$87.50. 28. \$54. 29. \$132. 30. \$78.75. 31. \$216. 32. \$37.50. 33. \$85. 34. \$48.13; \$57.75. \$51.56; \$61.88. \$63.02; \$75.63. 35. \$48.51; \$58.21. \$52.14; \$62.56. \$63.78; \$76.54. 36. \$24.44; \$29.33. \$244.44; \$293.33. \$2.44; \$2.93. \$30.56; \$36.67. \$305.56; \$366.67. 37. \$48.89; \$58.67. \$488.89; \$586.67. \$4.89; \$5.87. \$58.06; \$69.67. \$580.56; \$696.67. 38. \$30.75; \$38.44; \$46.13; \$26.91; \$34.59; \$42.28. 39. \$11.25. 40. \$372.75. 41. \$546.75. 42. \$1715.08. 43. \$2200.50. 44. \$2616.25.

**Page 109.** 1. \$393.75. 2. \$337.28. 3. \$104.58. 4. \$651.97. 5. \$191.96. 6. \$178.85.

**Page 111.** 1. Four hundred and seventy-five thousandths; four hundred seventy-five thousandths. 2. LXXIX; LXIV. 3. 17,000,000. 4. \$38.50. 5. 33 ft. 2 in. 6. \$207.65. 7. \$276.86. 8. \$236.79. 9. \$278.38. 10. \$113,263. 11. \$357,719.25. 12. 14.57. 13. 15.96. 14. 116.298. 15. 35.02. 16. \$52. 17. \$34.13. 18. \$27.48. 19. $\frac{7}{15}$. 20. $\frac{7}{15}$. 21. 24. 22. 37. 23. 1. 24. 2.

**Page 113.** 1. $3\frac{1}{2}$¢. 2. $4\frac{1}{2}$¢; \$16,43. 3. \$24.96. 4. \$29.20; \$70.59. 5. 75,862. 6. \$913.26; \$1286.74. 7. \$2511. 8. \$2069.16; 3.8¢. 9. 3.1¢. 10. 29.03%. 11. $1306\frac{2}{3}$.

**Page 116.** 1. \$0.85. 2. \$1. 3. \$2.45. 4. \$7.59. 5. \$3.55. 6. \$1.41. 7. \$41.67.

**Page 117.** 1. 24. 2. 23 in. 3. $36\frac{3}{8}$ in. 4. \$100. 5. $1\frac{1}{3}$. 6. \$0.20; \$0.20. 7. 25%. 8. $90\frac{5}{13}$%. 9. $19\frac{3}{13}$%; 520%.

**Page 118.** 1. 4 qt. 2. 25%; yes. 3. $5\frac{5}{7}$%. 4. $33\frac{1}{3}$%; 25%. 5. 5 qt. 6. 2 gal.

**Page 119.** 1. \$0.28. 2. \$0.36. 3. \$0.06. 4. \$1.62. 5. \$0.38. 6. \$0.24. 7. \$0.10. 8. \$1.14. 9. 500; 450.

**Page 120.** 1. No; \$0.40. 2. \$1.40; $26\frac{2}{3}$%; yes. 3. \$901.65.

**Page 121.** 1. 20; 34 in. 2. $1\frac{1}{4}$ yd.; 45 in. 3. 24. 4. 21. 5. 18. 6. $16\frac{2}{3}$%. 7. 52. 8. \$0.05$\frac{7}{9}$ cheaper by the piece.

**Page 122.** 1. \$18.92. 2. \$15.61. 3. 2. 4. $43\frac{1}{3}$. 5. 6. 6. \$2.23. 7. $\frac{14}{15}$; $0.93\frac{1}{3}$. 8. $2\frac{2}{3}$ yd. 9. \$1.97.

**Page 123.** 1. 10. 2. 4935. 3. 54 T. 4. $77\frac{8}{3}$. 5. 4 hr. 25 min. $50\frac{0}{1}$ sec. 6. \$15. 7. $12\frac{1}{2}$. 8. $11\frac{1}{2}$ T.

**Page 124.** 1. $33\frac{1}{3}\%$. 2. $101\frac{1}{3}\frac{8}{1}\%$. 3. $125\frac{5}{7}\%$. 4. 100%.

**Page 125.** 1. 84. 2. 4312. 3. $107\frac{4}{5}$. 4. $282\frac{1}{3}$ cu. ft.

**Page 126.** 1. \$2074; \$979.61. 2. 165.2%; 108.5%; 190.6%; 106.7%; 92.3%. 3. \$441.85; \$332.09; \$73.02; \$76.75; \$5.47; \$27.12; \$21.03; \$2.67; \$109.76; \$1632.15; \$647.52.

**Page 127.** 1. \$64.80. 2. 47; $43\frac{3}{4}$; 45; $37\frac{3}{4}$; $46\frac{1}{2}$; $22\frac{1}{4}$. $43\frac{1}{2}$; $41\frac{3}{4}$; $39\frac{1}{2}$; $42\frac{1}{4}$; $32\frac{3}{4}$; $242\frac{1}{4}$. \$21.75; \$20.88; \$21.73; \$23.24; \$9.01; \$117.86. 3. \$28.44; \$25.88; \$21.95; \$31.67; \$32.76. 4. \$30.38; \$28.35; \$27.13; \$25.79; \$20.52. 5. \$30; \$28.05; \$27.41; \$28.37; \$30.40.

**Page 128.** 1. \$264. 2. \$150. 3. \$265; \$1 more than annual rent. 4. 30%; \$900. 5. 10%. 6. $5\frac{1}{1}\frac{1}{2}\frac{0}{0}\%$.

**Page 129.** 1. \$105. 2. 4500; 25,500. 3. $\$0.49\frac{1}{2}$; \$0.05. 4. \$159. 5. The same.

**Page 131.** 1. 0.505; 500.005. 2. 44. 3. \$625.88; \$228.76. 4. \$1619.57; \$244.45. 5. \$1462.91; \$277.55. 6. \$948.76; \$851.26. 7. \$27,545.70. 8. \$42,461.94. 9. \$77,467.46. 10. 355 ft. 11. 54; 27; 1080. 12. 2.7; 27; 0.27. 13. 7 ft. 4 in. 14. 7 lb. 2 oz. 15. 0.5; 50%. 16. 0.25; 25%. 17. 0.75; 75%. 18. 0.375; $37\frac{1}{2}\%$. 19. 0.625; $62\frac{1}{2}\%$. 20. 0.875; $87\frac{1}{2}\%$. 21. $0.66\frac{2}{3}$; $66\frac{2}{3}\%$. 22. 0.6; 60%. 23. $0.83\frac{1}{3}$; $83\frac{1}{3}\%$. 24. 0.4375; $43\frac{3}{4}\%$.

**Page 133.** 1. 251.23 lb. 2. 56%; 24%; 13%; 7%. 3. 6054.4. 4. \$1684.80. 5. 1700 sq. ft.

**Page 134.** 1. \$1.40 gain. 2. \$8160. 3. 608; \$42,560. 4. \$3921.16. 5. 705,600. 6. 6,350,400. 7. 7200. 8. 22,464.

**Page 135.** 1. \$91.25. 2. \$465. 3. \$375. 4. \$231. 5. \$168.30. 6. \$911.10. 7. \$107.18. 8. \$1852.50. 9. \$73.20. 10. \$167.

**Page 139.** 1. \$3000; \$3038.76. 2. \$4720; \$4764.06. 3. \$3920; \$3937.02. 4. \$2520; \$2532.40. 5. \$3080; \$3095.15. 6. \$1933.75; \$1940.25. 7. \$713.07; \$717.77. 8. \$12,400; \$12,653.19. 9. \$502.09; \$507.42. 10. \$312.69; \$314.40. 11. \$225.23. 12. \$324.73. 13. \$731.59. 14. \$765.41. 15. \$900.94. 16. \$541.21. 17. \$1093.08. 18. \$1126.16. 19. \$2143.72. 20. \$3152.83. 21. \$9.48 more at simple interest. 22. \$35.16. 23. \$1380.85.

**Page 142.** 1. \$509. 2. \$1176.92. 3. \$524.23. 4. \$2462.35. 5. \$930.75. 6. \$211.44. 7. \$715.23. 8. \$1668. 9. \$246.80. 10. \$223.25.

**Page 144.** 1. \$155.25. 2. \$156. 3. \$159.23. 4. \$517.42. 5. \$798.88. 6. \$848.74.

**Page 146.** 1. $77.29. 2. $26.50. 3. $371. 4. $386.25. 5. $825. 6. $78.75. 7. $256.25. 8. $132.50. 9. $254.58. 10. $353.94. 11. $286.

**Page 147.** 1. $135. 2. $210. 3. $807.50. 4. $1567.50. 5. $1680. 6. $2025. 7. $3400. 8. $450. 9. $300. 10. $225. 11. $150. 12. $930. 13. $198. 14. $315. 15. $56.25. 16. $22.50. 17. $112.50. 18. $312.50.

**Page 148.** 1. $12,375. 2. $85,968.75. 3. $1125. 4. $7562.50. 5. $750.

**Page 150.** 1. $0.38; $74.62. 2. $0.45; $89.55. 3. $3.13; $246.87. 4. $3.44; $271.56. 5. $7.08; $417.92. 6. $4.50; $295.50. 7. $11.25; $363.75. 8. $6; $394. 9. $1.50; $148.50. 10. $1.83; $398.17. 11. $2; $198. 12. $1.31; $173.69. 13. $5.50; $294.50. 14. $2.92; $497.08. 15. $24.79; $2950.21. 16. $63.75; $4186.25. 17. $6.25; $1243.75. 18. $25; $2475. 19. $22.50; $1477.50. 20. $15.63; $1234.37. 21. $26.25; $1723.75. 22. $10.31; $2464.69. 23. $4.38; $870.62. 24. $12.19; $962.81. 25. $2.13; $422.87. 26. $21.40; $2546.60. 27. $12.50; $1237.50. 28. $18.75; $1481.25. 29. $32.06; $2532.94. 30. $44.69; $3205.31. 31. $15.94; $4234.06. 32. $1.63; $323.87. 33. $3.56; $423.94. 34. $0.99; $236.51. 35. $743.75. 36. $671.62. 37. $1231.25. 38. $1500. 39. $22.50; $1477.50; $1485. 40. $700; $696.50.

**Page 152.** 1. $2.50. 2. $4. 3. $9. 4. $3.75. 5. $8.75. 6. $13.88. 7. $1.75. 8. $4.75. 9. $7.88. 10. $11.63. 11. $6.25. 12. $37.50. 13. $112.50. 14. $57.29. 15. $160.42. 16. $309.38. 17. $145.83. 18. $312.50. 19. $208.33. 20. $347.22. 21. $0.75; $300.25. 22. $6.86; $799.81. 23. $2.78; $552.72. 24. $1.39; $331.74. 25. $5.03; $999.97. 26. $21.58; $1252.03. 27. $31.25; $3718.75. 28. $35.21; $4189.79. 29. $95; $9405. 30. $62.50; $7437.50. 31. $52.81; $4172.19. 32. $83.63; $5491.37. 33. $31.25; $7468.75. 34. $23.75; $4726.25. 35. $0.21; $42.29. 36. $0.14; $27.46. 37. $0.18; $35.32. 38. $0.14; $28.61. 39. $1.26; $124.24. 40. $2.75; $272.50. 41. $5.63; $369.87. 42. $6.76; $443.99. 43. $1.91; $455.59. 44. $1.24; $295.26. 45. $3.60; $284. 46. $4.69; $370.71. 47. $31.97; $2718.03. 48. $38.07; $3236.93. 49. $45.82; $4714.18. 50. $26.42; $2718.58. 51. $1250; $6500. 52. $1747.07.

**Page 155.** 1. $749.25. 2. $549.45. 3. $149.70. 4. $\frac{4}{15}\%$. 5. $\frac{1}{5}\%$. 6. $74.85. 7. $\frac{25}{361}\%$. 8. $249.75. 9. $99.90. 10. $0.75. 11. $971.10. 12. $348.42. 13. $718.35. 14. $1214.72. 15. $1656.42. 16. $615.13. 17. $1480.92. 18. $2059.20.

**Page 160.** 1. $300.60. 2. $3203.20; $2501.25. 3. $37.65; $63; $14.40; $86.80; $75.70. 4. $4001.60. 5. 0.3%. 6. $2750; $2.75. 7. $3750. 8. $250.25; $150.15; $100.10; $350.35. 9. Draft; 10¢. 10. Registered letter; 2¢.

**Page 162.** 1. $3756.70. 2. $751.50. 3. $2452.45. 4. $3496.50. 5. $6753.38. 6. $17,465. 7. $5161. 8. $3408.50. 9. Discount; $\frac{1}{8}\%$. 10. Discount; $\frac{1}{5}\%$. 11. $249.75. 12. $2447.50. 13. $379.75. 14. $3240.

**Page 163.** 1. $609.49. 2. $813.68. 3. $1505.19. 4. $2343.20. 5. $146.25. 6. Proceeds of $750 note; $13.25.

**Page 167.** 1. $3.44; $1.28. 2. $0.58, loss. 3. $1.53, gain. 4. 40; 160; 40. 5. 12; 24; 12. 6. Phosphoric acid. 12; 12. 80; 80. 7. 12 nitrogen; 24 phosphoric acid; no potash. No nitrogen; 24 phosphoric acid; 12 potash.

**Page 171.** 1. $40. 2. $174.40. 3. $63.75. 4. $68.45. 5. $240.90. 6. $62.10. 7. 9 mills.

**Page 172.** 1. $35.75. 2. $48.13. 3. $26.81. 4. $42.21. 5. $66. 6. $90.75. 7. $134.20. 8. $129.25. 9. $171.05. 10. $127.88. 11. $103.81. 12. $248.88. 13. $36. 14. $52. 15. $64. 16. $36.80. 17. $56. 18. $36.40. 19. $70. 20. $78. 21. $54. 22. $68. 23. $96. 24. $200. 25. $260. 26. $308. 27. $362. 28. $606. 29. $65.28. 30. $273.60. 31. $153.75. 32. $15,000; $18,000. 33. $71.20. 34. $70.

**Page 173.** 1. 44.06%. 2. 27.44%; 24.39%; 21.34%. 3. $57,333,333.33.

**Page 175.** 1. $870; ad valorem. 2. $525; specific. 3. $440. 4. $756.50. 5. $585. 6. $11.25. 7. $250. 8. $68.75. 9. $97.50. 10. $237.50. 11. $76.70. 12. $78.89. 13. $1500. 14. $1443.75.

**Page 176.** 1. $600. 2. $411.42. 3. $388.92. 4. $299.47. 5. $491.79. 6. $518.34. 7. $1142.95. 8. $759.26. 9. $6937.47. 10. $55,165.28.

**Page 177.** 1. $14,168. 2. $820.05. 3. $336.10. 4. $487.50. 5. $175. 6. $52.36.

**Page 180.** 1. $23.75. 2. $30.80. 3. $43.13. 4. $88. 5. $75. 6. $3500. 7. $2052. 8. $202.13. 9. $1.53; $0.51. 10. $1.20; $0.40. 11. $10,000. 12. $10,500; $16,800. 13. $6050. 14. $31.25; $1500.

**Page 183.** 1. $410.40; $4104. 2. $2739. 3. $171; $3420. 4. $243.75; $4875. 5. $66; $82.25. 6. $263.60; $6590. 7. $4076; $1108.80. 8. $2739; $4108.50.

**Page 184.** 1. $1.50. 2. $50. 3. $4.50. 4. $70; savings bank. 5. Yes; $245. 6. $1125; $450; the one at 6%. 7. $97.50. 8. $20. 9. $23.75. 10. $23. 11. $29. 12. $39. 13. $52.50. 14. $50.88. 15. $39.38. 16. $50. 17. $63. 18. $312.50. 19. $375. 20. $525. 21. $493.50. 22. $195. 23. $337.50. 24. $595. 25. $175. 26. Same. 27. Same. 28. $125; $300; the investment at 5%.

**Page 186.** 1. 7%. 2. $599.50. 3. $242. 4. $5\frac{1}{18}$%. 5. $622.60. 6. $315; $6300. 7. $560.

**Page 188.** 1. 77¢. 2. $1.90. 3. $1.50. 4. $2.25. 5. $9.50. 6. $557.

**Page 192.** 1. $1,250,000. 2. $500. 3. $270. 4. $135. 5. $2,750,000. 6. 5%.

**Page 196.** 1. $5000; $2500; $1250. 2. 75; 300. 3. $7218.75; $7706.25; $5146.88; $7500. 4. 4%. 5. 5% stock @ $139\frac{7}{8}$. 6. Same. 7. $3\frac{1}{2}$%. 8. $3\frac{49}{117}$%. 9. $81.25, loss. 10. $62.50, gain. 11. $118.75, loss. 12. $250, gain. 13. $137.50, loss. 14. No gain or loss. 15. $5\frac{1}{2}$% mortgage. 16. 7% stock @ $149\frac{7}{8}$.

**Page 199.** 1. LXXV; LXXXVI; XCIII; XXXIX. 2. 200.005; 0.205. 3. 2139.7; 1345.9. 4. 5152.17; 3214.67. 5. 4677.97; 3278.85. 6. 9363.63; 8640.51. 7. 248.832; 120. 8. 2088.603; 80.3. 9. 2949.7804; 76. 10. 32,649.202; 8.2. 11. 238.45095; 9.5. 12. 5182.9623; 5.03. 13. $6\frac{7}{30}$; $1\frac{7}{30}$. 14. $8\frac{3}{40}$; $1\frac{27}{40}$. 15. $10\frac{9}{16}$; $4\frac{11}{16}$. 16. $12\frac{11}{16}$; $5\frac{7}{16}$. 17. $10\frac{5}{32}$; $5\frac{9}{32}$. 18. $10\frac{7}{40}$; $3\frac{17}{40}$. 19. $\frac{8}{15}$; $\frac{5}{8}$. 20. $\frac{15}{32}$; $\frac{5}{8}$. 21. $\frac{5}{12}$; $\frac{15}{16}$. 22. $\frac{7}{160}$; $1\frac{3}{32}$. 23. $\frac{21}{64}$; $\frac{7}{12}$. 24. $\frac{7}{24}$; $\frac{21}{32}$. 25. $\frac{3}{5}$; $1\frac{7}{20}$. 26. $\frac{6}{25}$; $\frac{3}{8}$. 27. $\frac{3}{40}$; $\frac{15}{32}$. 28. 75 ft. 4 in.; 18 ft. 10 in. 29. 96 lb. 12 oz.; 24 lb. 3 oz. 30. 138 ft. 9 in.; 15 ft. 5 in.

**Page 201.** 1. \$255; \$60; \$22.50; \$420; \$127.50; \$90; \$225; \$225. 2. \$15; \$7.25; \$4.90; \$13.15; \$7.50; \$46.40; \$23.65. \$33.80; \$111.75, amount saved. 3. 25% and 75%, approximately. 4. \$26.50. 5. 20%; more; $40\frac{10}{21}$%; greater.

**Page 202.** 1. \$87.60. 2. \$106.80. 3. \$133. 4. \$97. 5. \$16,250. 6. \$9.76.

**Page 204.** 1. 25. 2. 18. 3. 22. 4. 27. 5. 24. 6. 45. 7. 36. 8. 33. 9. 8.1. 10. 3.5. 11. 6.4. 12. 121. 13. 2.2 in. 14. 1.4 ft. 15. 1.5 ft. 16. 2.1 in. 17. 66 in. 18. 125 in. 19. 77 yd. 20. 3.3 yd. 21. 0.35 ft. 22. 324 ft. 23. 440 in. 24. 420 ft. 25. 560 ft. 26. 48.4 in. 27. 576 in. 28. 512 in. 29. 520 ft. 30. 1440 in. 31. 32 rd.; 96 rd. 32. 52 rd.; 104 rd.; 2. 33. 8 rd.; 5 rd. 34. 68 rd.; 204 rd.; 3.

**Page 206.** 1. 57. 2. 41. 3. 61. 4. 35. 5. 63. 6. 53. 7. 71. 8. 59. 9. 82 ft. 10. 87 ft. 11. 95 ft. 12. 97 ft. 13. 89 rd. 14. 83 yd. 15. $\frac{12}{23}$. 16. $\frac{55}{57}$. 17. $\frac{31}{32}$. 18. $\frac{19}{91}$. 19. $\frac{33}{34}$. 20. $\frac{23}{69}$. 21. $\frac{44}{71}$. 22. $\frac{17}{97}$.

**Page 208.** 1. 111. 2. 234. 3. 10.5. 4. 89.7. 5. 4.41. 6. 0.53. 7. 34.3. 8. 9.07. 9. 251. 10. 4607. 11. 7008. 12. 9812. 13. 1.41. 14. 2.24. 15. 2.65. 16. 2.83. 17. 3.32. 18. 12.65.

**Page 209.** 1. 85 ft. 2. 2.83 ft. 3. 36.06 in. 4. 95 in. 5. 125 ft. 6. 535 in.; 85 in.; 153 ft.; $76\frac{1}{2}$ rd.; $42\frac{1}{2}$ yd. 7. 22 ft. 6 in. 8. 16 ft. 6 in. 9. 43.60 ft. 10. 88.24 ft. 11. 16.40 rd. 12. 76.20 ft. 13. 8.49 in. 14. 9.76 in. 15. 9.29 in. 16. 41.05 in. 17. 4.34 in. 18. 21.93 ft. 19. 3.46 in. 20. 50 ft. 21. 7.5 mi. 22. 8.94 mi. 23. 166. 24. 146. 25. 44 in. 26. 249. 27. 4299.2. 28. 10.61 ft. 29. 17.68 ft. 30. 1.87 ft. 31. 36 ft. 32. 20.62 ft. 33. 34.99 ft.

**Page 213.** 1. 214.34 in. 2. 151.8 ft. 3. $1329\frac{3}{7}$ in. 4. 16.06 ft. 5. 169.4 ft. 6. 20.02 in. 7. 14.74 ft. 8. 182.6 in. 9. 18.92 ft. 10. $42\frac{3}{7}$ ft. 11. $55\frac{11}{21}$ ft. 12. 202.4 ft. 13. 42 ft. 14. $18\frac{6}{11}$ ft. 15. 31 ft. 16. 56 in. 17. 245 ft. 18. $30\frac{4}{5}$ ft. 19. 21.7 in. 20. 1.26 ft. 21. 0.154 ft. 22. 63 in. 23. 4.9 in. 24. 0.035 mi. 25. 53.4072 in. 26. 40.8408 in. 27. 8.79648 in. 28. 13.728792 in. 29. 13.19472 in. 30. 10.9956 ft. 31. 7.0686 ft. 32. 19.3732 ft. 33. 17 in. 34. 27 in. 35. 31 in. 36. 30 ft. 37. 58.003 ft. 38. 44 ft. 39. 308 in. 40. 484 in. 41. 572 in. 42. 52.8 in. 43. 61.6 in. 44. 660 in. 45. 96.8 in. 46. 11 in. 47. 23.8 ft. 48. 6.6 in. 49. 1.98 in. 50. $14\frac{1}{2}$ in.

**Page 215.** 1. $3\frac{1}{7}$ sq. in. 2. 1386. 3. 1386. 4. 35 ft. 5. 39.82 in.; 126.13 sq. in. 6. 7.7 in.; 3.85 in.; 46.585 sq. in. 7. 78.54 sq. in. 8. 22 ft.; 38.5 sq. ft. 9. $7\frac{1}{4}$ in.

**Page 217.** 1. 10,875 cu. in. 2. 325.92 cu. in. 3. 178.383 cu. ft. 4. 0.341 cu. ft. 5. 310.98 cu. in. 6. 199.076 cu. in. 7. $8\frac{1}{9}$ in. 8. 8 in. 9. 9 in. 10. $9\frac{9}{7}$ in. 11. 999.6 cu. in. 12. 70.5 cu. in. 13. 113.5 cu. ft. 14. 43.2 cu. ft. 15. 100.3 cu. ft. 16. 2301.3 cu. in. 17. 226,981 cu. in. 18. 78,608 cu. in. 19. 21 in.; 11 in. 20. 225 sq. in. 21. 15 in.

**Page 218.** 1. $50,285\frac{5}{7}$ cu. ft. 2. $46_{12}^{23}$. 3. $169,273\frac{23}{45}$. 4. $144\frac{3}{8}$ cu. ft. 5. 2079 cu. in.

**Page 220.** 1. 105. 2. 231 sq. in. 3. $62\frac{6}{7}$. 4. 5092. 6. $188\frac{4}{7}$. 7. $244\frac{3}{4}$. 8. 110 sq. ft. 9. 27,456 sq. ft. 10. $1178\frac{4}{7}$.

**Page 221.** 1. 160.332 cu. in. 2. 600.9404 cu. ft. 3. 3933.384 cu. in. 4. 667.038 cu. ft. 5. 21 sq. in. 6. 27 sq. in. 7. 21 sq. in. 8. 18 sq. in. 9. 93,537,284 cu. ft.; 15,714,263,712 lb.

**Page 223.** 1. 1919; 1921; 89. 2. 305,395. 3. 14,883,835. 4. $44,073.48. 5. $112.03. 6. $1\frac{3}{8}$. 7. $1\frac{7}{24}$. 8. $10\frac{1}{4}$. 9. $9\frac{1}{8}$. 10. $9\frac{1}{6}$. 11. $9\frac{1}{10}$. 12. $5\frac{29}{40}$. 13. $8\frac{5}{24}$. 14. $\frac{1}{8}$. 15. $1\frac{5}{8}$. 16. $3\frac{5}{8}$. 17. $6\frac{5}{6}$. 18. $\frac{7}{10}$. 19. $1\frac{7}{16}$. 20. $8\frac{3}{4}$. 21. $17\frac{5}{32}$. 22. $1\frac{3}{4}$. 23. 2. 24. 3. 25. 5. 26. $4314.19. 27. 5.45. 28. 1.27.

**Page 225.** 1. $3.75. 2. $7,500,000. 3. $7\frac{2}{5}\%$. 4. 40%. 5. $640. 6. $10,810,000. 7. $930,000,000. 8. $375,000,000. 9. $29,040,000. 10. $98,900,000. 11. $6330.

**Page 227.** 1. $3\frac{1}{2}$ in. 2. 15 in. 3. $6\frac{5}{12}$. 4. $2.08. 5. 116; $4.64. 6. 12.

**Page 229.** 1. $27.50. 2. Yes. 3. $70; $257.50; $233\frac{1}{3}\%$; 118.4%. 4. $233\frac{1}{3}\%$; 75%; yes. 5. $1.10. 6. $45; $12.20.

**Page 231.** 1. $0.21. 2. $1; 20%. 3. $9; $5\frac{5}{9}\%$. 4. Honey. 5. $0.05; $1.23; $0.13. 6. $7\frac{17}{19}\%$. 11. $2.32. 12. $2.88.

**Page 232.** 1. 1102.5. 2. $346.08. 3. 16,800. 4. Second; $0.96. 5. 36.

**Page 233.** 1. 12; 43,560. 2. 20. 3. 48 min. 24 sec. 4. 6,969,600. 5. 800.

**Page 234.** 1. 1760. 2. $34.60. 3. $3600; 700 T.; $5.14. 4. 157 acre-feet. 5. 192.

**Page 237.** 1. $1126\frac{1}{4}$ lb. 2. $0.48. 3. 730 lb. 4. 413,952. 5. $2\frac{3}{4}$ in. 6. 290,400.

**Page 238.** 1. $777\frac{2}{5}$. 2. $54,545_{11}^{5}$ T. 3. $6\frac{1}{19}\frac{54}{91}\%$; $10_{19}^{90}\%$. 4. 1340. 5. 20,000 ft. 6. $118\frac{3}{4}\%$.

**Page 239.** 1. $2227\frac{1}{2}$. 2. $1\frac{1}{99}\%$. 3. 25%; 20%. 4. 64,800. 5. 5 ft. $6\frac{2}{3}$ in. 6. $51,397\frac{1}{2}$. 7. 2420 lb. 8. 15; $3\frac{3}{4}$.

**Page 240.** 1. 22 in. 2. $23\frac{4}{7}$ ft. 3. $25\frac{1}{7}$ in. 4. $909_{32}^{7}$ lb. 5. $2127\frac{9}{28}$ sq. in. 6. $58,834\frac{2}{7}$ lb. 7. $62,857\frac{1}{7}$ in. 8. $1885\frac{5}{7}$ ft. 9. 1200. 10. 17. 11. 1000.

**Page 241.** 1. 5 ft. 2. 14.32 ft. 3. $6\frac{5}{12}$ ft. 4. $86\frac{3}{7}$ sq. ft. 5. $1571\frac{3}{7}$. 6. $1484\frac{28}{243}$.

**Page 242.** 1. 3 ft. $7\frac{3}{64}$ in. 2. $5\frac{61}{63}$ in. 3. $481\frac{1}{4}$ lb.

**Page 243.** 1. \$449.42. 2. \$58.08. 3. \$85,312.50. 4. \$4. 5. $3\frac{39}{87}\%$ greater profit on article costing \$8.70. 6. $62{,}304\frac{1}{16}$ lb. 7. \$4255.20. 8. \$4837.50.

**Page 244.** 1. \$118.75. 2. 60. 3. 67,260 lb. 4. \$1345.20. 5. \$35. 6. $2.88\frac{16}{37}$; \$213.44. 7. 16; \$2.10.

**Page 245.** 1. 5040; 4800. 2. \$31,600; \$2633.33$\frac{1}{3}$. 3. $58\frac{1}{3}\%$. 4. \$33,120. 5. \$394.40. 6. \$265.60. 7. \$0.86$\frac{1}{4}$. 8. \$13.80.

**Page 246.** 1. \$687.96. 2. \$905.66. 3. \$1076.40. 4. 384; 1536; 55,296; 221,184. 5. \$642.05; \$2568.19. 6. \$1981.60. 7. \$1591.20.

**Page 247.** 1. 0.4%. 2. 0.5%. 3. $3\frac{1}{8}$; $3\frac{1}{8}$; $243\frac{3}{4}$. 4. $28\frac{7}{20}$; $28\frac{7}{20}$. 5. 299.04. 6. \$6.50. 7. \$1.58; \$8.83; \$19.17.

**Page 248.** 1. \$1125. 2. \$2500. 3. \$0.20. 4. Neither. 5. \$46.31. 6. 103. 7. \$5180.70. 8. \$85.65.

**Page 249.** 1. \$6.78. 2. $5404\frac{4}{9}$. 3. \$41.67 loss. 4. The note, \$6.90 better. 5. $4\frac{3}{4}$. 6. \$18,200. 7. 6 ft. 8. \$40. 9. 4.49%. 10. $35\frac{5}{8}$ yd.

**Page 250.** 1. 7.07 ft. 2. 177.18 da. 3. $89\frac{1}{4}$ A. 4. 121.6 lb. 5. \$75,000; \$28,750.75. 6. 308 bu. 7. 3 hr. 9 min. 8. 5.196 cu. in. 9. \$81.75. 10. 60 ft.

**Page 252.** 1. 2 hr. 9 min. 2. 700 gal. 3. 27 min. 4. 300. 5. 360 ft. 6. 48,906. 7. 418,288.75 T. 8. $28\frac{4}{7}\%$. 9. 2.65. 10. 21.

**Page 253.** 1. $61\frac{7}{8}$ A. 2. \$15. 3. 468; 91; 78; 13. 4. 673.75; 17.5; 183.75. 5. $940\frac{1}{2}$. 6. 234. 7. $218\frac{2}{11}$. 8. $68\frac{4}{7}$. 9. 106 ft. 6 in. 10. 3; 7; 11.

**Page 254.** 1. \$420. 2. 52.95 ft. 3. No gain or loss. 4. 32 min. $48\frac{3}{4}$ sec. 5. 17,000. 6. $42\frac{1}{2}$ mi. 7. $51\frac{1}{5}$. 8. 4. 9. 17 ft. 9 in. 10. 8316.

**Page 256.** 1. 118,800,000. 2. 0.7 in. 3. $3764\frac{4}{9}$. 4. \$237.50. 5. 1960. 6. 1320. 7. 71.00016 in.; 71.02857 in.; 71 in. 8. 150 in.; 149.94 in.; 150.0003 in. 9. \$195.46. 10. 849.15; 238.65; 22.2.

**Page 258.** 1. 240; 1040. 80; 1200. 480; 960. 105; 525. 300; 420. 132; 156. 2; 3. 28.2; 84.6. 150; 350. 7; 13. 11; 44. 33; 187. 2. \$900; \$45. 3. 390. 4. 52 ft. 5. 650 ft.; 65 ft.; 90%. 6. \$1700. 7. 3600; 216. 8. \$1250. 9. \$3000. 10. \$2709.68.

**Page 260.** 1. \$300. 2. $\frac{1}{4}$; 0.25. 3. \$460; $\frac{23}{100}$. 4. \$360. 5. 0.25. 6. 0.28. 7. \$728. 8. 0.26. 9. \$552; \$624; \$240; \$1416. 10. \$44; \$528.

**Page 261.** 1. 23 in. 2. \$73,920. 3. \$72; \$81. 4. $2828\frac{4}{7}$. 5. $30\frac{2}{5}$. 6. \$233.10. 7. 234.59 in. 8. 5 hr. 9. \$4500; \$1200; \$7200. 10. 15 mi.

**Page 262.** 1. 96. 2. 7 hr. 3. \$1800; \$2100; \$900. 4. $157\frac{1}{2}$. 6. \$40. 7. 256 lb. 8. $7\frac{1}{3}\frac{2}{3}$. 9. 2464; 18,432. 10. 3963.296 mi.; 3949.791 mi.; 13.505 mi.

**Page 263.** 1. 77.98 lb. 2. $481\frac{1}{4}$ lb. 3. $589\frac{2}{7}$ lb. 4. $2062\frac{1}{2}$ sq. in. 5. $54{,}308\frac{4}{7}$ lb. 6. 1053 lb. 7. $938\frac{9}{10}\frac{1}{8}$ lb. 8. $144\frac{3}{8}$ cu. ft. 9. $1497\frac{2}{9}$ lb. 10. 72 ft.

**Page 264.** 1. $7\frac{4}{6}\frac{7}{4}$. 2. $1\frac{7}{2}\frac{2}{0}$; $\frac{2}{2}\frac{0}{7}$; 1. 3. 3. 4. 5.67 bbl.; 3.54 cu. yd. 5. 2. 6. 4. 7. $\frac{1}{8}$; $\frac{1}{2}$; $\frac{9}{1}\frac{}{6}$. 8. $7\frac{1}{5}$. 9. $1\frac{2}{3}\frac{3}{7}$. 10. 7.35 mi.

**Page 267.** 1. 160 A. 2. 160 A. 3. 80 A. 4. 80 A. 5. 80 A. 6. 80 A. 7. 40 A. 8. 40 A. 9. $10,400. 10. $3000. 11. 160 A. 12. 320 A. 13. 480 rd. 14. 2 mi. 15. 3; $14.25.

**Page 268.** 1. $365.25. 2. $467.52. 3. $331.16. 4. $80.36. 5. £8. 6. £10. 7. £5. 8. £9. 9. $17.85. 10. $22.85. 11. $29.75. 12. $892.50. 13. 40 M. 14. 50 M. 15. 60 M. 16. 300 M. 17. $16.41. 18. $48.25. 19. $44.39. 20. $144.75.

**Page 270.** 1. 1 hr. 48 min. 17 sec. 2. 5 hr. 2 min. 2 sec. 3. 11 hr. 8 min. 10 sec. A.M. 4. 105°; 9 P.M. 5. 4 A.M. 6. 8 hr. 22 min. 42 sec. A.M.; 8 hr. 2 min. 42 sec. A.M.

**Page 271.** 1. 30° 47′ 30″. 2. 105° W. 3. 90° W. 4. 2° 5′ 40.5″. 5. 22° 45′ W.

**Page 274.** 1. $17.50. 2. $10.13; $7.13. 3. $7.50; $742.50. 4. $3.25; $646.75. 5. $36; $2364. 6. $18. 7. $19.05. 8. $7.63. 9. $38.63. 10. $54.38. 11. $21.25. 12. $7.50. 13. $9.06. 14. $7.08. 15. $7.33. 16. $22.31. 17. $50. 18. $35. 19. $46.88. 20. $2.78. 21. $7.81.

**Page 275.** 1. 9. 2. 11. 3. 8. 4. 12. 5. 14. 6. 25. 7. 16. 8. 18. 9. 2.4 in. 10. 22 in.

**Page 278.** 1. 5. 2. 7. 3. 8. 4. 9. 5. 2.646. 6. 3.606. 7. 5.385. 8. 6.856. 9. 8.185. 10. 9.539. 11. 2. 12. 3. 13. 4. 14. 2.351. 15. 2.571. 16. 2.668. 17. 3.037. 18. 3.583. 19. 3.708. 20. 4.626. 21. 15. 22. 23. 23. 31. 24. 46. 25. 72. 26. 83. 27. 88. 28. 93. 29. 95. 30. 98. 31. 18. 32. 21. 33. 27. 34. 35. 35. 44. 36. 46. 37. 81. 38. 93. 39. 15. 40. 19. 41. 22. 42. 29. 43. 38. 44. 41. 45. 62. 46. 69. 47. 82. 48. 93. 49. 14. 50. 28. 51. 42. 52. 87. 53. 13 in. 54. 3.39 ft. 55. 688.90 cu. in. 56. 14.69 cu. in.

**Page 280.** 1. 192 sq. in. 2. $229\frac{1}{2}$ sq. in. 3. $2\frac{3}{8}$ sq. ft. 4. $37\frac{1}{3}\frac{3}{2}$ sq. in. 5. 15 sq. ft. 6. $2\frac{3}{1}\frac{}{6}$ sq. ft. 7. 1490.72 cu. in. 8. 623.7 cu. in. 9. 33,264 cu. in. 10. 764,000 sq. ft. 11. 960 sq. ft. 12. $183\frac{1}{3}$ sq. ft.

**Page 282.** 1. $63\frac{9}{1}\frac{}{4}$ sq. in. 2. 2,359,727,071,428$\frac{4}{7}$ sq. mi. 3. $17\frac{1}{2}\frac{6}{4}\frac{3}{4}$ sq. in. 4. 2828$\frac{4}{7}$ sq. in. 5. $86\frac{3}{7}$ lb. 6. 19.404. 7. $1230\frac{1}{2}\frac{5}{4}\frac{}{2}$ oz. 8. 268,190,476,000 cu. mi. 9. 50 in. 10. 2040 sq. ft. 11. $115\frac{1}{2}$ sq. ft. 12. $2416\frac{3}{6}\frac{2}{3}$ lb. 13. 1,457,777$\frac{7}{9}$. 14. $\frac{2}{2}\frac{}{7}$ in.

**Page 284.** **1.** 1850.39 in. **2.** 1358.265 in. **3.** 2047.24 in. **4.** 2657.475 in. **5.** 127.5588 in. **6.** 1.114171 in. **7.** 184.2516 in. **8.** 118.11 in. **9.** 2.7559 in. **10.** 666.01 ft. **11.** 123.03 ft. **12.** 51.181 in. **13.** 125.4 mi. **14.** 106.8 mi. **15.** 61.20 fr., or 61 fr. 20 c. **16.** 27.94 mi. per hr.

**Page 285.** **1.** 7,500,000,000,000 cm.$^2$ **2.** 370,000 cm.$^2$ **3.** 42.96 cm.$^2$ **4.** 62,500 cm.$^2$ **5.** 19,750 m.$^3$ **6.** 0.00042765384 m.$^3$ **7.** 7 m.$^3$ **8.** 3.5 m.$^3$ **9.** 0.000937 m.$^3$ **10.** 0.486 m.$^3$

**Page 286.** **1.** 170 kg. **2.** $194\frac{6}{11}$ kg. **3.** $92\frac{8}{11}$ kg. **4.** 10 kg. **5.** 22 kg. **6.** $272{,}727\frac{3}{11}$ kg. **7.** 3.275 kg. **8.** 7.275 kg. **9.** 79 kg. **10.** 7.386 kg. **11.** 4.984 kg. **12.** 6.249 kg. **13.** 300 l. **14.** 750 l. **15.** 4 l. **16.** 1700 l. **17.** 225 l. **18.** 50 l. **19.** 5 l. **20.** 2.5 l. **21.** 2000 l. **22.** 1500 l. **23.** 72,400 l. **24.** 683 l.

**Page 288.** **1.** $3\frac{1}{3}\%$. **2.** $13\frac{1}{3}\%$. **3.** 5%. **4.** 20%. **5.** $33\frac{1}{3}\%$. **6.** $16\frac{2}{3}\%$. **7.** 58%. **8.** 48%; 52%. **9.** 253 ft. **10.** 20. **11.** 48.

8 1 6
3 5 7
4 9 2